

HOW THE CALCULATOR-ASSISTED INSTRUCTION ENHANCES TWO FIFTH GRADE STUDENTS' LEARNING NUMBER SENSE IN TAIWAN

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This paper reviews a calculator-assisted instruction programme devised for two fifth grade pupils, who were low achievers in elementary arithmetic, for learning number sense. The programme was designed in three phases: (1)pre-test for examining what kinds of number sense the subjects were lack of; (2)instruction activities to guide them to develop the relative number sense; (3)post-test for confirming if they have developed the relative number sense. The results reveal that appropriate calculator-assisted instruction could enhance low achievers in arithmetic learning number sense. By providing a non-counting dependent procedure, pupils can concentrate on thinking the problem without too much cognitive load under the teacher's guidance.

INTRODUCTION

Numbers and operations are one of the main topics in school mathematics. When teaching this topic, teachers should not only introduce numerical conceptions and computational algorithms, but also teach relationships, estimations, awareness of what numbers are,, etc. These relative conceptions are all about what should be called "number sense". Many research reports emphasise the importance of teaching and learning number sense in school (e.g. Markovits & Sowder, 1994; McIntosh, Reys, Reys, Bana & Farrel, 1997). National Council of Teachers of Mathematics [NCTM] even indicates directly that "central to the Number and Operations Standard is the development of number sense" (NCTM, 2000, p.32). Number sense refers to an intuitive reasoning for numbers and their various uses and interpretations, as well as an appreciation for various level of accuracy when one computes (Reys, 1994). In general, number sense reflects a person's general understanding of numbers and operations including the ability and inclination to use the understanding in a flexible way to make mathematical judgments and develop appropriate strategies (e.g. mental computation and estimation) for manipulating numbers and operations (Howden, 1989; McIntosh, Reys, & Reys, 1992; Reys, 1994; Reys & Yang, 1998; Sowder, 1992; Treffers, 1991). Students with number sense could develop a holistic perspective of numbers, who "naturally decompose numbers, use particular numbers as referents, solve problems using the relationships among operations and knowledge about the base-ten system, estimate a reasonable result for a problem, and have a disposition to make sense of numbers, problems, and results" (NCTM, 2000, p.32).

The TIMSS relevant technical reports reveal that Taiwanese student mathematics achievements are highly ranked (Mullis, Martin, Gonzalez, & Chrostowski, 2003). However, having good computation abilities do not ensure having good number sense (Reys and Yang, 1998), that seems to be an echo of Mack's study (1990), who

suggests that relying on computation rules too much affects pupil's thinking. For instance, Chih (1996) finds that when sixteen fourth grade Taiwanese students were asked to consider which one is wrong of the following questions: $47 \times 9 = 423$, $98 \times 16 = 948$, $38 \times 12 = 456$, only four out of the sixteen could get the correct answer directly while the other twelve were busy in computing with paper and pencil.

Gray and Pitta (1997) conduct a study about helping a low achiever in elementary arithmetic change the quality of imagery associated with numerical symbols by using a calculator. Gray and Pitta's study adds a further dimension to notions that the use of calculators not only does not affect student computational ability but supports their concept development (e.g., Goldenberg, 1991; Huinker, 1992, 2002; Shuard, Walsh, Goodwin, and Worcester, 1991; Shumway, 1990; ..., etc.). Further studies of the literature about calculator-assisted instruction also show that appropriate calculator-based programmes can help students develop problem solving ability (e.g., Dunham & Dick, 1994; Frick, 1989; Keller & Russell, 1997; ..., etc.) and enhance their learning motivation (e.g., Hembree & Dessart, 1992; ..., etc.). However, using calculator in the classroom is not popular in Taiwan. Two decades ago, students were prohibited from using calculator when learning mathematics (except statistics). Even in the newly published Grade 1-9 Curriculum, at the elementary level, the use of calculator is only for checking the correctness of the student's answer and dealing with the basic computation with large numbers.

Based on the above literature review, the study presented in this paper was designed to investigate how the calculator-assisted instruction helps two fifth grade students, who were low achievers in arithmetic, develop number sense.

RESEARCH DESIGN AND METHODOLOGY

In general, the whole research design includes picking two low achievers to be our subjects; developing a questionnaire with four questions accompanied with corresponding activity design; using the method of qualitative semi-structural interview to collect data while the two subjects meet pre-test, instruction activities, and post-test; as well as analysing and interpreting the collected data.

