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ON SOME INTEGRATION METHODS FOR CONNECTIONS ON FIBERED MANIFOLDS

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Abstract. In the paper relations between (Ehresmann) connections on jet prolongations of fibered manifolds are studied. Two indirect integration methods for these connections are proposed, generalizing both the "method of characteristics" from the theory of exterior differential systems and the "method of fields of extremals" (Hamilton-Jacobi integration method) from the classical calculus of variations to (non-variational) first and second-order partial or ordinary differential equations, explicitly solved with respect to the highest derivatives.

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1. Introduction

The purpose of this paper is to propose some integration methods for differential equations on fibered manifolds. We do not intend to deal with differential equations in general - this is subject of such theories as e.g. the theory of exterior differential systems ([1], [3], [5] and references therein) or the theory of diffieties as submanifolds of ∞-order jet spaces endowed with the Cartan distribution (e.g. [10] and references therein); an interesting approach to the geometry of differential equations can be found also in [25]. We restrict our attention to equations explicitly solved with respect to the highest derivatives i.e. equations for integral sections of *Ehresmann connections* on fibered manifolds. Equivalently, these equations are represented by horizontal distributions (i.e. by Pfaffian systems of a special kind).

There are several possibilities how to search for solutions to such systems. First of all, *direct* integration methods should be mentioned. These lead to a direct computation of maximal integral manifolds of the given distribution by finding "complete sets of independent first integrals", i.e. charts adapted to the integral submanifolds. An interesting method of this kind using symmetries of the corresponding distribution, can be found in [7]. Other methods arise from the fact that any vector distribution on a smooth manifold can be (at least locally) related with the characteristic distribution of a (in general non-unique) closed 2-form. Hence, one can use symmetries of this 2-form to solve the integration problem. In particular, one can find only "a part" of symmetries of the related 2-form and apply the generalized Liouville or Jacobi theorem [16]. Another possibility to solve differential equations is to apply some of the *indirect* integration methods, transferring the given integration problem to that of solving certain related

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equations to the original ones. A classical example of this procedure is provided by the Hamilton-Jacobi theory of the calculus of variations, where a system of second-order ordinary differential equations, arising from a regular lagrangian, is solved with help of the Hamilton-Jacobi equation, which is a first-order partial differential equation.

In this paper we propose two indirect integration methods for connections. The results are based on relations between various kinds of connections on fibered manifolds. The first of the two methods, explained in Sec. 4, could be referred to as a "method of characteristics for connections". We introduce to a given connection the *characteristic connection*, uniquely determined by the original connection and possessing similar properties as do the characteristic systems of general exterior differential systems. Sec. 5 deals with an integration theory for 2-connections, which, in a sense, could be viewed as a generalization of the Hamilton-Jacobi integration method of classical mechanics to arbitrary systems of second-order ordinary or partial differential equations, explicitly solved with respect to second derivatives. Roughly speaking, the classical idea consists in transferring the integration problem for a system of variational equations (which are nothing but equations for integral sections of a 2-connection) to the problem of finding a family of the so-called *fields of extremals* which are integral sections of a horizontal distribution called a *complete integral* (see e.g. [2], [8], [14], [15], [20]). These constructions, however, are strongly based on the existence of a lagrangian and therefore cannot be used in the "non-variational" situation. Here we show that they can be generalized to general 2-connections (without any reference to the calculus of variations). We introduce the notion of a *field of paths*, which is a connection representing a "non-variational field theory parallel" to the field of extremals, and we prove a generalized version of the *Van Hove Theorem*. We also introduce a concept of an *integral* of a 2-connection, which again is a connection, now generalizing the notion of a complete integral of the calculus of variations (in the sense of [2] or [14], [15]). We show that locally the integration problem for a 2-connection is equivalent to the problem of looking for an integral of this 2-connection, which, from the point of view of applications, means a reduction of the order of the equations.

Notation and main concepts used throughout the paper are briefly recalled in Sect. 2 and 3. For more details we refer to e.g. [4], [9], [11], [12], [17], [18], [19], [21].

All manifolds and mappings are supposed to be smooth and the summation convention is used as far as possible.

2. PRELIMINARIES

Throughout the paper, $\pi\colon Y \to X$ denotes a fibered manifold, $\dim X = n$, $\dim Y = n + m$. When speaking of *coordinates*, we have a *fibered* chart (V, ψ) with *fibered* coordinates $\psi = (x^i, y^\sigma)$, $i = 1, \dots, n$, $\sigma = 1, \dots, m$, in mind.

The *first jet prolongation of* π will be denoted by $J^1\pi$ (or J^1Y if the manifold structure rather than the fibered one is considered). The first jet prolongation of a local section γ of π is denoted by $j^1\gamma$. Fibered coordinates on J^1Y associated with (x^i, y^σ) are denoted by $(x^i, y^\sigma, y^\sigma_i)$. Recall that the manifold $J^1\pi$ can be viewed as the total space of two different fibered manifolds; $\pi_1 : J^1Y \to X$, and $\pi_{1,0} : J^1Y \to Y$. The latter one is an affine bundle modelled on the associated vector bundle $V_\pi Y \otimes \pi^*(T^*X) \to Y$, where T^*X is the cotangent bundle of X, $V_\pi Y$ is the π-vertical subbundle of the tangent bundle TY and $\pi^*(T^*X)$ is, by definition, the fibered product $Y \times_X T^*X$.

