POLUTION EMISSIONS OPTIMIZATION OF HYDRO-THERMAL SYSTEMS

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Abstract

This paper examines the applicability of the Ritz method to the optimal environmental dispatching of hydro-thermal systems. The algorithm proposed is aimed to reduce the production of atmospheric emissions such as NO_x and SO_2 caused by the operation of fossil-fueled thermal generation. Such a reduction is achieved by including the emissions as a function in the objective of the dispatching problem. The technique may also be applied to the study of other types of emissions or to a combination of objectives. The generic minimization algorithm, which is not difficult to construct on the basis of the Ritz method, has certain advantages in comperison with the conventional methods.

Key Words: Environmental Dispatch, NO_x and SO_2 control, Emission control, Ritz Method.

1. INTRODUCTION

The electric power systems are traditionally operated in such a way that the total fuel cost is minimized regardless of the emissions produced. The increasing requirements to the environmental protection give rise to the need in alternative strategies. This paper presents an algorithm of environmental dispatch in a hydro-thermal system and discusses the problem of minimization of emissions in the power flow (SO_2 and NO_x). The algorithm is constructed with the use of the Ritz method.

The form of the emission function model depends (among other things) on the emission type. It is generally acknowledged that for SO_2 the emissions are proportional to the thermal unit's fuel consumption. As a result, the sulfuric emission function will be of the same form as that of the fuel cost function used in the economic dispatch. The NO_x emissions function is less straightforward to represent because these emissions are highly nonlinear in P. Several models have been used to represent the emission levels: the sum of a quadratic and an exponential term, a second order polynomial, a combination of a straight line and an exponential term, etc. We take for the function $E(P)$ the second order polynomial,

$$E(P) = \alpha + \beta P + \gamma P^2,$$

where the parameters are computed via the least square criteria from several tests.

In this work, we apply the Ritz method for the actual computer implementation of the optimal strategies in this hydro-thermal scheduling (HTS) problem of short range. The hydro network is assumed to have several chains of hydro plants on different streams. The volume of water discharge for each hydro-plant is a prespecified constant amount over the optimization interval, and we consider the river transport delay. Finally, the algorithm developed for this problem has been successfully applied to one real system; these results are presented below.

2. MATHEMATICAL OPTIMIZATION TECHNIQUES

In this chapter, we discuss the problem of minimization of a quadratic functional.
Let F a heterogeneous quadratic functional over a domain $D(F) \subseteq H$, where H is a Hilbert space. The functional $F(x)$ assumes its minimum value F_0 at x^*. We seek an element x to approximate F_0. A simple and efficient method of approximate solution of this problem was proposed in 1908 by Ritz.
Let $g(u)$ be a homogeneous quadratic functional corresponding to the bilinear symmetric form $G(u,v)$, over the same domain $D(F)$and such that $g(u) = G(u,u)$.
Let $l(u)$ be a linear (that is, additive and homogeneous, but not necessarily bounded) functional such that its domain is $D(F)$.
Let F be the functional: $F(x) = g(x) - 2lx$, and let us assume that the functional g is positiv, that is: $g(x) \succ 0$ for $x \neq 0$, $x \in D(F)$.

Lemma 1

If the homogeneous quadratic functional g is *positive definite*, that is, there exists a constant $m_0 \succ 0$ such that: $g(x) \geq m_0 \left\| x \right\|^2$and the functional l is bounded, then the functional F is bounded from below.
In this case the problem of minimization of F makes sense.

Let us introduce now the following concept.

A sequence $\left\{ x^{(n)} \right\}_{n=1}^{\infty}$ of $D(F)$ is called *minimizing* for the functional F if:

$$F\left(x^{(n)}\right) \rightarrow F_0 = \inf F(x)$$

It is easy to prove that the Ritz approximate solutions form a minimizing sequence. Now we describe the Ritz process.

We take the *coordinate elements* $\{u_i\}_{i=1}^{\infty}$ in $D(F)$, and will assume that every finite set of coordinate elements is linearly independent. Then we introduce the subspaces:

$$H^{(n)} = L(u_1, ..., u_n) \in D(F)$$

We shall also use the following lemma.

Lemma 2

The functionals $g(x)$ and lx are continuous in each space $H^{(n)}$ and the functional $g(x)$ is positive definite in each space $H^{(n)}$, that is, there exist positive constants m_n such that:

$$g(x) \geq m_n \left\| x \right\|^2 \; (x \in H^{(n)})$$

Let

$$F_n = Inf_{x \in H^{(n)}} F(x)$$

Then $\forall\ n$ in $H^{(n)}$ there exists only one element where $F = F_n$.

The Ritz method consists in seeking the minimum of F not in H but in $H^{(n)}$, the n-dimensional subspace spanned by $\{u_i\}_{i=1}^{n}$.

