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CONTRACTUAL M-CORE AND EQUILIBRIUM ALLOCATIONS

Abstract. The paper deals with equilibrium characterization of the so-called totally contractual allocations [1–3]. As a consequence of the characterization obtained, a rather strong coalitional stability of equilibrium allocations is established. Due to the complicated logical structure of contractual blocking, we pay strong attention to the pure descriptive aspects of the concepts under consideration. Quite simple sufficient conditions guaranteeing coincidence of the totally contractual core and the set of Walrasian equilibrium allocations are established, and the structure of domination relations induced by several rules of breaking the contracts is studied. Game-theoretic approach elaborated in the paper rests on reduction of the original blocking to some more simple domination relations in cooperative games, associated with the contractual blocking in question.

1. Totally contractual set and M-core.

Below, we consider a slightly generalized pure exchange model

$$\mathcal{E} = \langle N, \{X_i, w^i, \alpha_i\}_N, \sigma\rangle, \tag{1}$$

where $N := \{1, \dots, n\}$ is a set of consumers, and $X_i \subseteq \Re^l$, $w^i \in \Re^l$, $\alpha_i \subseteq X_i \times X_i$ are their consumption sets, initial endowments, and individual preference relations, respectively. As to σ, which is a nonempty subset of 2^N, called a *coalitional structure* of $\mathcal{E}$, it defines a collection of admissible coalitions $S \subseteq N$, which may join efforts of their participants in order to improve (block) any current distribution of the total initial endowment $\sum_N w^i$.

To present the main definition of contractual domination (blocking) in $\mathcal{E}$, we modify first the relevant definitions from [1], aiming to clarify some features of the elementary interchange structure we deal with. For each $S \in \sigma$ we fix some subset $M_S \subseteq \Re^l$ such that $0 \in M_S$, and put $M_\sigma := \{M_S\}_\sigma$.

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Definition 1. *A contract (of type M_S) of coalition $S \in \sigma$ is a collection of vectors* $\mathrm{v} = \{\mathrm{v}^{ij}\}_{i,j \in S}$ *satisfying conditions:* (i) $\mathrm{v}^{ii} = 0$ *for any* $i \in N$, (ii) $\mathrm{v}^{ij} \in M_S$ *for any* $i, j \in S$, *and* (iii) $\mathrm{v}^{ij} = -\mathrm{v}^{ji}$ *for any* $i, j \in S$.

Coalition S entering into a contract v will be denoted by $S(\mathrm{v})$, as well.

Components of vector v^{ij}, appearing in definition of the contract $\mathrm{v} = \{\mathrm{v}^{ij}\}_{i,j \in S}$, indicate amount of the corresponding commodities used in the bilateral exchange between the participants $i, j \in S(\mathrm{v})$. Here nonnegative component $\mathrm{v}^{ij}_k \geqslant 0$ of v^{ij} denotes the amount of commodity k that agent j is obliged to deliver to agent i, and absolute value of negative component $\mathrm{v}^{ij}_r < 0$ measures the amount of commodity r agent i has to deliver to agent j.

As to the subsets M_S, they define admissible types of elementary exchanges within the contract $\mathrm{v} = \{\mathrm{v}^{ij}\}_{i,j \in S}$, e.g., in case $M_S := \{x \in \Re^l \mid p^S \cdot x = 0\}$ the only constraint concerning v^{ij}, $i, j \in S$ is that the bilateral exchanges v^{ij} should be equivalent w.r.t. the (fixed) prices p^S.

We call $\mathrm{v} = \{\mathrm{v}^{ij}\}_{i,j \in S}$ a proper contract, if either v is a trivial contract (i.e., $\mathrm{v}^{ij} = 0$ for all $i, j \in S(\mathrm{v})$), or for any $i \in S(\mathrm{v})$ there exist $j, k \in S(\mathrm{v})$ such that v^{ij} (v^{ik}) contains strictly positive (strictly negative) components. Note, that the properness of the contract v makes it possible (in principal) to rescind this contract by each member $i \in S(\mathrm{v})$ simply by not delivering the corresponding commodity to the participant $k = k(i)$. Any finite set $\mathcal{V} = \{\mathrm{v}_r\}_{\mathcal{R}}$, consisting of proper contracts v_r, is called a (proper) contract system of type M_σ (contractual M_σ-system, or c.s. (of type M_σ), for short). Let us stress at once that the elements of $\mathcal{R}$ are supposed to be the "titles" of any type (not necessary natural numbers), naming the contracts belonging to the set $\mathcal{V}$.

Thus, it is supposed that the members of any coalition $S \in \sigma$ may enter several contracts. Moreover, a c.s. $\mathcal{V}$ may contain several samples of the same contract v differing just by their titles (or, by their names, to be exact). To simplify the terms, we call a c.s. $\tilde{\mathcal{V}} = \{\tilde{\mathrm{v}}_r\}_{\tilde{\mathcal{R}}}$ a subsystem of c.s. $\mathcal{V} = \{\mathrm{v}_r\}_{\mathcal{R}}$ iff $\tilde{\mathcal{R}} \subseteq \mathcal{R}$, and $\tilde{\mathrm{v}}_r = \mathrm{v}_r$ for any $r \in \tilde{\mathcal{R}}$. Hence, we consider contract systems $\mathcal{V}$ and $\tilde{\mathcal{V}}$ to be identical iff $\mathcal{R} = \tilde{\mathcal{R}}$ and $\tilde{\mathrm{v}}_r = \mathrm{v}_r$ for any $r \in \mathcal{R}$.

Denote by $\Delta^i(\mathcal{V})$ the net outcome of the agent i resulting after a c.s. $\mathcal{V} = \{\mathrm{v}_r\}_{\mathcal{R}}$ is accepted and realized

$$\Delta^i(\mathcal{V}) := \begin{cases} \sum_{r \mid i \in S(\mathrm{v}_r)} \sum_{j \in S(\mathrm{v}_r)} \mathrm{v}^{ij}_r, & i \in \bigcup_{r \in \mathcal{R}} S(\mathrm{v}_r), \\ 0 & \text{otherwise.} \end{cases}$$

Since $\mathrm{v}_r^{ij} = -\mathrm{v}_r^{ji}$ for any $i,j \in S(\mathrm{v}_r)$, $r \in \mathcal{R}$, we have that $\sum_N \Delta^i(\mathcal{V}) = 0$ and, hence, $\sum_N x^i(\mathcal{V}) = \sum_N w^i$ with

$$x^i(\mathcal{V}) = x^i(\mathcal{V},\mathcal{E}) := w^i + \Delta^i(\mathcal{V})$$

to be the resulting outcome of the agent i, obtained by means of the entering into c.s. $\mathcal{V}$. In the sequel we call $x(\mathcal{V}) = x(\mathcal{V},\mathcal{E}) := (x^i(\mathcal{V},\mathcal{E}))_N$ a contractual M_σ-allocation of economy $\mathcal{E}$ (c.a. (of type M_σ), for short).