A *connection* (*Ehresmann connection, jet field*) on a fibered manifold $\pi\colon Y \to X$ is a section (possibly defined on an open subset of Y) $\Gamma\colon Y \to J^1Y$ of $\pi_{1,0}$. In local coordinates, the equations of Γ are $y_i^\sigma = \Gamma_i^\sigma(x^j, y^\lambda)$, where the functions Γ_i^σ are called the *components* of Γ. A connection Γ on π can be identified (among others) with the *horizontal form* $h_\Gamma \colon Y \to TY \otimes \pi^*(T^*X)$, which is in an obvious sense a projector on TY, locally expressed by

$$h_\Gamma = \left(\frac{\partial}{\partial x^i} + \Gamma_i^\sigma \frac{\partial}{\partial y^\sigma}\right) \otimes dx^i,$$

and which generates the decomposition $TY = V_\pi Y \oplus H_\Gamma$, where the n-dimensional π-horizontal distribution $H_\Gamma = \operatorname{Im} h_\Gamma$ on Y is given by

$$H_\Gamma = \operatorname{span}\left\{\frac{\partial}{\partial x^i} + \Gamma_i^\sigma \frac{\partial}{\partial y^\sigma}, 1 \le i \le n\right\} = \operatorname{span}\left\{dy^\sigma - \Gamma_j^\sigma\, dx^j, 1 \le \sigma \le m\right\}.$$

A (local) section γ of π is called a *path* or an *integral section* of the connection Γ if $j^1\gamma = \Gamma \circ \gamma$. This locally means a system of partial differential equations for γ

$$y_i^\sigma \circ j^1\gamma = \Gamma_i^\sigma \circ \gamma, \qquad \text{i.e.} \qquad \frac{\partial \gamma^\sigma}{\partial x^i} = \Gamma_i^\sigma(x^j, \gamma^\lambda).$$

Hence, from the point of view of applications, connections on π are global counterparts of *differential equations* (ordinary or partial according to the dimension of X) *explicitly solved with respect to the derivatives.*

The *curvature* R_Γ of Γ is a vector-valued 2-form $R_\Gamma = \frac{1}{2}[h_\Gamma, h_\Gamma] \colon Y \to V_\pi Y \otimes \Lambda^2 T^*X$, where $[\ ,\ ]$ is the Frölicher-Nijenhuis bracket. Recall that a connection Γ is called *flat* if $R_\Gamma \equiv 0$.

A connection Γ is called *integrable* if the distribution H_Γ is completely integrable (satisfying the Frobenius theorem). Since the images of integral sections of Γ coincide with integral manifolds of H_Γ of maximal dimension, the integrability of Γ means that to each point $y \in Y$ there exists a unique maximal integral section of Γ passing through y. The following equivalent assertions are easily proved by a direct computation:

(1) Γ *is integrable.*
(2) Γ *is flat.*
(3) *The components of* Γ *satisfy the Frobenius integrability conditions*

$$\frac{\partial \Gamma_i^\sigma}{\partial x^j} + \frac{\partial \Gamma_i^\sigma}{\partial y^\lambda}\Gamma_j^\lambda = \frac{\partial \Gamma_j^\sigma}{\partial x^i} + \frac{\partial \Gamma_j^\sigma}{\partial y^\lambda}\Gamma_i^\lambda.$$

Another integrability condition will be discussed in Sec. 5.

Remark 2.1. Notice a relationship between the horizontal distribution H_Γ of Γ and the canonical *Cartan distribution* $C_{\pi_{1,0}}$ on J^1Y, or more precisely the Cartan distribution $C^\Gamma_{\pi_{1,0}}$ of the equation $\Gamma(Y) \subset J^1Y$, defined by $C^\Gamma_{\pi_{1,0}} = C_{\pi_{1,0}} \cap T\Gamma(Y)$. Since $C_{\pi_{1,0}}$ is spanned by the well-known contact forms $dy^\sigma - y_i^\sigma\, dx^i$ for $\sigma = 1, \ldots, m$, it holds

$$C^\Gamma_{\pi_{1,0}} = \operatorname{span}\left\{\frac{\partial}{\partial x^i} + \Gamma_i^\sigma \frac{\partial}{\partial y^\sigma} + \left(\frac{\partial \Gamma_j^\sigma}{\partial x^i} + \frac{\partial \Gamma_j^\sigma}{\partial y^\lambda}\Gamma_i^\lambda\right)\frac{\partial}{\partial y_j^\sigma}, 1 \le i \le n\right\}.$$

It can be easily shown that if Γ is an integrable connection on π, then the following diagram commutes for each Γ-horizontal vector field ζ on Y :

$$\begin{tikzcd}
J^1\pi \arrow[r, "\mathcal{J}^1\zeta"] & C^{\Gamma}_{\pi_{1,0}} \\
Y \arrow[u, "\Gamma"] \arrow[r, "\zeta"] & H_{\Gamma}, \arrow[u, "T\Gamma"']
\end{tikzcd}$$

where $\mathcal{J}^1\zeta$ is the first jet prolongation of ζ. This can be expressed in terms of integral sections by the assertion that *a section* γ *of* π *is an integral mapping of the horizontal distribution* H_Γ *if and only if* $j^1\gamma$ *is an integral mapping of the distribution* $C^{\Gamma}_{\pi_{1,0}}$.

3. CONNECTIONS ON J^1Y

Let $\pi\colon Y \to X$ be a fibered manifold, $J^1\pi$ its first jet prolongation. By $J^2\pi$ we denote the *second (holonomic) jet prolongation* of π (considered as a fibered manifold $\pi_2\colon J^2\pi \to X$), and by $J^1\pi_1$ the *second anholonomic jet prolongation* of π, which is, by definition, the first jet prolongation of the fibered manifold $\pi_1\colon J^1\pi \to X$ (*repeated jets*). Fibered coordinates on $J^2\pi$ and $J^1\pi_1$, associated with fibered coordinates (x^i, y^σ) on Y, will be denoted by $(x^i, y^\sigma, y^\sigma_i, y^\sigma_{ij})$ and $(x^i, y^\sigma, y^\sigma_i, Y^\sigma_i, Y^\sigma_{ij})$, respectively. Note that $y^\sigma_{ij} = y^\sigma_{ji}$ but, in general, $Y^\sigma_{ij} \neq Y^\sigma_{ji}$. The canonical projections of these spaces to $J^1\pi$ will be denoted by $\pi_{2,1}\colon J^2\pi \to J^1\pi$ and $(\pi_1)_{1,0}\colon J^1\pi_1 \to J^1\pi$. It is worthwhile to note that there is a canonical inclusion $i\colon J^2\pi \to J^1\pi_1$ over $J^1\pi$, defined by $i(j^2_x\gamma) = j^1_x(j^1\gamma)$, and locally expressed by $Y^\sigma_i \circ i = y^\sigma_i$, $Y^\sigma_{ij} \circ i = y^\sigma_{ij}$. Thus we can, for the sake of brevity, identify $J^2\pi$ with the embedded submanifold $i(J^2\pi) \subset J^1\pi_1$. Note that, however, there is no natural projection $J^1\pi_1 \to J^2\pi$.

