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SOME REMARKS ON THE EXPONENTIAL MAP ON THE GROUPS $\mathrm{SO}(n)$ AND $\mathrm{SE}(n)$

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Abstract. The problem of describing or determining the image of the exponential map $\exp : \mathfrak{g} \to G$ of a Lie group G is important and it has many applications. If the group G is compact, then it is well-known that the exponential map is surjective, hence the exponential image is G. In this case the problem is reduced to the computation of the exponential and the formulas strongly depend on the group G. In this paper we discuss the generalization of Rodrigues formulas for computing the exponential map of the special orthogonal group $\mathrm{SO}(n)$, which is compact, and of the special Euclidean group $\mathrm{SE}(n)$, which is not compact but its exponential map is surjective, in the case $n \geq 4$.

1. Introduction. Lie Groups and the Exponential Map

Let G be a Lie group with its Lie algebra $\mathfrak{g}$. The exponential map $\exp : \mathfrak{g} \to G$ is defined by $\exp(X) = \gamma_X(1)$, where $X \in \mathfrak{g}$ and γ_X is the one-parameter subgroup of G induced by X. Recall the following general properties of the exponential map:

1. For every $t \in \mathbb{R}$ and for every $X \in \mathfrak{g}$, we have $\exp(tX) = \gamma_X(t)$
2. For every $s, t \in \mathbb{R}$ and for every $X \in \mathfrak{g}$, we have
$$\exp(sX)\exp(tX) = \exp(s+t)X$$
3. For every $t \in \mathbb{R}$ and for every $X \in \mathfrak{g}$, we have $\exp(-tX) = \exp(tX)^{-1}$
4. $\exp\colon \mathfrak{g} \to G$ is a smooth mapping, it is a local diffeomorphism at $0 \in \mathfrak{g}$ and $\exp(0) = e$, where e is the unity element of the group G
5. The image $\exp(\mathfrak{g})$ of the exponential map generates the connected component G_e of the unity $e \in G$