Definition 2. *A contractual system $\mathcal{V}$ is called feasible, if $x^i(\mathcal{V}) \in X_i$ for any $i \in N$.*

By definition, feasibility of a c.s. $\mathcal{V}$ guarantees the contractual M_σ-allocation $x(\mathcal{V})$ to belong to the set

$$X(N) = X_{\mathcal{E}}(N) := \{(x^i)_N \in \prod_N X_i \mid \sum_N x^i = \sum_N w^i\}$$

of attainable allocations of the economy $\mathcal{E}$.

Remind, that due to the properness, any contract $\mathrm{v} \in \mathcal{V}$ may, in principal, be broken by any participant $i \in S(\mathrm{v})$. To present a formal description of the outcomes of rescinding (breaking) some contracts v_r, $r \in \mathcal{R}'$ with $\mathcal{R}' \subseteq \mathcal{R}$, let us denote first by $A(\mathcal{V},\mathcal{R}',\mathcal{E})$ a collection of all feasible contract systems $\tilde{\mathcal{V}} = \{\mathrm{v}_r\}_{\tilde{\mathcal{R}}}$ of the model $\mathcal{E}$, satisfying requirements: (i) $\tilde{\mathcal{V}}$ is a subsystem of $\mathcal{V}$, (ii) $\tilde{\mathcal{R}} \subseteq \mathcal{R} \setminus \mathcal{R}'$, (iii) $\tilde{\mathcal{V}}$ is the maximal (by inclusion) subsystem of $\mathcal{V}$, satisfying condition (ii).

Thus, $A(\mathcal{V},\mathcal{R}',\mathcal{E})$ contains all the outcomes of the breaking (cancellation) procedure, consisting of two steps. At the first step we nullify all the contracts v_r, $r \in \mathcal{R}'$. In case $\mathcal{U} := \{\mathrm{v}_r\}_{\mathcal{R}\setminus\mathcal{R}'}$ is feasible we put $A(\mathcal{V},\mathcal{R}',\mathcal{E}) := \{\mathcal{U}\}$. Otherwise, at the second, final step, we continue to nullify the contracts with the titles belonging to some subset $\mathcal{R}''$ of $\mathcal{R}\setminus\mathcal{R}'$ in order to provide the feasibility of the subsystem $\tilde{\mathcal{U}} = \{\mathrm{v}_r\}_{\mathcal{R}\setminus(\mathcal{R}'\cup\mathcal{R}'')}$. The only requirement concerning $\mathcal{R}''$ is its minimality: according to condition (iii) we cancel as "small" collection of contracts (belonging to $\mathcal{U}$), as possible, provided that not cancelled ones constitute a feasible contractual system. Speaking differently, at the second step we choose some maximal (by inclusion) feasible subsystem of $\mathcal{V}$ with contractual titles, belonging to $\mathcal{R}\setminus\mathcal{R}'$. Since there may be either none, or several nonempty feasible subsystems of $\mathcal{V}$ of that type, the collection $A(\mathcal{V},\mathcal{R}',\mathcal{E})$ may be either trivial (i.e., consisting of the empty subsystem), or it may contain several nonempty feasible subsystems of $\mathcal{V}$.

Remark 1. Certainly, it might be reasonable to consider some other "rescinding rules," e.g., the rule, which allows to cancel the contracts from $\mathcal{U} = \{\mathrm{v}_r\}_{\mathcal{R}\setminus\mathcal{R}'}$ step by step, in accord with some ordering on $\mathcal{R}\setminus\mathcal{R}'$. This cancellation process should last until those contracts, which are not cancelled yet, constitute (for the first time) a feasible subsystem of $\mathcal{U}$. Another rules may allow either to take any feasible subsystem of $\mathcal{U}$, or to take $\mathcal{R}'$ from the very beginning in such a way that $\mathcal{U}$ is feasible (provided, probably, that $\mathcal{R}'$ satisfies some a priori given assumptions). We plan to elaborate these and some other approaches somewhere else.

To describe which way a coalition $S \in \sigma$ can improve upon a c.a. $x(\mathcal{V})$, generated by a feasible c.s. $\mathcal{V} = \{\mathrm{v}_r\}_{\mathcal{R}}$ of type M_σ, we suppose first that together with possibility to cancel some contracts v_r with $r \in \mathcal{R}^S = \mathcal{R}^S_{\mathcal{V}} := \{r \in \mathcal{R} \mid S(\mathrm{v}_r) \bigcap S \neq \emptyset\}$ it is allowed for the coalition S to enter some new contract w. Denote by $\mathcal{E}_{\mathrm{w}}$ the modification of $\mathcal{E}$ generated by w:

$$\mathcal{E}_{\mathrm{w}} = \langle N, \{X_i, w^i + \Delta^i(\mathrm{w}), \alpha_i\}_N, \sigma\rangle,$$

where, as before, $\Delta^i(\mathrm{w}) := \Delta^i(\{\mathrm{w}\})$, i.e.,

$$\Delta^i(\mathrm{w}) = \begin{cases} \sum_{j\in\in S(\mathrm{w})} \mathrm{w}^{ij}, & i \in S(\mathrm{w}), \\ & \text{otherwise}. \end{cases}$$

Definition 3. *We say that a coalition $S \in \sigma$ improves (blocks) a feasible contractual M_σ-system $\mathcal{V} = \{\mathrm{v}_r\}_{\mathcal{R}}$, if there exists a subset $\mathcal{R}' \subseteq \mathcal{R}^S$, a proper contract* w *of type M_S, and a system $\mathcal{V}' \in A(\mathcal{V}, \mathcal{R}', \mathcal{E}_{\mathrm{w}})$ such that* (i) *$S(\mathrm{w}) = S$, and* (ii) *$x^i(\mathcal{V}, \mathcal{E})\alpha_i x^i(\mathcal{V}', \mathcal{E}_{\mathrm{w}})$ for any $i \in S$, with $x^i(\mathcal{V}', \mathcal{E}_{\mathrm{w}}) \in \mathcal{P}_i(x^i(\mathcal{V}, \mathcal{E}))$ for at least one $i \in S$.*

(Here and below, as usual, $\mathcal{P}_i(z) := \{x^i \in \alpha_i(z) \mid (x^i, z) \notin \alpha_i\}$ is the set of those bundles from X_i, which are strictly preferred to z.)