In other words, $x \in H^{(n)}$ can be written as

$$x = \sum_{i=1}^{n} a_i u_i$$

where the $a_{1,...,} a_n$ are constants, and we thus seek the minimum of the function of n variables:

$$F\left(\sum_{i=1}^{n} a_i u_i\right) = F(a_{1,....,} a_n)$$

In this case

$$F(a_{1,....,} a_n) = \sum_{i,j=1}^{n} a_i a_j G(u_i, u_{j)} - 2\sum_{i=1}^{n} a_i l u_i$$

As is easy to see, to minimize F amounts to solving the system of equations $0 = \frac{\partial F}{\partial a_k} = \sum_{j=1}^{n} a_j G(u_k, u_j) + \sum_{i=1}^{n} a_i G(u_i, u_k) - 2lu_k = 2\left[\sum_{i=1}^{n} a_i G(u_i, u_k) - lu_k\right]$ If we take for the $(a_i)_{i=1}^{n}$ the solution of the latter system, then the elements $x^{(n)}$, depending on the coordinate elements $(u_1, ..., u_n)$, will be called the *Ritz approximate solutions* of the basic optimization problem and $(a_i)_{i=1}^{n}$ will be called the *Ritz coefficients.*

Note: It worth noting that the approximate solution does not change if the functional is of the general form

$$F(x) = g(x) - 2lx + c,$$

where c is constant.

Let us now consider the problem of determining the minimum of the functional

$$F(x) = (Ax, x) - 2(y^*, x) \ \ (x \in D(A))$$

which is called the *energy functional.* Later on, it will be our cost functional. The next sections will be devoted to its construction.

The following is supposed:

a) the operator A is symmetric, that is, $\forall u, v \in D(A)$ we have

$$(Au, v) = (u, Av)\,;$$

b) the operator A is positive definite, that is, there exists a constant $m_0 \succ 0$ such that $\forall u \in D(A)$

$$(Au, u) \succeq m_0 \left\|u\right\|^2;$$

c) $y^* \in R(A)$

Let us assume that $D(F) = D(A), G(u,v) = (Au,v), lu = (u,y^*)$ for, $u,v \in D(F)$, then the *energy functional* can be written as

$$F(x) = g(x) - 2lx.$$

We now make use of the following construction due to K. Friedrichs. Define a new scalar product (the *scalar energy product*) setting

$$[u,v] = (Au,v)\,\mathrm{for} u,v \in D(A),$$

and the new *energy norm*:

$$|u| = [u,u]\,.$$

Let us complete $D(A)$ with respect to this norm $|*|$. It is not difficult to verify that in the result of this completion $D(A)$ becomes a new Hilbert space H_A which is called the *energy space.* Moreover, if $u_i \in D(A)$, which is dense in H_A, we obtain:

$$H^{(n)} \subset D(A) \subset H_A.$$

We thus arrive at the following theorem.

Theorem

If the coordinate system is complete in H_A, then $x^{(n)}$ converges in H and in H_A *to the element x^* which yields the minimum of F.*

3. HYDRO-THERMAL SYSTEM MODELS

The aim of this chapter is to present some fundamental aspects of modeling various parts of the electric power system. An electric power system with m thermal plants and (n-m) hydro plants is studied. The following models are considered:

1) The transmission losses.

The transmission losses are assumed to be represented by the model of Kirchmayer, with the following loss equation:

$$P_L = K_{LO} + \sum_{i=0}^{n} B_{io}P_i + \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j,$$

where K_{LO}, B_{io} and B_{ij} are known parameters of the hydro-thermal system.

2) Hydro network model.

The hydro network is assumed to have several chains of hydro plants on different streams as well as hydraulically isolated plants. Let us denote by R_h the set of all hydro plants in the system . The hydro network configuration is shown in the figure 1.

We can see that the rate of discharge at the upstream plant affects the behaviour at the downstream plants. We say that the hydraulic system has hydraulic coupling. The time delay of water discharge between two consecutive plants is assumed to be a constant τ (function of distance between plants).