The *second semiholonomic jet prolongation* of π will be denoted by $\widehat{J}^2\pi$. Equations of this submanifold of $J^1\pi_1$ are $y^\sigma_i = Y^\sigma_i$ and the corresponding canonical projection $\widehat{\pi}_{2,1}\colon \widehat{J}^2\pi \to J^1\pi$ is the restriction $\widehat{\pi}_{2,1} \equiv (\pi_1)_{1,0}\colon \widehat{J}^2\pi \to J^1\pi$.

According to the decomposition of the corresponding associated vector bundles, there exists a canonical bundle isomorphism

$$\widehat{J}^2\pi \cong J^2\pi \times_{J^1\pi} \left[\pi^*_{1,0}(V_\pi Y \otimes \pi^*(\Lambda^2 T^*X))\right]. \tag{3.1}$$

This splitting gives rise to affine projections $r\colon \widehat{J}^2\pi \to \pi^*_{1,0}(V_\pi Y \otimes \pi^*(\Lambda^2 T^*X))$ and $s\colon \widehat{J}^2\pi \to J^2\pi$ expressing the antisymmetric or symmetric part of every fibered coordinate Y^σ_{ij}, respectively.

In keeping with Sec. 2, a connection on π_1 is a section $\Sigma\colon J^1\pi \to J^1\pi_1$ of $(\pi_1)_{1,0}$. In fibered coordinates, $Y^\sigma_i \circ \Sigma = \Sigma^\sigma_i$, $Y^\sigma_{ij} \circ \Sigma = \Sigma^\sigma_{ij}$, where the components Σ^σ_{ij} are not in general symmetric in i, j. Equivalently, Σ is represented by the π_1-horizontal n-dimensional distribution H_Σ on J^1Y

$$\begin{aligned}
H_\Sigma &= \operatorname{span}\left\{\frac{\partial}{\partial x^i} + \Sigma^\sigma_i \frac{\partial}{\partial y^\sigma} + \Sigma^\sigma_{ij} \frac{\partial}{\partial y^\sigma_j}, 1 \le i \le n\right\} \\
&= \operatorname{span}\left\{dy^\sigma - \Sigma^\sigma_j\, dx^j, dy^\sigma_i - \Sigma^\sigma_{ij}\, dx^j, 1 \le i \le n, 1 \le \sigma \le m\right\}.
\end{aligned}$$

Using the fact that $J^2\pi \equiv i(J^2\pi) \subset \widehat{J}^2\pi \subset J^1\pi_1$, we get the following kinds of connections.

A *semiholonomic connection* on π_1 is a section $\widehat{\Gamma}^{(2)}\colon J^1\pi \to \widehat{J}^2\pi$ of $\widehat{\pi}_{2,1}$. Thus $\widehat{\Gamma}^{(2)}$ is locally given by $Y_i^\sigma \circ \widehat{\Gamma}^{(2)} = y_i^\sigma \circ \widehat{\Gamma}^{(2)} = y_i^\sigma$, $Y_{ij}^\sigma \circ \widehat{\Gamma}^{(2)} = \widehat{\Gamma}_{ij}^\sigma$, where the components $\widehat{\Gamma}_{ij}^\sigma$ again need not be symmetric in i, j. Equivalently, $\widehat{\Gamma}^{(2)}$ is represented by the π_1-horizontal n-dimensional distribution on J^1Y

$$\begin{aligned} H_{\widehat{\Gamma}^{(2)}} &= \operatorname{span}\left\{\frac{\partial}{\partial x^i} + y_i^\sigma\,\frac{\partial}{\partial y^\sigma} + \widehat{\Gamma}_{ij}^\sigma\,\frac{\partial}{\partial y_j^\sigma}, 1 \le i \le n\right\} \\ &= \operatorname{span}\left\{dy^\sigma - y_j^\sigma\,dx^j,\, dy_i^\sigma - \widehat{\Gamma}_{ij}^\sigma\,dx^j, 1 \le i \le n, 1 \le \sigma \le m\right\}. \end{aligned}$$

Similarly, a 2-*connection* on π, (a *semispray connection* on π, or a *holonomic connection* on π_1), is a section $\Gamma^{(2)}\colon J^1\pi \to J^2\pi$ of $\pi_{2,1}$. Locally, the equations of $\Gamma^{(2)}$ are $Y_i^\sigma \circ \Gamma^{(2)} = y_i^\sigma \circ \Gamma^{(2)} = y_i^\sigma$, $Y_{ij}^\sigma \circ \Gamma^{(2)} = y_{ij}^\sigma \circ \Gamma^{(2)} = \Gamma_{ij}^\sigma$, where $\Gamma_{ij}^\sigma = \Gamma_{ji}^\sigma$. The corresponding n-dimensional π_1-horizontal distribution $H_{\Gamma^{(2)}} = \operatorname{Im} h_{\Gamma^{(2)}}$ is given by

$$\begin{aligned} H_{\Gamma^{(2)}} &= \operatorname{span}\left\{\frac{\partial}{\partial x^i} + y_i^\sigma\,\frac{\partial}{\partial y^\sigma} + \Gamma_{ij}^\sigma\,\frac{\partial}{\partial y_j^\sigma}, 1 \le i \le n\right\} \\ &= \operatorname{span}\left\{dy^\sigma - y_j^\sigma\,dx^j,\ dy_i^\sigma - \Gamma_{ij}^\sigma\,dx^j, 1 \le i \le n, 1 \le \sigma \le m\right\}. \end{aligned}$$