Definition 4. *A feasible contractual M_σ-system $\mathcal{V}$ is called quasi-stable, if there is no coalition $S \in \sigma$, which improves $\mathcal{V}$.*

For any $x \in X(N)$ denote by $\mathcal{W}(x) = \mathcal{W}_{M_\sigma}(x)$ the set of all feasible c.s. $\mathcal{V}$ of type M_σ such that $x = x(\mathcal{V})$. To isolate the allocations $x \in X(N)$ with $\mathcal{W}_{M_\sigma}(x) \neq \emptyset$, whose coalitional stability is independent on the concrete representation via some c.s. $\mathcal{V}$ of type M_σ, we introduce

Definition 5. *An allocation $x \in X(N)$ is called a totally contractual M_σ-allocation (t.c.a. (of type M_σ), for short), if $\mathcal{W}_{M_\sigma}(x)$ is nonempty* *set, and it contains only quasi-stable contractual systems (i.e., for every $\mathcal{V} \in \mathcal{W}_{M_\sigma}(x)$ it holds: $\mathcal{V}$ is quasi-stable).*

The set $D_0^{M_\sigma}(\mathcal{E})$ of all the totally contractual M_σ-allocations of $\mathcal{E}$ is called the totally contractual set (of type M_σ) of economy $\mathcal{E}$.

Note that any coalitional structure σ admits the (unique) representation as the union of (locally) irreducible coalitional substructures, inscribed into the corresponding components of N. Therefore in what follows we suppose σ to be irreducible itself (recall [3], that coalitional structure σ is called irreducible, if for any nontrivial partition $\{N_1, N_2\}$ of N there is $S \in \sigma$ such that $S \bigcap N_i \neq \emptyset$, $i = 1, 2$; as usual, a partition $\{N_1, N_2\}$ is called *nontrivial*, if $N_1, N_2 \neq \emptyset$). Besides, everywhere below it is assumed that all the subsets M_S are identical and equal to some linear subspace $M \subseteq \Re^l$ satisfying the following sign-assumption: every nonzero $z \in M$ contains both strictly positive and strictly negative components.

Remark 2. Observe, that from the bipolar theorem and Minkovski separation theorem it follows immediately that a linear subspace $M \neq \Re^l$ satisfies the above mentioned sign-assumption iff its polar subspace $M^0 := \{p \in \Re^l \mid p \cdot z = 0, z \in M\}$ meets the requirement

$$M^0 \bigcap \operatorname{int} \Re^l_+ \neq \emptyset. \tag{2}$$

It is clear that in this case, there exists a nonempty finite subset $P_M \subseteq \Re^l$ containing at least one strictly positive vector, such that $M = \{z \in \Re^l \mid p \cdot z = 0,\ p \in P_M\}$. Hence, the economical meaning of the constraint imposed on the type of the contracts we consider below is as follows: elementary exchanges bundles v^{ij} should be compatible with all the fixed-price vectors p, belonging to some nonempty finite subset $P_M \subseteq \Re^l$, containing at least one strictly positive price vector $\bar{p}$.

Under sign-assumption it can easily be shown also, that for any $S \subseteq N$ with $|S| \geqslant 3$ there exists a proper "zero-contract" w of type M (i.e., a proper contract $\mathrm{w} \neq 0$ of type M such that (i) $S(\mathrm{w}) = S$, and (ii) $\Delta^i(\mathrm{w}) = 0$ for any $i \in S$). It means that the properness of any contract v of type $M \neq \{0\}$ with $|S(\mathrm{v})| \geqslant 3$ is guaranteed "automatically:" $\mathrm{u} := \mathrm{v} + \lambda\ \mathrm{w}$ is a proper contract of type M with $S(\mathrm{u}) = S(\mathrm{v})$ and $\Delta^i(\mathrm{u}) = \Delta^i(\mathrm{v})$, $i \in N$, provided that $\mathrm{w} \neq 0$ is a proper "zero-contract" of type M with $S(\mathrm{w}) = S(\mathrm{v})$, and $\lambda > 0$ is large enough. Hence, everywhere below we may (and will) deal with some contracts and contract systems of type M, not necessarily satisfying properness-assumption for not-two-person coalitions (at least, in those situations, where only properness "modulo zero-contract" matters).

Denote by $\mathcal{W}_M$ the collection of all c.s. of type M_σ with $M_S = M$ for any $S \in \sigma$. In what follows, the contracts v of type $M_S = M$, as well as the systems $\mathcal{V} \in \mathcal{W}_M$, and allocations $x = x(\mathcal{V})$, will be called M-contracts, M-systems and M-allocations, respectively. To present more detailed description of some undominated (in the sense of Definition 3) M-allocations $x(\mathcal{V})$, we characterize first all feasible allocations, attainable by entering into the contract systems of type M. Put

$$M_{\mathcal{E}}(N) := \{x \in \prod_N X_i \mid \exists \mathcal{V} \in \mathcal{W}_M : x = x(\mathcal{V})\}.$$

Proposition 1. *If σ is irreducible coalition structure, then*

$$M_{\mathcal{E}}(N) = \{x \in X(N) \mid \exists \Delta \in \Delta_M(N) : x = w + \Delta\},$$

where $\Delta_M(N) := \{\Delta = (\Delta^i)_N \in M^N \mid \sum_N \Delta^i = 0\}$, $w := (w^i)_N$.

Proof. If $x \in M_{\mathcal{E}}(N)$, then $x = x(\mathcal{V}) = w + \Delta(\mathcal{V})$ for some $\mathcal{V} \in \mathcal{W}_M$, where $\Delta(\mathcal{V}) := (\Delta^i(\mathcal{V}))_N$. By definition of M-contract it follows that $\Delta^i(\mathcal{V}) \in M$, $\sum_N \Delta^i(\mathcal{V}) = 0$ and, consequently, we get desired: $x = w + \Delta$ for some $\Delta \in \Delta_M(N)$.