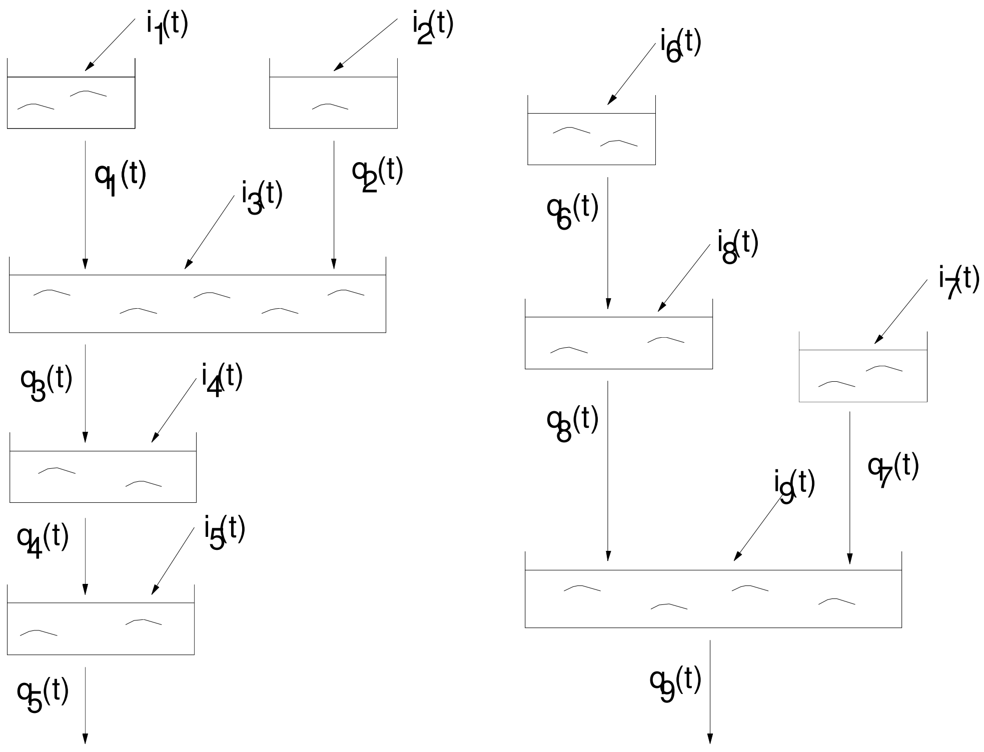

Figure 1

The variables of the system are:

$i_i(t)$: Rate of natural water inflow to the i reservoir.

$q_i(t)$: Rate of water discharge by i plant.

τ_i: Time delay of water between i and downstream reservoir.

$s_i(t)$: Volume of water stored.

The forebay elevation $y_i(t)$ is related to the storage $s_i(t)$ according to the following formula:

$$y_i(t) = y_{i0} + B_{yi} s_i(t) \qquad i = m+1, m+2, ..., n.$$

The tailwater elevation varies with the rate of water discharge according to the following relation:

$$y_{Ti}(t) = y_{Ti0} + B_{Ti}q_i(t) \qquad\qquad i = m+1, m+2, ..., n.$$

The effective hydraulic head at the ith hydro plant is:

$$h_i(t) = y_i(t) - y_{T_i}(t) \qquad\qquad i = m+1, m+2, ..., n$$

The ith hydro plant's active power generation is given by the expression

$$P_{hi}(t) = q_i(t)h_i(t)/G_i \qquad\qquad i = m+1, m+2, ..., n.$$

The reservoir's dynamics are described in the following way:

(1) *Upstream plants*:

$$\dot{s_i}\ (t) = i_i(t) - q_i(t) \tag{1}$$

(2) *Intermediate plants:* We denote by R_{hi} the set of all plants immediately upstream and feeding into the ith hydro plant.

The flow from each plant has a transport delay of $\tau_j (j \in R_{hi})$ to the ith plant. Then the ith reservoir's dynamics are expressed by the formula

$$\dot{s_i}\ (t) = i_i(t) + \sum_{j\in R_{hi}} q_j(t-\tau_j) - q_i(t)). \tag{2}$$

(3) *Downstream plants*: The reservoir dynamics are described by the same relation derived for the intermediate plants.

(4) *Isolated plants*: The reservoir dynamics for this type are described by the same relation that for the upstream plants.

For convenience of formulation, we introduce the following auxiliary variables:

$$\begin{aligned} D_i(t) &= s_i(0) + \int_0^t i_i(x)dx \\ Y_j(t,\tau_j) &= \int_0^t q_j(x-\tau_j)dx \\ Q_i(t) &= \int_0^t q_i(x)dx \end{aligned}$$

The first of them, $D_i(t)$, is the storage value at the i reservoir if both its own plant and the upstream plant are desactived (no discharge).

The second variable $Y_j(t,\tau_j)$ is the volume of the water discharged delayed τ_j.

$Q_i(t)$ is the volume of water discharge at the plant i up to and including the time instant t.

