Seventh International Conference on
Geometry, Integrability and Quantization
June 2–10, 2005, Varna, Bulgaria
Iva"ılo M. Mladenov and Manuel de León, Editors
SOFTEX, Sofia 2006, pp 154–175

NEW INTEGRABLE MULTI-COMPONENT NLS TYPE EQUATIONS ON SYMMETRIC SPACES: $\mathbb{Z}_4$ AND $\mathbb{Z}_6$ REDUCTIONS

GEORGI G. GRAHOVSKI, VLADIMIR S. GERDJIKOV
NIKOLAY A. KOSTOV[†] and VICTOR A. ATANASOV

*Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences
72 Tsarigradsko chaussée, 1784 Sofia, Bulgaria*

[†]*Institute of Electronics, Bulgarian Academy of Sciences
72 Tsarigradsko chaussee, 1784 Sofia, Bulgaria*

Abstract. The reductions of the multi-component nonlinear Schrödinger models related to **C.I** and **D.III** type symmetric spaces are studied. We pay special attention to the MNLS related to the $\mathfrak{sp}(4)$, $\mathfrak{so}(10)$ and $\mathfrak{so}(12)$ Lie algebras. The MNLS related to $\mathfrak{sp}(4)$ is a three-component MNLS which finds applications to Bose–Einstein condensates. The MNLS related to $\mathfrak{so}(12)$ and $\mathfrak{so}(10)$ Lie algebras after convenient $\mathbb{Z}_6$ or $\mathbb{Z}_4$ reductions reduce to three and four-component MNLS showing new types of $\chi^{(3)}$-interactions that are integrable. We briefly explain how these new types of MNLS can be integrated by the inverse scattering method. The spectral properties of the Lax operators L and the corresponding recursion operator Λ are outlined. Applications to spinor model of Bose–Einstein condensates are discussed.

1. Introduction

When spinor **Bose–Einstein condensates** (BEC's) are trapped in magnetic potential, the spin degree of freedom is frozen. However, in the condensate trapped by an optical potential, the spin is free. We consider BEC's of alcali atoms in the $F = 1$ hyperfine state, elongated in x direction and confined in the transverse directions y, z by purely optical means. Then, in the absence of external magnetic fields is characterized by the magnetic quantum number which has three allowed values $m_F = 1, 0, -1$.Thus the assembly of atoms in the $F = 1$ hyperfine state can be described by a normalized spinor wave function $\mathbf{\Phi}(x,t) = (\Phi_1(x,t), \Phi_0(x,t), \Phi_{-1}(x,t))^T$ whose components are labelled by the values of m_F. In short the dynamics of