During the procedure of finding the two subjects, we firstly tried to find a case class of grade five whose teacher has strong will to collaborate in the research. Then we defined the nine low achievers by classifying those whose last four years mathematics marks were the last 20% from the whole class. We consulted the teacher about these nine pupils' computational abilities and interviewed them all. Finally we picked Howard and Jane who could not do arithmetic well but had good oral expression abilities which should be beneficial to our data collection (through qualitative semi-structural interviews).

Central to the whole research is the design of the questionnaire with corresponding calculator-assisted instruction activities. There were four main ways we would like to help Howard and Jane to learn in the study: (1) using a specific number as a referent to make estimation; (2) discovering the relation between division and numbers; (3) recognising number patterns; (4) discovering the relation between multiplication

and numbers. Based on the above four points, four groups of questions were designed in the questionnaire for examining if Howard and Jane have the abilities. Some examples, corresponding to the four points respectively, are as follows:

(1) Before having the mid-term examination, Andrew's parents set a standard of award for encouraging him to study harder. The standard is: If the total marks of the six subjects are above 450, Andrew can win a present from Mum; if above 550, an extra present will be given from Dad. What is the lowest average mark Andrew has to get to win the extra present? What is the highest average mark Andrew gets when he wins nothing? What are the average marks Andrew can only win one present?

(2) What are the relations between the numerator and the denominator when the quotient is greater than, equal to, and less than 1?

(3) Using calculator to find (a) 15×2 , 15×4 , 15×8 ; (b) 24×5 , 24×15 , 24×25 ; (c) 555×12 , 555×36 , 555×72 .

(4) Which of the following is the biggest: 15×0.699 , 2×0.699 , 18×0.699 , 16×0.699 ?

Consulting Wheatley and Clements (1990) and Waits and Demana (1998), four corresponding activities were also designed (Table 1). During the development of the research implements (questionnaire and instruction activity design), another mathematics educator and two senior mathematics teachers (with more than ten years teaching experiences) were consulted for the affirmation of the content validity.

Table 1. Instruction activity design

Activity	Content abstract	Number sense to develop
Range game	Finding the multiples of a given number that will fall in a given range.	Using a specific number as a referent to make estimation
Number guessing	Interviewer A picks a number x in his mind, the interviewee B guesses the number as y and keys it in in the calculator, then A shows B the answer of $y \div x$. Duplicating the procedures by modifying the guessing number y , until B finds the number x .	Discovering the relation between division and numbers
Observing the pattern	Discovering the regularity of the products of a series of two numbers.	Decomposing and integrating numbers, developing multiple and flexible strategies
Multiplying your expectation	Estimating the products of two numbers which are greater or less than 1.	Discovering the relation between multiplication and numbers

The whole study was conducted through one-to-one interviews during the lunch time (about an hour) of four days every week for ten weeks, while one week for pre-test, eight weeks for instruction, and one week for post-tests. In the pre-test, Howard and Jane were asked to answer the questionnaire under prohibition of using calculator without the time limit. The purpose of the pre-test is to find out what kinds of number sense they were lack of, thus the interviewer might interrupt the subject's answering the questionnaire in order to understand what he/she was thinking about. After analysing their answers to the questionnaire, the first author started to conduct the

calculator-assisted instruction with Howard and Jane one by one personally for helping them develop the relevant number sense. In the instruction phase, the main teaching strategy is to guide the two subjects to probe. That is not only to find the answers, but also to think of the questions holistically. Finally, we used the same questionnaire given to the two subjects in the post-test to examine if the instruction activities we designed were effective. Of course, they were prohibited to use a calculator in the post-test. Probably there will be doubt that the pupils might be able to remember some of the questions when taking the post-test. However, we do not consider this is a possibility as the way the study was conducted should not motivate them to do so. In addition, they are only in year five and very naïve. Nevertheless, we would pay attention to whether it will happen in the post-test especially.