It is to be noted here that the variable $D_i(t)$ is an uncontrolled variable whereas $Q_i(t)$ and $Y_j(t,\tau_j)$ are controlled variables. Integrating (1) and (2) gives the following expression for the storage:

$$s_i\ (t) = D_i(t) - Q_i(t),$$
$$s_i\ (t) = D_i(t) + \sum_{j\in R_{hi}} Y_j(t,\tau_j) - Q_i(t).$$

The ith hydro plant's active power generation is given by

$$P_{hi}(t) = \frac{q_i(t)}{G_i}\left[\alpha_{yi} + B_{yi}s_i(t) - B_{Ti}q_i(t),\right]$$

where G_i is the efficiency constant of the hydro plant, and α_{yi}, B_{yi} and B_{Ti} are parameters corresponding to the geometry.

Others variables that can be calculated for each reservoir are:

$$A_i(t) = -\frac{1}{G_i}\left[\alpha_{yi} + B_{yi}D_i(t)\right]; B_i = \frac{B_{yi}}{G_i}; C_i = \frac{B_{Ti}}{G_i}.$$

Later on, we shall see that $Y_j(t,\tau_j)$ is related to the new function:

$$\Psi_j(t,\tau_j) = \int_{-\tau_j}^{t-\tau_j} q_j(x)dx \quad j \in R_{hi} \quad t \leq \tau_j \quad .$$

This function represents the volume of water discharge by the immediately upstream plants between $-\tau_j$ and $t-\tau_j$ instants. Note that this is an initially known function more useful than $Y_j(t,\tau_j)$. It is easy to see that

$$Y_j(t,\tau_j) = \begin{cases} \Psi_j(t,\tau_j) & \text{for} \quad 0 \leq t \leq \tau_j \\ \Psi_j(\tau_j,\tau_j) + Q_j(t-\tau_j) & \text{for} \quad \tau_j \prec t \prec T_f \end{cases} \qquad j \in Rhi$$

Finally, it is assumed that the volume of water discharge b_i for each hydro plant is a given constant amount from the optimization interval $[0,\mathrm{T}]_f]$, and is given by:

$$\int_0^{T_f} q_i(t)dt = b_i$$

3) Emission function model.

For economy operation problems treated in many works the fuel cost of the thermal plants is approximated by:

$$F(P_s) = \alpha^* + \beta^* P_s + \gamma^* P_s^2$$

Here α^*, β^* and γ^* are known constants for each thermal plant and P_s is the power generation of this plant. For the emissions, let us modelize the emission curve by a second-order polinomial in the active power generation.

We use a function $E(P)$ (tons/h) of total NO_x and SO_2 emissions which is assumed to be a second order polynomial:

$$E(P) = \alpha + \beta P + \gamma P^2,$$

where the parameters were calculated via the least square criterion from several tests (Suárez 1993) in thermal plant of Aboño (Spain).

4. STATEMENT OF THE PROBLEM

An electric power system with m thermal plants and (n-m) hydro plants in W chains of hydro plants on different streams is considered. In order to simplify the formulation, let us consider only one chain, and generalize the results at the end. The assumptions on the thermal subsystem and the electric network model are the same that we have defined above. The hydro network configuration is shown in the previous figure (Fig. 0.1)

The problem is to determine the power generation of each plant in order to minimize the emissions of the thermal plants in the optimization interval [0,T$_f$].

Thus, it is required to obtain the active generation schedules which will minimize the cost functional:

$$J = \int_0^{T_f} \sum_{i=1}^{m} (\alpha_i + \beta_i P_{si}(t) + \gamma_i P_{si}^2(t) dt.$$

The problem is to be solved under the following conditions. First, the generation schedule must satisfy the active power balance equation (APBE):

$$\sum_{i=1}^{n} P_i(t) = P_D(t) + P_L(t).$$

Here $P_D(t)$ is the system power demand. With the use of the transmission power loss equation, the (APBE) can be written in the following way:

$$K_{LO} + \sum_{i=1}^{n} \sum_{j=1}^{n} P_i(t) B_{ij} P_j(t) + \sum_{i=0}^{n} (B_{io} - 1) P_i(t) + P_D(t) = 0. \tag{3}$$

The other constraints follow from the equations of the network hydro model. Under the above assumptions, the active power generated and the rate of water discharge are given by:

$$q_{m+i}(t) - \dot{Q}_{m+i}(t) = 0 \,(i \in R_h) \tag{4}$$

$$\begin{gathered} P_{hm+i}(t) + A_{m+i}(t) q_{m+i}(t) + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t) = 0 \,(i \in R_{hCA}) \\ P_{hm+i}(t) + A_{m+i}(t) q_{m+i}(t) - B_{m+i} q_{m+i}(t) \sum_{j \in R_{hi}} Y_j(t, \tau_j) + \\ + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t) = 0 \,(i \in R_{hID}) \end{gathered} \tag{5}$$

where R_{hCA} is the set of all upstream and isolated plants and R_{hID} is the set of intermediate, and downstream hydro plants. The fourth constraint is the identity:

$$Y_j^2(t, \tau_j) = \begin{cases} \Psi_j^2(t, \tau_j) & \text{for} \quad 0 \le t \le \tau_j \\ \Psi_j^2(\tau_j, \tau_j) + Q_j^2(t - \tau_j) + 2\Psi_j(\tau_j, \tau_j) Q_j(t - \tau_j) & \text{for} \quad \tau_j \prec t \prec T_f \end{cases} \quad j \in Rhi \tag{6}$$