Let now $x = w + \Delta$ for some $\Delta \in \Delta_M(N)$. To prove that there is $\mathcal{V} \in \mathcal{W}_M$ such that $\Delta = \Delta(\mathcal{V})$, we apply induction on the number $|N|$ of the economical agents of $\mathcal{E}$. It is clear, that in case $|N| = 2$ we have : $\mathcal{V} = \{\mathrm{v}\}$, with $\mathrm{v}^{12} := \Delta^1$ and $\mathrm{v}^{21} := \Delta^2$. Suppose that our assertion is valid for all economies of type (1) with $|N| \leqslant m$. Fix some economy $\mathcal{E}$ with $|N| = |N_{\mathcal{E}}| = m + 1$. If $N \in \sigma$, then, modulo "zero-contract," according to Remark 2, a c.s. required consists of the only M-contract v, defined by the formula: $\mathrm{v}^{ii+1} := \sum_{k=1}^{i} \Delta^k$, $i = 1, \dots, m$; $\mathrm{v}^{ij} = 0$, $j - i > 1$ (note that due to the equalities $\mathrm{v}^{ij} = -\mathrm{v}^{ji}$ to give a complete description of the contract v it is sufficient to indicate all the vectors v^{ij} with $i, j \in S(\mathrm{v})$ satisfying inequalities $i < j$).

Consider the case $N \notin \sigma$. Entering into the trivial contracts for some coalitions $S \in \sigma$, if necessary, we may assume without loss of generality, that σ is minimal (w.r.t. inclusion in 2^N) irreducible coalitional structure. Fix some $S^0 \in \sigma$. It is not hard to prove that there exists a partition $\eta^0 = \{N_1^0, N_2^0\}$ of N such that (i) coalitions $S_k^0 := S^0 \bigcap N_k^0$, $k = 1, 2$, are nonempty; (ii) for any $T \in \sigma \setminus \{S^0\}$ either $T \subseteq N_1^0$, or $T \subseteq N_2^0$; and (iii) coalitional structures $\sigma_k^0 := \{T \in \sigma \mid T \subseteq N_k^0\} \bigcup \{S_k^0\}$, $k = 1, 2$, are irreducible in N_1^0, N_2^0, respectively. Indeed, existence of a partition η^0, satisfying conditions (i) and (ii), follows immediately from the minimality of the irreducible coalitional structure σ (since in case $\sigma \setminus \{T\}$ is irreducible for some $T \in \sigma$ we get a contradiction). As to the irreducibility of σ_1^0, for example, it can easily be shown by consideration of three cases for any nontrivial partition $\{N_1^1, N_1^2\}$ of N_1^0: 1) $S^0 \bigcap N_1^k \neq \emptyset$, $k = 1, 2$; 2) $S^0 \bigcap N_1^1 \neq \emptyset$, $S^0 \bigcap N_1^2 = \emptyset$; and 3)$S^0 \bigcap N_1^1 = \emptyset$, $S^0 \bigcap N_1^2 \neq \emptyset$. Specifically, considering partitions $\{N_1^2, N_1^1 \bigcup N_2\}$ and $\{N_1^1, N_1^2 \bigcup N_2\}$ in cases 2) and 3), respectively, we get required coalitions from σ_1^0 on the basis of condition (ii). In fact, since in both these cases coalition S^0 doesn't intersect one of the elements of the corresponding nontrivial partition of N, by (ii) and irreducibility of σ, in each of these cases there exists a coalition $S \in \sigma_1^0$ such that $S \bigcap N_1^k \neq \emptyset$ for any $k = 1, 2$.

Now, by making use of the partition η^0, we divide initial economy $\mathcal{E}$ into two appropriate "smaller" economies $\mathcal{E}_{(k)} := \langle N_{(k)}, X_i^{(k)}, w_{(k)}^i, \alpha_i^{(k)}\}_{N_{(k)}}, \sigma_{(k)}\rangle$, $k = 1, 2$, satisfying inductive hypothesis and "implementing" the allocation $x = w + \Delta$ with $\Delta = (\Delta^1, \dots, \Delta^n) \in \Delta_M(N)$ by means of some contractual M-system (as a result of a suitable exchange, both within and between these economies). To this end we fix some economic agents $i_k \in S_k^0$, $k = 1, 2$, and put $N_{(k)} := N_k^0$, $k = 1, 2$; $w_{(k)}^i := w^i$, $i \in N_{(k)}$, $k = 1, 2$, and $X_i^{(k)} := X_i$ for any $i \in N_{(k)} \setminus \{i_k\}$, $k = 1, 2$. As to the consumption sets $X_{i_1}^{(1)}$ and $X_{i_2}^{(2)}$, we put $X_{i_1}^{(1)} := \{w^{i_1} - \sum_{i \in N_{(1)} \setminus \{i_1\}} \Delta^i\}$, and $X_{i_2}^{(2)} := \{w^{i_2} + \sum_{i \in N_{(1)} \cup \{i_2\}} \Delta^i\}$. Finally, we take $\sigma_{(k)}$ to be equal to σ_k^0, $k = 1, 2$, and put $\alpha_i^{(k)} := \alpha_i$ for any $i \in N_{(k)}$ and $k = 1, 2$.

To apply inductive hypothesis, take $\Delta_{(k)} \in \Delta_M(N_{(k)})$, $k = 1, 2$ with

$$\Delta_{(1)}^{i_1} := -\sum_{i \in N_{(1)} \setminus \{i_1\}} \Delta^i, \quad \Delta_{(2)}^{i_2} := \sum_{i \in N_{(1)} \cup \{i_2\}} \Delta^i,$$

and $\Delta_{(k)}^i := \Delta^i$ for any $i \in N_{(k)} \setminus \{i_k\}$, $k = 1, 2$. Further, for each $k = 1, 2$, put $w_{(k)} := (w^i)_{N_{(k)}}$ and $\Delta_{(k)} := (\Delta_{(k)}^i)_{N_{(k)}}$. It is clear, that $x_{(1)} := w_{(1)} + \Delta_{(1)}$ and $x_{(2)} := w_{(2)} + \Delta_{(2)}$ are attainable allocations of the economies $\mathcal{E}_{(1)}, \mathcal{E}_{(2)}$, respectively. Hence, considerations, given above, implies: economies $\mathcal{E}_{(1)}, \mathcal{E}_{(2)}$, and corresponding allocations $x_{(1)}, x_{(2)}$ satisfy our inductive hypothesis, and, consequently, there exist M-systems $\mathcal{V}_{(1)}$ and $\mathcal{V}_{(2)}$ such that $x_{(k)} = x(\mathcal{V}_{(k)}, \mathcal{E}_{(k)})$, each $k = 1, 2$.