Note that both $H_{\widehat{\Gamma}^{(2)}}$ and $H_{\Gamma^{(2)}}$ are subdistributions of the Cartan distribution $C_{\pi_{1,0}}$, which means that the integral sections of these connections are holonomic (i.e. they are prolongations $j^1\gamma$ of sections γ of π). This enables one to define a *path* or an *integral section* of a connection $\Gamma^{(2)}$ on π to be a section γ satisfying $j^2\gamma = \Gamma^{(2)} \circ j^1\gamma$ on its domain. Expressing this relation in fiber coordinates we see that 2-connections on π are geometric counterparts of systems of second-order differential equations solved with respect to the highest derivatives of the form

$$\frac{\partial^2\gamma^\sigma}{\partial x^i \partial x^j} = \Gamma_{ij}^\sigma\left(x^k, \gamma^\lambda, \frac{\partial \gamma^\lambda}{\partial x^k}\right).$$

Having in mind the splitting (3.1), the *torsion* [19] of a semiholonomic connection $\widehat{\Gamma}^{(2)}$ is defined as the mapping $T_{\widehat{\Gamma}^{(2)}} = r \circ \widehat{\Gamma}^{(2)}\colon J^1\pi \to \pi_{1,0}^*\left(V_\pi Y \otimes \pi^*(\Lambda^2 T^*X)\right)$. Hence we get that there is a canonical splitting of each semiholonomic connection on π_1 to its torsion $T_{\widehat{\Gamma}^{(2)}} = r \circ \widehat{\Gamma}^{(2)}$ and the 2-connection $\Gamma^{(2)} = s \circ \widehat{\Gamma}^{(2)}$. Obviously, $\widehat{\Gamma}^{(2)}$ is holonomic if and only if $T_{\widehat{\Gamma}^{(2)}} = 0$.

Another fibered manifold under consideration will be the first jet prolongation $J^1\pi_{1,0}$ of $\pi_{1,0}\colon J^1\pi \to Y$, i.e. the fibered manifold $(\pi_{1,0})_1\colon J^1\pi_{1,0} \to Y$. By definition, it is a set of 1-jets of local connections on π. The induced coordinates on $J^1\pi_{1,0}$ are $(x^i, y^\sigma, y_i^\sigma, z_{ij}^\sigma, z_{i\lambda}^\sigma)$, where

$$z_{ij}^\sigma(j_y^1\Gamma) = \frac{\partial \Gamma_i^\sigma}{\partial x^j}(y), \quad z_{i\lambda}^\sigma(j_y^1\Gamma) = \frac{\partial \Gamma_i^\sigma}{\partial y^\lambda}(y).$$

A connection on $\pi_{1,0}$ is a (local) section $\Xi\colon J^1\pi \to J^1\pi_{1,0}$ of $(\pi_{1,0})_{1,0}$. The distribution $H_\Xi = \operatorname{Im} h_\Xi$ of Ξ is then an $(n+m)$-dimensional $\pi_{1,0}$-horizontal distribution on J^1Y, whose *integral sections of maximal dimension are local connections on* π. It holds

$$\begin{aligned} H_\Xi &= \operatorname{span}\left\{\frac{\partial}{\partial x^j} + \Xi_{ij}^\sigma\,\frac{\partial}{\partial y_i^\sigma},\ \frac{\partial}{\partial y^\lambda} + \Xi_{i\lambda}^\sigma\,\frac{\partial}{\partial y_i^\sigma}, 1 \le j \le n,\ 1 \le \lambda \le m\right\} \\ &= \operatorname{span}\left\{dy_i^\sigma - \Xi_{ij}^\sigma\,dx^j - \Xi_{i\lambda}^\sigma\,dy^\lambda, 1 \le i \le n,\ 1 \le \sigma \le m\right\}, \end{aligned}$$

where Ξ^σ_{ij}, $\Xi^\sigma_{i\lambda}$ are the components of Ξ. A connection $\Gamma\colon Y \supset V \to J^1Y$ is then an integral section of Ξ if and only if $j^1\Gamma = \Xi \circ \Gamma$, which in fibred coordinates gives a system of $(n+m)$ first-order partial differential equations of the form

$$\frac{\partial \Gamma^\sigma_i}{\partial x^j} = \Xi^\sigma_{ij}(x^k, y^\nu, \Gamma^\nu_k), \qquad \frac{\partial \Gamma^\sigma_i}{\partial y^\lambda} = \Xi^\sigma_{i\lambda}(x^k, y^\nu, \Gamma^\nu_k)$$

for the components of Γ.

4. CHARACTERIZABLE CONNECTIONS

In this section we will study relations between connections on $\pi_{1,0}$ and 2-connections on π which lead to the *theory of characteristics for connections.*

We start with a theorem, proved in [6] using the theory of natural bundles and operators [9], [12].

Theorem 4.1. *All natural transformations of* $J^1\pi_{1,0}$ *into* $J^1\pi_1$ *over the identity of* $J^1\pi$ *form a 1-parameter family* $\{f_a\}_{a\in\mathbb{R}}$, *where the mapping* f_a *is in fibered coordinates expressed by*

$$\begin{aligned} &Y^\sigma_i \circ f_a = y^\sigma_i \\ &Y^\sigma_{ij} \circ f_a = z^\sigma_{ij} + z^\sigma_{i\lambda} y^\lambda_j + a(z^\sigma_{ij} - z^\sigma_{ji} + z^\sigma_{i\lambda} y^\lambda_j - z^\sigma_{j\lambda} y^\lambda_i). \end{aligned} \tag{4.1}$$

Note that f_0 ($a = 0$ in (4.1)) is nothing but the canonical maping $J^1\pi_{1,0} \to J^1\pi_1$, defined for each local connection $\Gamma\colon Y \supset V \to J^1\pi$ on π and each $y \in V$ by $j^1_y\Gamma \mapsto j^1_x(\Gamma \circ \gamma)$, where γ is a local section of π such that $\Gamma(y) = j^1_x\gamma$. If γ is an integral section of Γ then obviously $j^1_x(\Gamma \circ \gamma) = j^2_x\gamma \in J^2\pi$.