Furthermore, the water discharge at each hydro plant has to satisfy the following constraint on the volume of water used, which is taken from the optimization interval:

$$\int_0^{T_f} q_i(t) dt = b_i \tag{7}$$

To satisfy these constraints, we introduce the unknown functions of time $l(t)$, $m_i(t)$, $n_i(t)$, and $r_i(t)$ which correspond to the constraints (3), (3), (5), and (6), respectively. The constraint (7) is associated with the unknown constant factor μ_i.

These unknown functions, the constants μ_i, and some suitable conditions have to be included in the cost functional forming the *augmented cost functional*:

$$J = \int_0^{T_f} \{ \sum_{i=1}^{m} \left(\alpha_i + \beta_i P_{si}(t) + \gamma_i P_{si}^2(t) \right) +$$

$$+ l(t) \left[K_{LO} + \sum_{i=1}^{n} \sum_{j=1}^{n} P_i(t) B_{ij} P_j(t) + \sum_{i=0}^{n} (B_{io} - 1) P_i(t) + P_D(t) \right] +$$

$$+ \sum_{i \in R_h} m_{m+i}(t) \left[q_{m+i}(t) - \dot{Q}_{m+i}(t) \right] +$$

$$+ \sum_{i \in R_{hCA}} n_{m+i}(t) \left[P_{hm+i}(t) + A_{m+i}(t) q_{m+i}(t) + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t) \right] +$$

$$+ \sum_{i \in R_{hID}} n_{m+i}(t) \left[\begin{array}{c} P_{hm+i}(t) + A_{m+i}(t) q_{m+i}(t) - B_{m+i} q_{m+i}(t) \sum_{j \in R_{hi}} Y_j(t, \tau_j) + \\ + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t) \end{array} \right] +$$

$$+ \sum_{i \in R_{hID}} \sum_{i \in R_{hi}} r_j(t) Y_j^2(t, \tau_j) \} dt - \sum_{i \in R_{hID}} \sum_{i \in R_{hi}} \int_0^{\tau_j} r_j(t) \Psi_j^2(t, \tau_j) dt -$$

$$- \sum_{i \in R_{hID}} \sum_{i \in R_{hi}} \int_{\tau_j}^{T_f} r_j(t) \left[\Psi_j^2(\tau_j, \tau_j) + Q_j^2(t - \tau_j) + 2\Psi_j(\tau_j, \tau_j) Q_j(t - \tau_j) \right] +$$

$$+ \int_0^{T_f} \sum_{i \in R_h} \mu_{m+i} q_{m+i}(t) dt - \sum_{i \in R_h} \mu_{m+i} b_{m+i}$$

This cost functional can be written in matrix form. Let us define the control vector,

$$u(t) = \left[\begin{array}{c} P(t) \\ W(t) \end{array} \right],$$

where $P(t)$ is a column vector of all active power generations, and

$$W(t) = col \left[W_i(t) : i \in R_h \right].$$

Each of the hydro-plants offers a subvector $W_i(t)$, and each of the hydro subvectors has a dimension and a definition that depend on the category of the plant. Thus we have:

$$W_i(t) = \left[\begin{array}{c} Q_i(t) \\ q_i(t) \end{array} \right] \qquad i \in R_{hCA},$$

$$W_i(t) = \left[\begin{array}{c} Q_i(t) \\ q_i(t) \\ Y_{iw}(t) \end{array} \right] \quad \text{with} \quad Y_{iw}(t) = col \left[Y_j(t, \tau_j) : j \in R_{hi} \right] \quad i \in R_{hID}.$$

With the above preliminary definitions, the problem is readily formulated as:

$$J \left[u(t) \right] = \int_0^{T_f} \left[u^T(t) B(t) u(t) \right] dt + \int_0^{T_f} \left[L^T(t) u(t) \right] dt + c$$

where the matrix $B(t)$, the vector $L(t)$, and the constant c are defined and calculated in (Bayón,1995).

Let us define

$$G(u, u) = (Au, u) = \int_0^{T_f} \left[u^T(t) B(t) u(t) \right] dt$$

$$l \cdot u = \int_0^{T_f} \left[L^T(t)u(t) \right] dt$$

$$c = - \sum_{i \in R_h} \mu_i b_i$$

The cost functional can be written as

$$F(u) = (Au, u) - 2lu + c \quad (u \in D(A))$$

With these preliminaries at hand, we conclude that the problem is ready for casting as a problem of the minimum of a quadratic functional.