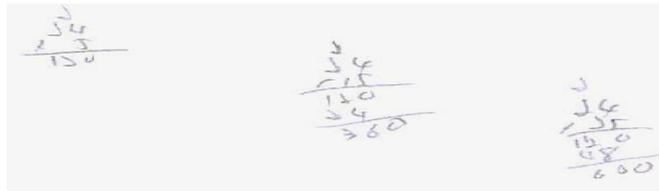
RESULTS

In the pre-test, Howard and Jane both passed the second question since they could directly give the answer that when the numerator and the denominator are the same, the quotient will be 1; when the numerator is greater than the denominator, the quotient will be greater than 1; when the numerator is less than the denominator, the quotient will be less than 1. However, they both failed the other three questions. It should be noticed that both Howard and Jane just immediately started to use paper-and-pencil computation to deal with the questions one by one without showing using other strategies when facing the three questions. The following is an episode of the interview with Howard when he was answering Question 3:

Interviewer: How do you solve the three sub-questions?

(1) $24 \times 5 =$ (2) $24 \times 15 =$ (3) $24 \times 25 =$

Howard wrote the answers by using paper-and-pencil computation immediately.



Interviewer: Jolly good, you make a correct answer. But do you have any other strategies?

Howard: (Thinking for about 30 seconds.) No, I don't.

Interviewer: Do you notice there are three sub-questions of this question?

Howard: Yes.

Interviewer: And how do you think about it?

Howard: What should I think?

Interviewer: You could see the three sub-questions as a whole.

Howard: (Keeping pondering for a while, and saying nothing.)

Interviewer: So can you think of any other strategies now?

Howard: No! (Keeping shaking his head.)

In the following instruction phase, we only focused on three activities which were corresponding to Questions 1, 3, and 4 in the questionnaire. Another episode of

Jane's interview, as an example, in the activity "Observing the patterns" is presented as follows. It can be noticed in the conversation that following the interviewer's guidance, Jane could observe the pattern of the series of questions (relation of multiples) and progressively applied it to answering the following questions. She even tried to use mental calculation although the answer she got was incorrect.

(Jane was asked to answer the following series of questions: 15×4 , 30×4 , 45×16 , 375×8 , 375×48 , 15×308 , 1395×16 , 150×24 , 120×16 , 495×36 .)

Interviewer: Well, you just use calculator to get all these answers. Perhaps mental calculation is more useful for some of the questions.

Jane: I'm not good at mental calculation.

Interviewer: O.K. Have a look at $15 \times 4 = 60$ and $30 \times 4 = 120$, can you notice any relation between the questions and their answers?

Jane: They are multiples!

Interviewer: What multiples? Can you explain it more clearly?

Jane: Just like "30 is double of 15"!

Interviewer: Right! How about the answer?

Jane: It's double as well! (Note: She means 120 is double of 60.)

Interviewer: If $15 \times 4 = 60$ is given, can you answer 30×4 without using paper-and-pencil calculation?

Jane: Let me think. (Pondering for about two minutes.)

Interviewer: You can try to think of the relation of multiples.

Jane: I got it! 120.

Interviewer: How does the answer come from?

Jane: Since 30 is double of 15, so is the answer. Therefore, two 60s make 120.

Interviewer: Great! Let's try one more question, O.K. If given $15 \times 4 = 60$, can you answer 15×308 ?

Jane: Do I need to apply the same strategy?

Interviewer: Yes, if you think it's useful.

Jane: O.K. Let me see. 308 is 77 times of 4 (by using calculator), so the answer of 15×308 must multiply 77.

Interviewer: Can you find out how much it is?

Jane: No problem! (Using the calculator to count 60×77) 4620.

Interviewer: O.K. Can you count 375×48 if given $375 \times 8 = 3000$?

Jane: (keying in the calculator $48 \div 8$) 6 times. So the answer of 375×48 must be 6 times of 3000, it's 1800 (Using mental calculation.).

Interviewer: How come the answer of 375×48 is less than $375 \times 8 (=3000)$.

Jane: (Pondering for a while and using the calculator to check the answer.) Aha! I see. There's a zero missing.

Howard's performance in the series of "range game" activity is also worth of mentioning. The activity was about finding the multiples of a given number (e.g. 20) which will fall in a given range (e.g., 250-430). At the beginning of the instruction activity, Howard was just trying to compute 20×1 , 20×2 , 20×3 , ..., etc. The interviewer posed some probing questions to guide him to think of a more efficient way. Thus Howard started to change his strategy by trying 20×10 as a referent number to make estimation. The following conversation was happened at the end of

the activity. It is obviously that Howard became quite skilful in finding a specific number as referent to make estimation.