Consider the mappings $s\colon \widehat{J}^2\pi \to J^2\pi$ and $r\colon \widehat{J}^2\pi \to \pi^*_{1,0}(V_\pi Y \otimes \pi^*(\Lambda^2 T^*X))$ defined in the previous section and put

$$\begin{aligned} &S\colon J^1\pi_{1,0} \to J^2\pi, \quad S := s \circ f_0 \\ &R\colon J^1\pi_{1,0} \to \pi^*_{1,0}(V_\pi Y \otimes \pi^*(\Lambda^2 T^*X)), \quad R := r \circ f_0. \end{aligned}$$

The fibered coordinates expressions of S and R are

$$S^\sigma_{ij} = \frac{1}{2}\left(z^\sigma_{ij} + z^\sigma_{ji} + z^\sigma_{i\lambda} y^\lambda_j + z^\sigma_{j\lambda} y^\lambda_i\right), \quad R^\sigma_{ij} = \frac{1}{2}\left(z^\sigma_{ij} - z^\sigma_{ji} + z^\sigma_{i\lambda} y^\lambda_j - z^\sigma_{j\lambda} y^\lambda_i\right).$$

Then we can write (4.1) in the form $\{f_b\}_{b\in\mathbb{R}} = \{Y^\sigma_i = y^\sigma_i, Y^\sigma_{ij} = S^\sigma_{ij} + b\, R^\sigma_{ij}\}_{b\in\mathbb{R}}$. Note, that for each $a \in \mathbb{R}$ we get $s \circ f_a = S$ and $R_a := r \circ f_a = (1+2a)R$.

As a direct consequence of Theorem 4.1 we get

Theorem 4.2. *(1) All natural transformations transforming connections on* $\pi_{1,0}$ *into (in fact, semiholonomic) connections on* π_1 *are of the form* $f_a \circ \Xi$.
(2) There is a unique natural transformation transforming connections on $\pi_{1,0}$ *into 2-connections on* π; *it is of the form* $S \circ \Xi$.

Theorem 4.2 leads to the following important conclusions:

Corollary 4.1. *Let Ξ be a connection on $\pi_{1,0}$. There is a unique 2-connection $\Gamma^{(2)}$ on π naturaly assigned to Ξ. This connection is defined by $\Gamma^{(2)} = S \circ \Xi$. The components of $\Gamma^{(2)}$ are in fibered coordinates expressed by*

$$\Gamma^\sigma_{ij} = \frac{1}{2}\left(\Xi^\sigma_{ij} + \Xi^\sigma_{ji} + \Xi^\sigma_{i\lambda}\, y^\lambda_j + \Xi^\sigma_{j\lambda}\, y^\lambda_i\right),$$

where Ξ^σ_{ij}, $\Xi^\sigma_{i\lambda}$ are the components of Ξ.

Corollary 4.2. *Let Ξ be a connection on $\pi_{1,0}$. If $R \circ \Xi = 0$, then $R_a \circ \Xi = 0$ for all $a \in \mathbb{R}$. Locally, $R \circ \Xi = 0$ means*

$$\frac{1}{2}\left(\Xi^\sigma_{ij} - \Xi^\sigma_{ji} + \Xi^\sigma_{i\lambda}\, y^\lambda_j - \Xi^\sigma_{j\lambda}\, y^\lambda_i\right) = 0. \tag{4.2}$$

Clearly, $R \circ \Xi = T_{f_0 \circ \Xi}$. As a direct consequence of the properties of the distributions $H_{\Gamma^{(2)}}$ and H_Ξ we get:

Proposition 4.1. *Let Ξ be a connection on $\pi_{1,0}$, let $\Gamma^{(2)} = S \circ \Xi$. Then $H_{\Gamma^{(2)}} \subset H_\Xi \iff R \circ \Xi = 0$. Consequently, $H_{\Gamma^{(2)}} \subset H_\Xi \iff \Gamma^{(2)} = f_0 \circ \Xi$.*

The above results enable us to introduce the following concepts:

Definition 4.1. A connection Ξ on $\pi_{1,0}$ will be called *characterizable* if $R \circ \Xi = 0$. The 2-connection $\Gamma^{(2)} = S \circ \Xi$ will be then called the *characteristic connection* of Ξ. The distribution $H_{\Gamma^{(2)}}$ of the characteristic connection $\Gamma^{(2)}$ of Ξ will be called the *characteristic distribution* of Ξ and the integral manifolds of $H_{\Gamma^{(2)}}$ of maximal dimension will be called the *characteristics* of the connection Ξ.

Clearly, the maximal integral manifolds of H_Ξ (integral sections of Ξ) are foliated by the maximal integral manifolds of the characteristic distribution (characteristics, 1-jet prolongations of integral sections of the characteristic connection). Accordingly, integral sections of Ξ are "glued together" from the characteristics.

Since Ξ is characterizable if and only if its components satisfy (4.2), the components of the characteristic connection are $\Gamma^\sigma_{ij} = \Xi^\sigma_{ij} + \Xi^\sigma_{i\lambda}\, y^\lambda_j$. The *equations for characteristics* of a characterizable connection Ξ on $\pi_{1,0}$ are

$$\frac{\partial^2 \gamma^\sigma}{\partial x^i \partial x^j} = \Xi^\sigma_{ij}(x^k, \gamma^\nu, \frac{\partial \gamma^\nu}{\partial x^k}) + \Xi^\sigma_{i\lambda}(x^k, \gamma^\nu, \frac{\partial \gamma^\nu}{\partial x^k})\, \frac{\partial \gamma^\lambda}{\partial x^j}.$$

It is clear that for the characteristic distribution $H_{\Gamma^{(2)}}$ of Ξ it holds $H_{\Gamma^{(2)}} \subset H_\Xi \cap C_{\pi_{1,0}}$. Notifying that

$$\begin{aligned} H_\Xi \cap C_{\pi_{1,0}} &= \operatorname{span}\left\{dy^\sigma - y^\sigma_j\, dx^j, dy^\sigma_i - \Xi^\sigma_{ij}\, dx^j - \Xi^\sigma_{i\lambda}\, dy^\lambda, 1 \le i \le n,\ 1 \le \sigma \le m\right\} \\ &= \operatorname{span}\left\{\frac{\partial}{\partial x^i} + y^\sigma_i\, \frac{\partial}{\partial y^\sigma} + \left(\Xi^\sigma_{ji} + \Xi^\sigma_{j\lambda}\, y^\lambda_i\right) \frac{\partial}{\partial y^\sigma_j}, 1 \le i \le n\right\}, \end{aligned}$$

we immediately get the following proposition.