In considering the problem formulated, we will have the following assertion which is a simple consequence of the definitions given; see, for example, (El Hawary, 1979):

i) $g(u)$: symmetric and positive definite functional.

ii) $l(u)$: linear and bounded functional.

Under the above conditions every minimizing sequence for the functional $F(u)$ converges in H_A to the element which yields the minimum of $F(u)$.

Let us choose the energy space: $L_{2B}^n[0, T_f] \equiv H_A$.

The vector $u(t)$ is considered as an element of the Hilbert energy space of n vector-valued square-integrable functions defined on $[0, T_f]$, whose inner product is given by

$$[v(t), u(t)] = (Av, u) = \int_0^{T_f} \left[v^T(t)B(t)u(t) \right] dt.$$

The question of a rational choice of the coordinate functions plays a fundamental role in the present work. For the sake of simplicity, we choose as the coordinate functions the system $\{t^i\}_{i=0}^{\infty}$, which satisfy the required boundary conditions. It is not difficult to verify (see, for example, Miklhin) that the above-formulated conditions are satisfied. That is, the elements are linearly independent and form a complete system in the energy space H_A, which completes the study of the questions related to the construction of approximate solutions and to verification of their convergence .

The approximate solutions are usually constructed as the linear combination of two or even one coordinate function.

5. OPTIMAL SOLUTION

In this section, we present a solution of the basic optimization problem under the above-formulated assumptions. The Ritz method consists in seeking the minimum of F not in H_A, but in $H^{(n)}$, the n -dimensional subspace spanned by $\{w_i\}_{i=1}^n$.

In other words, $x \in H^{(n)}$ can be written as

$$x = \sum_{i=1}^{n} a_i w_i.$$

Let us now consider the approximations to our control variables. We note here that the augmented and modified cost functional (Bayón 1995) depends only on two control variables: the thermal power generation $P_{si}(t)$, and the rate of water discharge by hydro plants $q_i(t)$.

In general, with the coordinate functions $w_i(t)$, the approximations are given by:

$$P_{si}(t) = P_{si}(0) + \sum_{l=1}^{f} d_l^i w_l(t) \qquad i = 1, 2, ..., m$$

$$q_{m+i}(t) = q_{m+i}(0) + \sum_{l=1}^{f} c_l^{m+i} w_l(t) \qquad i = 1, 2, ..., n-m,$$

where f is the number of coordinate functions and the dimension of the subspace $H^{(n)}$.

The results of computational experiments with a number of algorithms and samples show a very fast convergence of the minimizing sequence to the exact solution.

In order to avoid instability of the Ritz process, and for simplicity of calculations, we choose the system of coordinate functions $w_i(t) \equiv \{t^i\}_{i=0}^{\infty}$ and take for the approximate Ritz-representation (solution) only one coordinate function.

Under this assumption, we have:

$$P_{si}(t) = P_{si}(0) + d_l^i t \qquad i = 1, 2, ..., m$$

$$q_{m+i}(t) = q_{m+i}(0) + c_l^{m+i} t \qquad i = 1, 2, ..., n-m$$

Now, we adopt a variational calculus approach that employs Lagrange multipliers $l(j)$ and μ_i.This approach simply transforms the constrained problem into a nonlinear unconstrained problem, where the function depends of the $n \times f \times q$ Ritz coefficients, (f for each thermal or hydro plant of the system and one for each node).

We now consider the problem of the minimum of the function of n variables:

$$J = F\left(a_1, ..., a_{n.f.q}, l(1), ..., l(q), \mu_{m+1}, ..., \mu_n\right)$$

As is shown above, the minimization of F reduces the problem to seek one element

$$\left((a_i^*)_{i=1}^{n.f.q}, (l^*(j))_{j=1}^{q}, (\mu_{m+i}^*)_{i=1}^{n-m}\right)$$

in the $\Re^{n.f.q+q+n-m}$ space, where a_i^* are the Ritz coefficients of the approximate solutions $x^{(n.f.q)}$ with the coordinate elements $(w_1, ...w_{n.f.q})$.

The set of equations defining the optimality can be obtained simply by setting the gradients of the augmented cost J with respect to the unknowns

$$a_1, ..., a_{n.f.q}, l(1), ..., l(q), \mu_{m+1}, ..., \mu_n$$

equal to zero.