Now we design a contract $\mathrm{v}_{(S^0)}$, which coalition S^0 should enter to in order to organize (together with the M-systems $\mathcal{V}_{(1)}$ and $\mathcal{V}_{(2)}$) an

M-system $\mathcal{V}$, satisfying equality $x = x(\mathcal{V}) = x(\mathcal{V}, \mathcal{E})$. To do so, we put $\mathrm{v}^{i_1 i_2}_{(S^0)} := \sum_{i \in N_{(1)}} \Delta^i$, $\mathrm{v}^{i_2 i_1}_{(S^0)} := -\sum_{i \in N_{(1)}} \Delta^i$, and $\mathrm{v}^{ij}_{(S^0)} := 0$ for any $i, j \in S^0$ such that $\{i, j\} \neq \{i_1, i_2\}$. It is clear, that the system $\mathcal{V} := \mathcal{V}_{(1)} \bigcup \mathcal{V}_{(2)} \bigcup \{\mathrm{v}_{(S^0)}\}$, with $\mathrm{v}_{(S^0)}$ thus defined (and modified in the spirit of of Remark 2, if necessary), meets our requirement, which proves the proposition. □

To characterize opportunities available to some coalition $S \in \sigma$ for to improve the allocations from $M_{\mathcal{E}}(N)$, we describe first increments of the initial coalitional endowment $\sum_S w^i$, provided by the breaking contracts belonging to some c.s. $\mathcal{V} \in \mathcal{W}_M$. Put

$$\mathcal{A}(\mathcal{V}, \mathcal{R}', S) := \bigcup_{\mathrm{w} \mid S(\mathrm{w}) = S} A(\mathcal{V}, \mathcal{R}', \mathcal{E}_{\mathrm{w}}),\ \mathcal{A}(\mathcal{V}, S) := \bigcup_{\mathcal{R}' \subseteq \mathcal{R}^S} \mathcal{A}(\mathcal{V}, \mathcal{R}', S).$$

Then all the increments, mentioned above, may be presented as the vectors $\Delta(\mathcal{V}', S) := \sum_S \Delta^i(\mathcal{V}')$ with $\mathcal{V}' \in \mathcal{A}(\mathcal{V}, S)$. Note, that from the very definition of $\mathcal{A}(\mathcal{V}, S)$ it follows that $x_S(\mathcal{V}')$ belongs to $\prod_{i \in S} X_i$ and, hence, $\sum_S w^i + \Delta(\mathcal{V}', S) \in \sum_S X_i$ for any $\mathcal{V}' \in \mathcal{A}(\mathcal{V}, S)$ (as usual, x_S is a restriction of $x = (x^1, \ldots, x^n)$ to S: $x_S = (x^i)_S$ for any $S \subseteq N$).

As to the set $\mathcal{X}_{\mathcal{V}}(S)$ of coalitional allocations available to $S \in \sigma$ by means of breaking procedures, applied to some M-system $\mathcal{V}$, it is described as follows

$$\mathcal{X}_{\mathcal{V}}(S) = \{z \in \prod_S X_i^M \mid \exists \mathcal{V}' \in \mathcal{A}(\mathcal{V}, S) : z = w_S + \Delta_S(\mathcal{V}')\},$$

where $X_i^M := X_i \bigcap (w^i + M)$, $i \in N$, and, as above, $w_S = (w^i)_S$, $\Delta_S(\mathcal{V}') = (\Delta^i(\mathcal{V}'))_S$. To describe all the blocking possibilities of a coalition S at a state (allocation) $x \in M_{\mathcal{E}}(N)$, we have to take into account all the sets $\mathcal{X}_{\mathcal{V}}(S)$, $\mathcal{V} \in \mathcal{W}(x)$.

Observe that the unions of these sets, $Z_x(S) := \bigcup_{\mathcal{V} \in \mathcal{W}(x)} \mathcal{X}_{\mathcal{V}}(S)$, are independent on the choice of c.a. x.

Proposition 2. *For any $x, \tilde{x} \in M_{\mathcal{E}}(N)$ and $S \in \sigma$ it holds $Z_x(S) = Z_{\tilde{x}}(S)$.*