Proposition 4.2. *Let Ξ be a characterizable connection on $\pi_{1,0}$. The following conditions are equivalent:*
(1) $H_{\Gamma^{(2)}}$ is the characteristic distribution of Ξ.
(2) $H_{\Gamma^{(2)}} = H_\Xi \cap C_{\pi_{1,0}}$.

We will show in Sec. 5 that if Ξ is a characterizable and integrable connection on $\pi_{1,0}$, then its characteristic connection is integrable. The following proposition is not much surprising in this respect.

Proposition 4.3. *Let Ξ be an integrable connection on $\pi_{1,0}$. Then Ξ is characterizable if and only if any integral section Γ of Ξ is integrable.*

Proof. Let z be an arbitrary point from the domain of Ξ, let $\Gamma\colon Y \supset V \to J^1\pi$ be the uniquely determined integral section of Ξ passing through z, i.e. $\Gamma(y) = z$. Then evidently $R \circ \Xi(z) = R \circ j^1\Gamma(y) = R_\Gamma(y)$. $\square$

Remark 4.1. It is easy to see that knowing an integral manifold M of H_Ξ of dimension k one can use the characteristics to construct an integral manifold of dimension $\geq k$, containing M ("the Cauchy initial problem").

Remark 4.2. There are some additional interesting results concerning the geometry related to the characterizable connections on $\pi_{1,0}$. Roughly speaking, each such a connection splits into the direct sum of its characteristic connection and the so–called *reduced connection*, represented by a *strong horizontal subbundle* in TJ^1Y. The latter one is closely related to the theory of symmetries. We would like to discuss this problem in some of the forthcoming papers.

Example 4.1. Suppose $\dim X = 1$, denote by (t, q^σ), $1 \leq \sigma \leq m$, (local) fiber coordinates on π. Let Ξ be a connection on $\pi_{1,0}$ with the components Ξ^σ, Ξ^σ_λ. The distribution H_Ξ is locally spanned by $(m+1)$ vector fields

$$\frac{\partial}{\partial t} + \Xi^\sigma \frac{\partial}{\partial q^\sigma_1}, \ \frac{\partial}{\partial q^\lambda} + \Xi^\sigma_\lambda \frac{\partial}{\partial q^\sigma_1}, 1 \leq \lambda \leq m,$$

where $(t, q^\sigma, q^\sigma_1)$ are the associated coordinates on J^1Y. The equations for integral sections of Ξ (which are local connections on π) are

$$\frac{\partial \Gamma^\sigma}{\partial t} = \Xi^\sigma(t, q^\nu, \Gamma^\nu), \qquad \frac{\partial \Gamma^\sigma}{\partial q^\lambda} = \Xi^\sigma_\lambda(t, q^\nu, \Gamma^\nu)$$

for the components Γ^σ of Γ. Evidently, if $\dim X = 1$ then *every connection Ξ on $\pi_{1,0}$ is characterizable.* The components of its characteristic connection $\Gamma^{(2)}$ are then $\Gamma^\sigma_{(2)} = \Xi^\sigma + \Xi^\sigma_\lambda\, q^\lambda_1$, and thus the one-dimensional (consequently integrable) characteristic distribution $H_{\Gamma^{(2)}}$ is spanned by *semisprays*

$$\frac{\partial}{\partial t} + q^\sigma_1 \frac{\partial}{\partial q^\sigma} + (\Xi^\sigma + \Xi^\sigma_\lambda q^\lambda_1) \frac{\partial}{\partial q}$$

Corollary 5.1. *Let* $\Gamma\colon Y \supset V \to J^1\pi$ *be a field of paths of* $\Gamma^{(2)}$. *Then* H_Γ *defines a foliation of* V *each leaf of which is the image of an integral section of* $\Gamma^{(2)}$.

Proposition 5.3. *Let* Ξ *be a characterizable connection on* $\pi_{1,0}$ *and* $\Gamma^{(2)}$ *its characteristic 2-connection on* π. *Then each integral section* Γ *of* Ξ *is a field of paths of* $\Gamma^{(2)}$.

Proof. Let Γ be an integral section of Ξ . Then $\Gamma^{(2)} \circ \Gamma = S \circ \Xi \circ \Gamma = S \circ j^1\Gamma$. Since $S \circ j^1\Gamma = J^1(\Gamma, \mathrm{id}_X) \circ \Gamma$ for any integrable connection Γ on π, we have (5.2). □

The following Corollary generalizes to our "non-variational field theory situation" the *Van Hove Theorem* of the classical calculus of variations (saying that every extremal can be locally embedded in a field of extremals).

Corollary 5.2. (Embedding Theorem). *Let* Ξ *be an integrable characterizable connection on* $\pi_{1,0}$ *and* $\Gamma^{(2)}$ *its characteristic 2-connection on* π. *Then* $\Gamma^{(2)}$ *is integrable, and each integral section of* $\Gamma^{(2)}$ *is locally embedded in a field of paths* Γ, *which is an integral section of* Ξ.

By the above results, the problem of finding all the integral sections of a given integrable 2-connection $\Gamma^{(2)}$ (i.e. the problem of finding the foliation defined by $H_{\Gamma^{(2)}}$) is equivalent to the problem of solving all fields of paths of $\Gamma^{(2)}$. In what follows, we will show that, under some assumptions, it is sufficient to solve a certain "minimal" family of fields of paths of $\Gamma^{(2)}$ to obtain a complete local solution of $\Gamma^{(2)}$. In this respect, characteristic 2-connections have the same properties as possess variational equations of classical mechanics (cf. [14], and [15] for higher-order equations).