The following nonlinear system of equations are the first order partial derivatives of the augmented cost functional equation J:

$$\frac{\partial F}{\partial d_l^{i,j}} = 0;\ i = 1, 2, ..., m;\ l = 1, 2, ..., f;\ j = 1, 2, ..., q$$

$$\frac{\partial F}{\partial c_l^{m+i,j}} = 0;\ i = 1, 2, ..., n-m;\ l = 1, 2, ..., f;\ j = 1, 2, ..., q$$

$$\frac{\partial F}{\partial l(j)} = 0;\ j = 1, 2, ..., q$$

$$\frac{\partial F}{\partial \mu_{m+i}} = 0;\ i = 1, 2, ..., n - m$$

We know that Newton's method for solving nonlinear equations has an unfortunate tendency to wander off into the wild blue yonder if the initial guess is not sufficiently close to the root. A global method is the one that converges to a solution from almost any starting point.

To resolve this system we will develop an algorithm that combines the rapid local convergence of Newton's method with a globally convergent strategy that will guarantee some progress towards the solution at each iteration. The algorithm (Bayón, 1995) is close to the quasi-Newton method of minimization and we call it the *backtracking globally convergent (BGC) method.*

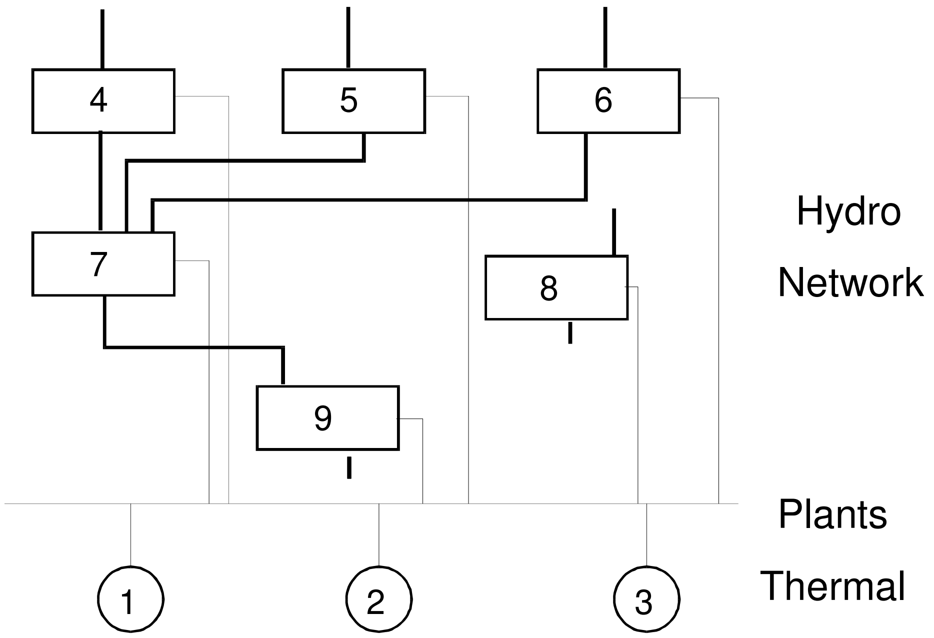

Figure 2

6. A NUMERICAL EXAMPLE

A computer program (Fortran) was written to apply the results obtained in this paper to practical power system whose data are summarized in Table 1. The system consists of three thermal plants and six hydro-plants and the hydro-network is as shown in the figure 2.

Thermal Plants

1	2	3	Coef.
0	0	0	$\alpha(\$)$
0.16	0.17	0.18	$\beta(\$/MW.h)$
0.0004	0.0005	0.0006	$\gamma(\$/MW^2.h)$
0.00016	0.00010	0.00016	$B(MW^{-1})$
31.58	66.98	87.73	$P(0)(MW)$

Hydro Plants

4	5	6	$Coef.$
$526{,}315 \cdot 10^3$	$536{,}315 \cdot 10^3$	$546{,}315 \cdot 10^3$	$G\,(m^4/h.MW)$
$141{,}6 \cdot 10^5$	$141{,}60 \cdot 10^5$	$141{,}60 \cdot 10^5$	$b\,(m^3)$
$220{,}0 \cdot 10^{-6}$	$200{,}0 \cdot 10^{-6}$	$240{,}0 \cdot 10^{-6}$	$B\,(MW^{-1})$
$101{,}952 \cdot 10^5$	$101{,}952 \cdot 10^5$	$101{,}952 \cdot 10^5$	$i\,(m^3/h)$
$203{,}904 \cdot 10^9$	$203{,}904 \cdot 10^9$	$203{,}904 \cdot 10^9$	$s\,(m^3)$
$581{,}740 \cdot 10^{-8}$	$571{,}740 \cdot 10^{-8}$	$581{,}740 \cdot 10^{-8}$	$B_T\,(m^{-2}.h)$
$149{,}5 \cdot 10^{-12}$	$159{,}5 \cdot 10^{-12}$	$149{,}5 \cdot 10^{-12}$	$B_y\,(m^{-2})$
$537{,}35 \cdot 10^3$	$531{,}57 \cdot 10^{-12}$	$537{,}30 \cdot 10^{-12}$	$q(0)\,(m^3)$
1	1	1	$\tau\,(h)$