Proof. Fix some $S \in \sigma$, x, $\tilde{x} \in M_{\mathcal{E}}(N)$ and demonstrate that $Z_x(S) \subseteq Z_{\tilde{x}}(S)$. To this end consider some nonempty contractual systems $\mathcal{V} = \{\mathrm{v}_r\}_{\mathcal{R}} \in \mathcal{W}(x)$ and $\mathcal{V}_0 = \{\mathrm{u}_r\}_{\mathcal{R}_0} \in A(\mathcal{V}, \mathcal{R}', \mathcal{E}_{\mathrm{w}}) \subseteq \mathcal{A}(\mathcal{V}, S)$, where w is an M-contract of the coalition S, and $\mathcal{R}'$ is some subset of $\mathcal{R}^S$. To show there is a c.s. $\mathcal{V}' \in \mathcal{W}(\tilde{x})$ yielding (after rescinding some suitable contracts from $\mathcal{V}'$) the same coalitional allocation for S, as $\mathcal{V}_0$ does, we consider first an arbitrary c.s. $\tilde{\mathcal{V}} = \{\tilde{\mathrm{v}}_r\}_{\tilde{\mathcal{R}}} \in \mathcal{W}(\tilde{x})$. Since M is a linear subspace, satisfying sign-assumption, we may assume w.l.g. (due to Remark 2), that $\tilde{\mathcal{R}} = \mathcal{R}_0$ with $S(\tilde{\mathrm{v}}_r) = S(\mathrm{u}_r)$ and $\tilde{\mathrm{v}}_r^{ij} \neq \mathrm{u}_r^{ij}$ for any $r \in \mathcal{R}_0$ and $i, j \in S(\mathrm{u}_r)$ such that $i \neq j$. Put $\mathcal{V}' := \{\mathrm{v}_{(r,0)}\}_{r\in\mathcal{R}_0} \bigcup \{\mathrm{v}_{(r,1)}\}_{r\in\mathcal{R}_0}$, where $\mathrm{v}_{(r,0)}^{ij} := \mathrm{u}_r^{ij}$, $\mathrm{v}_{(r,1)}^{ij} := \tilde{\mathrm{v}}_r^{ij} - \mathrm{u}_r^{ij}$. Directly by construction of $\mathcal{V}'$ we get $\Delta(\mathcal{V}') = \Delta(\tilde{\mathcal{V}})$. Hence, $\mathcal{V}'$ belongs to $\mathcal{W}(\tilde{x})$. Put $\mathcal{R} := \{(r,1) \mid r \in \mathcal{R}_0^S\}$ and denote by $\mathcal{R}_1$ some maximal subset of $\mathcal{R}_0 \setminus \mathcal{R}_0^S$, satisfying requirement: c.s. $\tilde{\mathcal{V}}_0 := \{\mathrm{v}_{(r,0)}\}_{r\in\mathcal{R}_0} \bigcup \{\mathrm{v}_{(r,1)}\}_{r\in\mathcal{R}_1}$ belongs to $A(\mathcal{V}', \mathcal{R}, \mathcal{E}_{\mathrm{w}})$ (note, that it can happen $\mathcal{R}_1$ to be empty set, since it may be necessary to dissolve all the contracts $\mathrm{v}_{(r,1)}$, $r \in \mathcal{R}_0 \setminus \mathcal{R}_0^S$, at the second step). Obviously, by definition of $\tilde{\mathcal{V}}_0$ we have the equality $\Delta_S(\tilde{\mathcal{V}}_0) = \Delta_S(\mathcal{V}_0)$. But this means that rescinding the contracts of c.s. $\mathcal{V}' \in \mathcal{W}(\tilde{x})$ with the "names" from $\mathcal{R}$ makes it possible for the coalition S to get the same coalitional allocation, as the realization of $\mathcal{V}_0 \in A(\mathcal{V}, \mathcal{R}', \mathcal{E}_{\mathrm{w}})$ does at the state x.

Due to the arbitrariness of allocations x, $\tilde{x} \in M_N(\mathcal{E})$, M-systems $\mathcal{V} \in \mathcal{W}(x)$, $\mathcal{V}_0 \in A(\mathcal{V}, \mathcal{R}', \mathcal{E}_{\mathrm{w}})$ with $\mathcal{R}' \subseteq \mathcal{R}^S$, and M-contract w, the argumentation given above proves Proposition 2. □

Thus, the set of coalitional allocations, accessible by coalition $S \in \sigma$ at the current state $x \in M_{\mathcal{E}}(N)$ as a result of breaking procedure, depends neither on M-allocation x, nor on $\mathcal{W}(x)$. According to Propositions 1 and 2 this set is determined completely by the cooperative game (in strategic form): $S \mapsto G_{\mathcal{E}}^M(S)$, $S \in \sigma \bigcup \{N\}$, where

$$G_{\mathcal{E}}^M(S) := Pr_S M_{\mathcal{E}}(N).$$

Definition 6. *We say, that an allocation $x \in M_{\mathcal{E}}(N)$ is M-dominated by an allocation $y \in M_{\mathcal{E}}(N)$, if there exists a coalition $S \in \sigma$ such that $x^i \alpha_i y^i$ for every $i \in S$ and, besides, $y^i \in \mathcal{P}_i(x^i)$ for at least one $i \in S$.*

The core of the M-domination relation, introduced above (i.e., the collection of all the allocations $x \in M_{\mathcal{E}}(N)$ that are not M-dominated by any allocation from $M_{\mathcal{E}}(N)$), will be called the contractual M-core of the economy $\mathcal{E}$ and denoted by $C_0^M(\mathcal{E})$. Observe, that from Propositions 1 and 2, and from the very definitions of the contractual M-core and totally contractual set $D_0^M(\mathcal{E})$ of the economy $\mathcal{E}$ it follows

Proposition 3. *For any exchange model $\mathcal{E}$ with irreducible coalitional structure σ, and for any linear subspace M satisfying sign-assumption it* *holds*

$$D_0^M(\mathcal{E}) = C_0^M(\mathcal{E}).$$

2. Totally contractual set and equilibrium allocations.

Reduction, realized by Proposition 3, makes it possible to investigate the existence conditions, as well as to present an equilibrium characterization of the totally contractual allocations of a rather broad class of exchange models by means of conventional tools of cooperative game theory.

Below, we restrict ourselves to the case, when M is a hypersubspace of $\Re^l$ (i.e., we assume that $\dim M = l - 1$). In this situation it turns out that any t.c. allocation is an equilibrium allocation under fairly natural regularity assumptions. Vice versa, under essentially weaker assumptions any equilibrium allocation turns out to be a t.c. allocation, provided that a subspace M is properly chosen. Recall, that $\bar{x} \in X(N)$ is called an equilibrium allocation, if there exists a nonzero vector $\bar{p} \in \Re^l$ such that

$$\mathcal{P}_i(\bar{x}^i) \bigcap B_i(\bar{p}) = \emptyset \tag{3}$$

for any $i \in N$, where $B_i(\bar{p}) := \{x^i \in X_i \mid \bar{p} \cdot x^i \leqslant \bar{p} \cdot w^i\}$ is the budget set of an agent i at prices $\bar{p}$.

Denote by $W(\mathcal{E})$ the set of all equilibrium allocations of economy $\mathcal{E}$. As usual, any vector $\bar{p}$, satisfying (3), is called an equilibrium price vector, supporting the allocation $\bar{x}$.

Theorem 1. *Let $\bar{x}$ be an equilibrium allocation of the economy $\mathcal{E}$. If there is a strictly positive equilibrium price vector $\bar{p}$, supporting $\bar{x}$, then $\bar{x} \in D_0^M(\mathcal{E})$ with $M := \{z \in \Re^l \mid \bar{p} \cdot z = 0\}$.*

Proof. Since $\bar{x} \in W(\mathcal{E})$ implies inclusion $\bar{x} \in X(N) \bigcap \prod_N B_i(\bar{p})$, we have: $\Delta^i := \bar{x}^i - w^i$ belongs to M for any $i \in N$. It is clear, that irreducibility of σ and Proposition 1 imply: $\bar{x} = w + \Delta \in M_{\mathcal{E}}(N)$, where, as before, $w = (w^i)_N$, and $\Delta = (\Delta^i)_N$. Suppose, the allocation $\bar{x}$ is M-dominated by some $x \in M_{\mathcal{E}}(N)$. Then, by definition of M-domination, there exist $S \in \sigma$ and $i \in S$ such that $x^i \in \mathcal{P}_i(\bar{x}^i)$. By definition of equilibrium allocation, the latter inclusion implies the inequality $\bar{p} \cdot x^i > \bar{p} \cdot w^i$, which contradicts to the inclusion $x^i \in X_i^M$ yielding $\bar{p} \cdot x^i = \bar{p} \cdot w^i$.