Definition 5.2. Let $\Gamma^{(2)}$ be an integrable 2-connection on π. A (local) connection Ξ on $\pi_{1,0}$ will be called an *integral* of $\Gamma^{(2)}$ if Ξ is integrable and $\Gamma^{(2)}$ is its characteristic connection.

We shall show that the existence of integrals of an integrable 2-connection is a direct consequence of the integrability property, and that one can construct an integral of $\Gamma^{(2)}$ by means of a set of independent first integrals of $H_{\Gamma^{(2)}}$. To this purpose recall that if Δ is a distribution on a manifold M then a function f on an open subset V of M is said to be a *first integral* of Δ on V if the form df belongs to the anihilator of Δ. A family $f^1, \dots, f^k$ of functions on V is called a *complete set of independent first integrals* of Δ if the forms $df^1, \dots, df^k$ are linearly independent at each point of V and it holds $\Delta = \mathrm{span}\{df^1, \dots, df^k\}$ on V.

Theorem 5.1. *Let* $\Gamma^{(2)}$ *be an integrable 2-connection on* π, *let* $\{a^1, \dots, a^{nm}\}$ *be a set of independent first integrals of* $\Gamma^{(2)}$, *defined on an open subset* $W \subset J^1\pi$. *Suppose that* $W \subset \pi_{1,0}^{-1}(V)$, *where* (V, ψ) *is a fiber chart on* Y. *If*

$$\det\left(\frac{\partial a^K}{\partial y^\sigma_j}\right) \neq 0 \tag{5.3}$$

on W, *then by*

$$H_\Xi = \mathrm{span}\{da^1, \dots, da^{nm}\} \tag{5.4}$$

an integral $\Xi\colon J^1\pi \supset W \to J^1\pi_{1,0}$ *of* $\Gamma^{(2)}$ *is defined.*

Proof. Consider the distribution (5.4) where $a^1, \dots, a^{nm}$ are independent first integrals of $\Gamma^{(2)}$. This distribution is obviously integrable. Denote by (Λ^σ_{jK}), where σ, j label rows and K label columns, the matrix inverse to $(\partial a^K/\partial y^\sigma_j)$. Computing the vector fields spanning H_Ξ on W we get

$$H_\Xi = \operatorname{span}\left\{ \frac{\partial}{\partial x^i} + \Xi^\sigma_{ji}\,\frac{\partial}{\partial y^\sigma_j},\ \frac{\partial}{\partial y^\lambda} + \Xi^\sigma_{j\lambda}\,\frac{\partial}{\partial y^\sigma_j}, 1 \leq i \leq n,\, 1 \leq \lambda \leq m \right\},$$

where

$$\Xi^\sigma_{ji} = -\Lambda^\sigma_{jK}\frac{\partial a^K}{\partial x^i}, \quad \Xi^\sigma_{j\lambda} = -\Lambda^\sigma_{jK}\frac{\partial a^K}{\partial y^\lambda}.$$

Further denote by Γ^σ_{ij} the components of $\Gamma^{(2)}$ in the chart (V,ψ). It holds

$$\begin{aligned}\Xi^\sigma_{ij} - \Xi^\sigma_{ji} + \Xi^\sigma_{i\lambda}\,y^\lambda_j - \Xi^\sigma_{j\lambda}\,y^\lambda_i &= -\Lambda^\sigma_{iK}\frac{\partial a^K}{\partial x^j} + \Lambda^\sigma_{jK}\frac{\partial a^K}{\partial x^i} - \Lambda^\sigma_{iK}\frac{\partial a^K}{\partial y^\lambda}\,y^\lambda_j + \Lambda^\sigma_{jK}\frac{\partial a^K}{\partial y^\lambda}\,y^\lambda_i = \\ &= -\Lambda^\sigma_{iK}\left(\frac{\partial a^K}{\partial x^j} + \frac{\partial a^K}{\partial y^\lambda}\,y^\lambda_j\right) + \Lambda^\sigma_{jK}\left(\frac{\partial a^K}{\partial x^i} + \frac{\partial a^K}{\partial y^\lambda}\,y^\lambda_i\right) = \\ &= \Lambda^\sigma_{iK}\frac{\partial a^K}{\partial y^\lambda_\ell}\,\Gamma^\lambda_{\ell j} - \Lambda^\sigma_{jK}\frac{\partial a^K}{\partial y^\lambda_\ell}\,\Gamma^\lambda_{\ell i} = \Gamma^\sigma_{ij} - \Gamma^\sigma_{ji} = 0.\end{aligned}$$

Since

$$\Xi^\sigma_{ij} + \Xi^\sigma_{i\lambda}\,y^\lambda_j = -\Lambda^\sigma_{iK}\frac{\partial a^K}{\partial x^j} - \Lambda^\sigma_{iK}\frac{\partial a^K}{\partial y^\lambda}\,y^\lambda_j = \Lambda^\sigma_{iK}\frac{\partial a^K}{\partial y^\lambda_\ell}\,\Gamma^\lambda_{\ell j} = \Gamma^\sigma_{ij},$$

the proof is completed. □

Notice that if $\Gamma^{(2)}$ is integrable then the existence of a set of independent first integrals satisfying the condition (5.3) is ensured by definition (since $H_{\Gamma^{(2)}}$ is horizontal).