(The question is “What are the multiples of 25 which will fall in the range 230-590?”.)

Howard: (Keying in 25×10 and getting 250, then murmuring ...) 10, 11, 12, let’s try 20 (Keying in 25×20). Yah! It works. So 21, 22, (Pondering for about ten seconds.) 22 is 550, so we could only have one more, 23. That’s all.

Interviewer: How do you know 23 is one of the numbers?

Howard: Because 25×20 is 500. Then add 50 is 550. So 21, 22 are the answer. Then I add 25 to 550 is 575. It also works. But add another 25 will be over 590. Therefore, I know 23 is the biggest number.

Interviewer: Brilliant! But how do you know 10 is the smallest number in the answer?

Howard: 25×10 is just adding a zero to 25. But $250 - 25$ is ... (Pondering a while then using mental computation.) 225. It’s less than 230. So 10 is the smallest one.

After the eight weeks instruction, Howard and Jane were given the same questionnaire without Question 2 the second time for the post-test. They both passed Questions 3 and 4, but unfortunately failed Question 1 which was beyond our calculation (p.s. further analysis will appear in the following section). That means both of them appeared to have developed the ability for decomposing and integrating numbers to use multiple and flexible strategies, as well as discovering the relation between multiplication and numbers after having the calculator assisted instruction. The following conversation presents an episode of Howard who was answering Question 3 in the post-test. Compared with his former performance in the pre-test (see the earlier quoted conversation), Howard could observe the pattern and use the relation to consider the whole series of questions this time.

Interviewer: Can you do this series of questions. ($15 \times 2 =$, $15 \times 4 =$, $15 \times 8 =$)

Howard: 30, 60, 120. (*Writing down the answers immediately.*)

Interviewer: Very good! Could you tell me how you can do that so quickly?

Howard: Just because 15 times 2 is 30, 4 is double of 2, thus 30 times 2 is 60. And next is the same, 60 times 2 is 120.

Table 2 shows the simple statistic of Howard and Jane’s performance in the pre- and post-tests and the conduction of calculator-assisted instruction activities.

Table 2. Howard and Jane’s performance and the conduction of instruction

	Pre-test				Instruction Activity				Post-test			
	Q1	Q2	Q3	Q4	A1	A2	A3	A4	Q1	Q2	Q3	Q4
Howard	×	○	×	×	✓	—	✓	✓	×	—	○	○
Jane	×	○	×	×	✓	—	✓	✓	×	—	○	○

DISCUSSION AND CONCLUSION

One of the results that both Howard and Jane failed Question 1 (corresponding to “range game”) in the post-test was beyond our expectations. It was clear to notice that they both had developed the ability to find a specific number as a referent to make estimation gradually in the “range game” activity. After our discussion which

also included two senior primary mathematics teachers, we considered that the problem seemed to be the contexts of the word problems of Question 1 in the questionnaire. For fifth grade pupils, word problems should be the most difficult question style. We also noticed that both Howard and Jane were struggling hard to mathematise the problem context into a mathematics problem, but unfortunately they were stuck in the procedure of mathematisation and unable to apply the ability of finding a referent to make estimation.

The research results suggest that the calculator-assisted instruction programme could provide the pupil, especially the low achiever in arithmetic, an alternative, non-counting dependent procedure to develop number sense. This kind of findings seems echo Gray and Tall's "procept theory" (Gray and Tall, 1994). We probably could extend the concept of "procept" in some way, which was originally applied to a symbol, say "+", and the process of "addition" and the concept of "sum". Number sense can be considered as a general procept, it contains the corresponding computation as the process and some specific number sense as the concept. Teachers who appreciate computational ability too much but neglect the development of the concept of number sense will make the pupils unable to construct the whole "procept" of number sense. In addition, pupils who are low achievers in arithmetic could also develop the concept of number sense through using a calculator. However, it should be stressed that computational ability is also important for developing the whole procept of number sense since "procept" contains "process" and "concept".

Although, based on the research results, all positive indications suggest that calculator-assisted instruction could enhance the two research subjects' learning number sense, the calculator is definitely not a panacea. The teacher plays a very crucial role in the calculator-assisted instruction activities. Including the well-designed activities, the teacher needs to foster the skill of posing probing questions to guide the student to concentrate on thinking the mathematics problem.

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