Thus, $\bar{x} \in C_0^M(\mathcal{E})$, which means (according to Proposition 3) that the allocation $\bar{x}$ is a t.c. M-allocation of the exchange model $\mathcal{E}$. □

To prove the reverse inclusion $D_0^M(\mathcal{E}) \subseteq W(\mathcal{E})$ we have to add some compatibility assumptions concerning the coalitional structure σ (as well, as traditional convexity and monotonicity assumptions, imposed on the consumption sets X_i and individual preference relations α_i). Namely, it is supposed that for any $i \in N$ it holds: (i) X_i is a convex set; (ii) α_i is a reflexive binary relation; (iii) for any $x^i \in \widehat{X}_i := \mathrm{Pr}_{\{i\}} X(N)$ there exists $z \in \mathrm{int}\Re^l_+$ such that $x^i + z \in \mathcal{P}_i(x^i)$; (iv) for any $x^i \in X_i$ the set $\mathcal{P}_i(x^i)$ is convex and, besides, $(x^i, y^i] \subseteq \mathcal{P}_i(x^i)$ for any $y^i \in \mathcal{P}_i(x^i)$, where $(x^i, y^i] := \{(1-t)x^i + ty^i \mid t \in (0,1]\}$.

Recall [3], that a subset $T \subseteq N$ is said to be σ-divisible, if for any $i \in N$ there is $S \in \sigma_i$ such that $T \bigcap (N \setminus S) \neq \emptyset$ (as before, $\sigma_i := \{S \in \sigma \mid i \in S\}$). Further, for any $x \in M_{\mathcal{E}}(N)$ put $N_x := \{i \in N \mid x^i \in \mathrm{int}_M X_i\}$, where $\mathrm{int}_M X_i$ is the (relative) interior of X_i^M in the affine subspace $w^i + M$.

The main result of this subsection is as follows.

Theorem 2. *Suppose $\bar{x}$ belongs to $D_0^M(\mathcal{E})$ for some M satisfying sign-assumption. If $N_{\bar{x}}$ is a σ-divisible subset of N, then $\bar{x}$ is an equilibrium allocation of $\mathcal{E}$.*

Proof. Let $\bar{x} \in D_0^M(\mathcal{E})$ and i be any agent of the economy $\mathcal{E}$. In order to show that $\bar{x}^i$ is a local maximum on X_i^M w.r.t. the binary relation α_i, we choose first some coalition $S_0 \in \sigma_i$ such that $k \notin S_0$ for some $k \in N_{\bar{x}}$ (recall that $N_{\bar{x}}$ is a σ-divisible set). Since $\bar{x}^k \in \mathrm{int}_M X_k$, there is a symmetric neighbourhood of zero in M, say U, such that $\bar{x}^k + U \subseteq X_k^M$. Suppose, there exists $z \in U$ satisfying inclusion $\bar{x}^i + z \in \mathcal{P}_i(\bar{x}^i)$. By Proposition 1, the allocation $y \in X(N)$, defined by the formula: $y^k := \bar{x}^k - z$, $y^i := \bar{x}^i + z$, $y^j := \bar{x}^j$ $(j \neq i, k)$, belongs to $M_{\mathcal{E}}(N)$. But directly from the construction of y and definition of M-domination it follows that $\bar{x} \notin C_0^M(\mathcal{E})$. Indeed, since α_j is a reflexive binary relation for any $j \in N$, we have that the allocation y M-dominates $\bar{x}$ via coalition S_0. Because the latter contradicts to Proposition 3, we get, finally, $\mathcal{P}_i(\bar{x}^i) \bigcap (\bar{x}^i + U) \bigcap X_i = \emptyset$. Therefore, the intersection $\mathcal{P}_i(\bar{x}^i) \bigcap X_i^M$ is empty, as well. To check the latter assertion, just mention, that for any $y^i \in \mathcal{P}_i(\bar{x}^i) \bigcap X_i^M$ (if such y^i exists) the bundle $\bar{z}(t) := ty^i + (1-t)\bar{x}^i$ belongs to the neighbourhood $\bar{x}^i + U$ of $\bar{x}^i$ for any $t \in (0,1)$ small enough. But this fact (together with the convexity assumption (iv)) contradicts to the local maximality of $\bar{x}^i$, which has been already proven above.

Summarizing, we have that for every $i \in N$ the consumption bundle $\bar{x}^i$ is maximal on X_i^M w.r.t. α_i. Fix now some $\bar{p} \in M^0 \bigcap \mathrm{int}\Re^l_+$ and prove that $\mathcal{P}_i(\bar{x}^i) \bigcap B_i(\bar{p}) = \emptyset$ for any $i \in N$. Let $y^i \in \mathcal{P}_i(\bar{x}^i)$ for some $i \in N$. Since $\bar{p} \cdot x^i = \bar{p} \cdot w^i$ for any $x^i \in X_i^M$ (in particular, $\bar{p} \cdot \bar{x}^i = \bar{p} \cdot w^i$, each

$i \in N$), from the maximality of $\bar{x}^i$ on X_i^M it follows that $\bar{p} \cdot y^i \neq \bar{p} \cdot w^i$. Suppose, that $\bar{p} \cdot y^i < \bar{p} \cdot w^i$. Because of the inclusion $\bar{x} \in X(N)$ we have, due to the monotonicity assumption (iii): there is $z \in \mathrm{int}\Re^l_+$ such that $\bar{y}^i := \bar{x}^i + z \in \mathcal{P}_i(\bar{x}^i)$. It is clear, that $\bar{p} \cdot \bar{y}^i > \bar{p} \cdot w^i$. But then there exists $\bar{t} \in (0,1)$ such that the bundle $z(\bar{t}) := \bar{t}y^i + (1-\bar{t})\bar{y}^i$ belongs to X_i^M. On the other hand, due to the convexity of $\mathcal{P}_i(\bar{x}^i)$ we have: $z(\bar{t}) \in \mathcal{P}_i(\bar{x}^i)$, which contradicts to the maximality of $\bar{x}^i$ on X_i^M w.r.t. α_i.