Theorem 5.1 suggests an integration method (parallel to the "method of fields of extremals" from the classical calculus of variations), transferring the problem of solving second-order equations related to $\Gamma^{(2)}$ to the problem of solving a family of first-order equations for integral sections of Ξ:

Let Ξ be an integral of $\Gamma^{(2)}$, defined on an open subset of $J^1\pi$ and let $H_\Xi = \operatorname{span}\{da^1, \dots, da^{nm}\}$. Then $a^1, \dots, a^{nm}$ are independent first integrals of $\Gamma^{(2)}$. Let $x \in \operatorname{dom}\Xi$ be a point, (V,ψ), $\psi = (x^i, y^\sigma)$ a fiber chart on Y such that $x \in \pi^{-1}_{1,0}(V)$. Since Ξ is a connection on $\pi_{1,0}$, the condition (5.3) is satisfied. Hence, there is a chart (W,χ) at x such that $W \subset \pi^{-1}_{1,0}(V)$, $\chi = (x^i, y^\sigma, a^K)$, $1 \leq K \leq nm$. Consider the nontrivial components $y^\sigma_i(x^j, y^\nu, a^K)$ of the overlap mapping and put for each fixed $\mathbf{a} = (a^K)$, $\Gamma^\sigma_{(\mathbf{a})i} = y^\sigma_i(x^j, y^\nu, a^K)$. Then for each $\mathbf{a}$, $\Gamma_{\mathbf{a}}\colon \pi_{1,0}(W) \to W$, $\Gamma_{\mathbf{a}}(x^i, y^\sigma) = (x^i, y^\sigma, \Gamma^\sigma_{(\mathbf{a})i})$ is a connection on π. We will say that $\{\Gamma_{\mathbf{a}}\}$ is an *nm-parametric system of connections.* Clearly, for each $\mathbf{a}$, $\Gamma_{\mathbf{a}}$ is an integral section of Ξ. Consequently, a section $\gamma\colon U \to Y$ such that $j^1\gamma(U) \subset W$ is an integral section of $\Gamma^{(2)}$ if and only if γ is an integral section of $\Gamma_{\mathbf{a}}$ for some $\mathbf{a}$. This means that *the problem of solving* $\Gamma^{(2)}$ *on* W *is equivalent to the problem of solving an nm-parametric system of first-order equations* of the form

$$\frac{\partial \gamma^\sigma}{\partial x^i} = \Gamma^\sigma_{(\mathbf{a})i}(x^i, y^\nu) \tag{5.5}$$

where $\Gamma^\sigma_{(\mathbf{a})i}$ are the components of the connection $\Gamma_{\mathbf{a}}$ defined by

$$H_{\Gamma_{\mathbf{a}}} = \operatorname{span}\left\{ \frac{\partial}{\partial x^i} + y^\sigma_i(x^j, y^\nu, a^K)\frac{\partial}{\partial y^\sigma} \right\}, \quad \mathbf{a} = \text{const}.$$

Example 5.1. *Classical mechanics.* Since the above integration method works for any connection (in mechanics spanned by a semispray = "a second-order differential equation"), it is applicable to *general systems of regular equations of classical mechanics* (without any reference to the existence of lagrangians). We shall illustrate briefly that in fact it is a generalization of some techniques and notions known from the first-order Lagrangean dynamics (cf. e.g. [4], [8], [11], [13] - [15]).

Let $\pi\colon Y \to X$ be a fibered manifold over one-dimensional base X. Consider a regular lagrangian λ on J^1Y and the corresponding Euler-Lagrange form E_λ on J^2Y; in a fiber chart (V,ψ), $\psi = (x^i, y^\sigma)$, we get $\lambda = L dt$, $E_\lambda = (A_\sigma + B_{\sigma\nu} q_2^\nu) dq^\sigma \wedge dt$, where the functions A_σ, $B_{\sigma\nu}$ depend on $(t, q^\lambda, q_1^\lambda)$ only, $(B_{\sigma\nu})$ is a regular matrix at each point of $\pi_{2,0}^{-1}(V)$, and A_σ, $B_{\sigma\nu}$ are expressed by

$$B_{\sigma\nu} = -\frac{\partial^2 L}{\partial q_1^\sigma \partial q_1^\nu}, \quad A_\sigma = \frac{\partial L}{\partial q^\sigma} - \frac{\partial^2 L}{\partial t \partial q_1^\sigma} - \frac{\partial^2 L}{\partial q^\nu \partial q_1^\sigma} q_1^\nu.$$

The Euler-Lagrange equations are equations for sections γ of π of the form $E_\lambda \circ j^2\gamma = 0$, or equivalently $j^1\gamma^* i_{J^1\zeta} d\Theta_\lambda = 0$ for each π-projectable vector field on Y, where Θ_λ is the Cartan form of λ. Equivalently, they are equations for integral sections of the 2-connection $\Gamma^{(2)}$ on π defined by $\Gamma^{(2)*}E_\lambda = 0$. The distribution $H_{\Gamma^{(2)}}$ of $\Gamma^{(2)}$ is spanned by semisprays

$$\frac{\partial}{\partial t} + q_1^\sigma \frac{\partial}{\partial q^\sigma} - B^{\sigma\nu} A_\nu \frac{\partial}{\partial q_1^\sigma},$$

where $(B^{\sigma\nu})$ is the inverse matrix to $(B_{\sigma\nu})$, or equivalently, by the Pfaffian system $i_\zeta d\Theta_\lambda$, where ζ runs over all vector fields on J^1Y. Hence, $H_{\Gamma^{(2)}}$ is the characteristic distribution of the closed 2-form $d\Theta_\lambda$. A section $\varphi\colon Y \to J^1Y$ is called a *field of extremals* of λ if it is an integral section of $d\Theta_\lambda$, i.e. if $\varphi^* d\Theta_\lambda = 0$; the latter equation is called the *eikonal equation* (*Hamilton-Jacobi equation*). It is easy to see that a field of extremals satisfies the condition (5.1), i.e. it is a field of paths of $\Gamma^{(2)}$. (In fact, the condition it satisfies is stronger, expressing that it is related to a given lagrangian). If a function S, depending on time t, position $(q^1, \dots, q^m)$ and m parameters $(a^1, \dots, a^m)$ is a solution of the Hamilton-Jacobi equation, then the distribution spanned by $da^1, \dots, da^m$ is an integral of $H_{\Gamma^{(2)}}$, and the functions $b^i = \frac{\partial S}{\partial a^i}$, $i = 1, \dots, m$ form a complete set of independent first integrals of the corresponding m-parametric system of first order equations for extremals (5.5).

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