7	8	9	$Coef.$
$570{,}834 \cdot 10^3$	$580{,}834 \cdot 10^3$	$590{,}834 \cdot 10^3$	$G\,(m^4/h.MW)$
$991{,}20 \cdot 10^5$	$991{,}20 \cdot 10^5$	$991{,}20 \cdot 10^5$	$b\,(m^3)$
$160{,}0 \cdot 10^{-6}$	$170{,}0 \cdot 10^{-6}$	$160{,}0 \cdot 10^{-6}$	$B\,(MW^{-1})$
$301{,}952 \cdot 10^5$	$401{,}952 \cdot 10^5$	$501{,}952 \cdot 10^5$	$i\,(m^3/h)$
$407{,}808 \cdot 10^8$	$407{,}808 \cdot 10^8$	$407{,}808 \cdot 10^8$	$s\,(m^3)$
$219{,}597 \cdot 10^{-8}$	$239{,}597 \cdot 10^{-8}$	$219{,}597 \cdot 10^{-8}$	$B_T\,(m^{-2}.h)$
$149{,}5 \cdot 10^{-11}$	$159{,}5 \cdot 10^{-11}$	$149{,}5 \cdot 10^{-11}$	$B_y\,(m^{-2})$
$3815{,}12 \cdot 10^{-12}$	$3802{,}6 \cdot 10^{-12}$	$3751{,}5 \cdot 10^{-12}$	$q(0)\,(m^3)$
2	2		$\tau\,(h)$

Table 1

The system's power demand and the computed optimal active power generations for the sample system are shown in the figure 3.

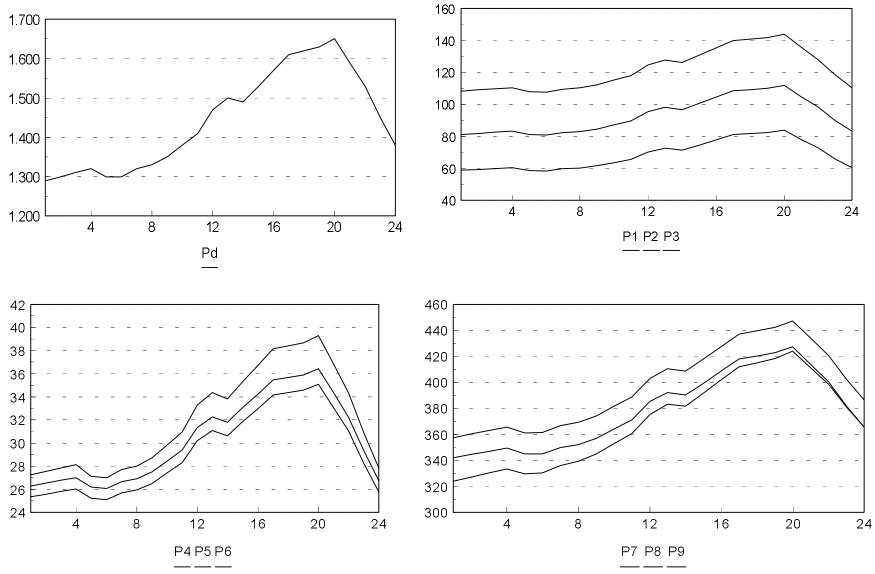

Figure 3

7. CONCLUSIONS

In this paper, a formulation of the environmental operation of hydro-thermal electric power systems is given. The optimal solution for the problem is introduced with the use of the Ritz method in this HTS problem of short range. The simplest way to arrive at the required equations is to use the calculus of variations. The optimality conditions are obtained by taking the partial derivatives with respect to the decision variables.

The formulation includes a general layout of hydro-plants that may form multi-chains of reservoir network.

Time-delays are included and the electric network is considered by using the active power balance equation. The volume of water discharge for each hydro-plant is a given constant amount from the optimization interval.

A computer program was written to solve the nonlinear system of optimal equations. The program used the above-mentioned backtracking globally convergent (BGC) method. This iterative technique shows a rapid convergence for such complex problems and the great advantage of BGC method is that the method is sure to converge to a solution from almost any initial approximation.

Finally, this strategy can be applied to another problems: the usual minimum total quadratic fuel cost objective, to combinations of the loss and cost objectives, multi-objective optimization, etc.

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