Contradiction obtained proves the inequality $\bar{p} \cdot y^i > \bar{p} \cdot w^i$, which implies the required relation: $y^i \notin B_i(\bar{p})$. □

Note that the cardinality of $N_{\bar{x}}$ may be very small. It is clear, however, that σ-divisibility condition takes the simplest form in case $N_{\bar{x}} = N$.

Corollary 1. *If $\sigma \setminus \{N\}$ is a covering system (i.e., $\bigcup_{S\in\sigma | S\neq N} S = N$), then $D_{00}^M(\mathcal{E}) \subseteq W(\mathcal{E})$, where*

$$D_{00}^M(\mathcal{E}) := D_0^M(\mathcal{E}) \bigcap X_0^M,\ X_0^M := \prod_N \mathrm{int}_M X_i.$$

The results obtained allows us to find some rather large classes of exchange models, admitting an equilibrium characterization for any t.c. allocation $\bar{x}$, independently on the structure of the set $N_{\bar{x}}$. To give some examples, introduce first additional characteristics of the exchange model $\mathcal{E}$. Denote by $\mathcal{M}$ the set of all hyperspaces $M \subseteq \Re^l$ satisfying sign-assumption, and for any $M \in \mathcal{M}$ put

$$S_{\mathcal{E}}^M := \\ := \left\{ i \in N \mid \alpha_i \text{ is complete, transitive, and } \alpha_i(w^i) \bigcap X_i^M \subseteq \mathrm{int}_M X_i \right\}.$$

Definition 7. *An exchange model $\mathcal{E}$ is called C_M-regular, if $\alpha_i(w^i) \bigcap G_{\mathcal{E}}^M(\{i\}) \neq \emptyset$ for any $i \in S_{\mathcal{E}}^M$.*

Definition 8. *An exchange model $\mathcal{E}$ is called $C_{\mathcal{M}}$-regular, if it is C_M-regular for any $M \in \mathcal{M}$.*

By applying, mutatis mutandis, argumentation used in the proof of Theorem 2 we obtain the following result.

Theorem 3. *Let $\mathcal{E}$ be C_M-regular exchange model. If $S_{\mathcal{E}}^M$ is σ-divisible and, besides, $\{i\} \in \sigma$ for any $i \in S_{\mathcal{E}}^M$, then $D_0^M(\mathcal{E}) \subseteq W(\mathcal{E})$.*

Below, we demonstrate some of the quite clear implications of Theorem 3.

Corollary 2. *If $\mathcal{E}$ is C_M-regular, $|S^M_{\mathcal{E}}| \geqslant 2$, and $\{i\} \in \sigma$ for any $i \in N$, then every totally contractual M-allocation is an equilibrium allocation.*

Corollary 3. *Let w^i belong to X_i for any $i \in N$. If $S^M_{\mathcal{E}}$ is σ-divisible and, besides, $\{i\} \in \sigma$ for any $i \in S^M_{\mathcal{E}}$, then every totally contractual M-allocation is an equilibrium allocation.*

Corollary 4. *If $\mathcal{E}$ is $C_{\mathcal{M}}$-regular, $S^M_{\mathcal{E}}$ is σ-divisible for any $M \in \mathcal{M}$, and $\{i\} \in \sigma$ for every $i \in \bigcup_{M\in\mathcal{M}} S^M_{\mathcal{E}}$, then $D_0(\mathcal{E}) \subseteq W(\mathcal{E})$, where*

$$D_0(\mathcal{E}) := \bigcup_{M\in\mathcal{M}} D^M_0(\mathcal{E}).$$

To present a more easy verifiable regularity condition, similar to that given in Definition 8, we introduce another characteristic of the exchange model under consideration:

$$S_{\mathcal{E}} := \{i \in N \mid \alpha_i \text{ is complete, transitive, and } \alpha_i(w^i) \bigcap \check{X}_i \subseteq \operatorname{int} X_i\},$$

where $\check{X}_i := X_i \setminus (w^i + \operatorname{int}\Re^l_+),\ \ i \in N$.

Definition 9. *An exchange model $\mathcal{E}$ is called C-regular, if $\alpha_i(w^i) \bigcap G^M_{\mathcal{E}}(\{i\}) \neq \emptyset$ for any $i \in S_{\mathcal{E}}$ and $M \in \mathcal{M}$.*

It is clear, that a slight modification of the proof of Theorem 3 and Corollaries 2–4 yields the following analogs of Corollary 4.

Theorem 4. *If $\mathcal{E}$ is C-regular, $S_{\mathcal{E}}$ is σ-divisible, and $\{i\} \in \sigma$ for every $i \in S_{\mathcal{E}}$, then $D_0(\mathcal{E}) \subseteq W(\mathcal{E})$.*

Corollary 5. *If $\mathcal{E}$ is C-regular, $|S_{\mathcal{E}}| \geqslant 2$, and $\{i\} \in \sigma$ for any $i \in N$, then $D_0(\mathcal{E}) \subseteq W(\mathcal{E})$.*

Corollary 6. *Let w^i belong to X_i for any $i \in N$. If $S_{\mathcal{E}}$ is σ-divisible and, besides, $\{i\} \in \sigma$ for any $i \in S_{\mathcal{E}}$, then $D_0(\mathcal{E}) \subseteq W(\mathcal{E})$.*

As usual, we say that α_i is locally monotonic, if for any $x^i \in \operatorname{Pr}_{\{i\}} X(N)$ there exists $\delta(x^i) > 0$ such that $x^i + z \in \mathcal{P}_i(x^i)$, whenever $z \in \Re^l_+$ and $0 < \|z\| < \delta(x^i)$. Taking account that local monotonicity guarantees equilibrium prices to be strictly positive, and summarizing the results, which has been proven above, we obtain the following core equivalence result.

Theorem 5. *Suppose $\mathcal{E}$ satisfies the assumptions of either Theorem 4, or one of the Corollaries 4–6. If α_i is locally monotonic for at least one agent of the economy $\mathcal{E}$, then $D_0(\mathcal{E}) = W(\mathcal{E})$.*

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