Записки научных
семинаров ПОМИ
Том 291, 2002 г.

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# VECTOR PARTICLES IN QUASILOCAL QUARK MODELS

Abstract. We consider the Quasilocal Quark Model of NJL type including vector and axial-vector four-fermion interaction with derivatives. The mass spectrum for the ground and first excited states is obtained. The chiral symmetry restoration sum rules in these channels are imposed as matching rules to QCD at intermediate energies and a set of constraints on parameters of QQM is performed.

## 1. Introduction and motivation

The main purpose of this paper is to describe the physics of as ground vector (V) and axial-vector (A) meson resonances at low energies as excited states with increasing masses at intermediate energies in the framework of effective action of Quasilocal Quark Models (QQM) which were introduced in [1]. We will see that these effective QCD QQM are sufficiently general and from the physical point of view evident to allow a good description of wide set of mass relations for vector and axial-vector mesons. Moreover, using properties of two-point QCD VA correlators of quark densities at high energies (asymptotic freedom and Operator Product Expansion (OPE)) and two-resonance ansatz for VA meson correlators of QQM, we will obtain a number of constraints for parameters of QQM from the so-called Chiral Symmetry Restoration (CSR) sum rules.

It is well known that the effective QCD-inspired quark models with four-fermion interactions of Nambu–Jona–Lasinio (NJL) type are often applied to describe the low-energy QCD in the hadronization regime [2–11]. The appearance of effective quark theory is connected with one of specific property of QCD – the dynamical Chiral Symmetry Breaking (CSB). It was noted by authors [2] that a phenomenon of CSB in local four-fermion models occurs because of self-interaction of fermion fields due to strong attraction in the scalar channel. As a consequence, the

This work was supported by Grant RFBR 01-02-17152, INTAS Call 2000 Grant (Project 587), and The Program "Universities of Russia: Fundamental Investigations" (Grant UR.02.01.001).

dynamical quark mass $m_{\rm din}$ is created, as well as an isospin multiplet of pions, massless in the chiral limit, and a massive scalar meson with mass $m_\sigma = 2m_{\rm din}$ appear. However, it is known from experiment that there are series of meson states with equal quantum numbers [12]. Due to confinement, one expects an infinite number of such excited states with increasing masses. In such a way, the problem arises how to describe the physics of those resonances at intermediate energies. To solve this problem one can extend the NJL quark model taking into account higher-dimensional quark operators with derivatives, i.e. quasilocal quark interactions [1]. For sufficiently strong couplings the new operators promote the formation of additional meson states. Such a quasilocal approach (see also [13–17]) represents a systematic extension of the NJL-model towards the complete effective action of QCD where many-fermion vertices with derivatives possess the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the perturbative QCD effective action. In the effective action of Quasilocal Quark Models (QQM) of NJL type the low-energy gluon effects are hidden in the coupling constants. The alternative schemes including the condensates of low-energy gluons can be found in [7, 18].

At the same time in the large-$N_c$ approach, which is equivalent to planar QCD [19], the correlators of color-singlet quark currents are saturated by narrow meson resonances. In particular, the two-point correlators of vector and axial-vector quark currents are represented by the sum of related meson poles in Euclidean space:

$$\begin{aligned} \Pi^C(p^2) &= \int d^4x \exp(ipx)\langle \bar{q}\Gamma q(x)\bar{q}\Gamma q(0)\rangle \\ &= \sum_n \frac{Z_n^C}{p^2+m_{C,n}^2} + D_0^C + D_1^C p^2, \qquad (1) \\ C \equiv V, A; &\qquad \gamma_\mu, \gamma_\mu\gamma_5; \qquad D_0, D_1 = \text{const}\,. \end{aligned}$$

The last two terms represent a perturbative contribution, with $D_0$ and $D_1$ being contact terms required for the regularization of infinite sums. On the other hand, their high-energy asymptotics is provided [20] by the perturbation theory and OPE due to asymptotic freedom of QCD. Therefrom the above correlators increase at large $p^2$,

$$\Pi^C(p^2)|_{p^2\to\infty} \sim p^2 \ln\frac{p^2}{\mu^2}\,. \qquad (2)$$

When comparing the two expressions above one concludes that the infinite series of resonances with the same quantum numbers should exist in order to reproduce the perturbative asymptotics. So at intermediate energies the correlators of QQM can be matched [21] to the OPE of QCD correlators. This matching realizes the correspondence to QCD and improves the predictability of QQM at intermediate energies.

Meantime the differences of correlators of opposite-parity currents rapidly decrease at large momenta $p^2 \to \infty$ [20, 22, 23]

$$\Pi^V(p^2) - \Pi^A(p^2) \equiv \frac{\Delta_{VA}}{p^6} - \frac{m_0^2 \Delta_{VA}}{p^8} + O\left(\frac{1}{p^{10}}\right), \ \Delta_{VA} \simeq -16\pi\alpha_s \langle \bar{q}q \rangle^2, \tag{3}$$

where $m_0^2 = 0.8 \pm 0.2\,\mathrm{GeV}^2$ [24], $\alpha_s \approx 0.3$ (at 1 GeV), and we have defined in the $V, A$ channels

$$\Pi^{V,A}_{\mu\nu}(p^2) \equiv (-\delta_{\mu\nu} p^2 + p_\mu p_\nu) \Pi^{V,A}(p^2). \tag{4}$$

Therefore the chiral symmetry is restored at high energies and the difference (3) represents an order parameter of CSB in QCD. As it decreases rapidly at large momenta one can perform the matching of QCD asymptotics by means of few lowest lying resonances that gives a set of constraints for QQM parameters from the CSR sum rules.

In the present work the vector QQM model is considered with two channels where two pairs of vector and axial-vector mesons are generated. Respectively it is expected to reproduce the lower part of QCD meson spectrum in the planar limit and the leading asymptotics of chiral symmetry restoration for higher energies. In the Sec. 2 we define the VA, $SU(2)$ QQM with two pairs of VA-mesons and the corresponding mass spectrum for VA boson states is obtained. The Sec. 3 is devoted to the $U(3)$ extension of VA QQM. With the help of two-resonance ansatz for VA-mesons the correlators of QQM are matched to the OPE of QCD correlators and a number of constraints for parameters of QQM from CSR sum rules are performed in the Sec. 4.

## 2. VA, $SU(2)$ Quasilocal Quark Model

The $SU(2)$ QQM Lagrangian for the two-channel vector (V) and axial-vector (A) case in the chiral limit $m_q = 0$ has the form [25] (in Euclidean space):

$$\mathcal{L}_{VA} = \bar{q}i\not{\partial}q + \frac{1}{4N_f N_c \Lambda^2} \sum_{k,l=1}^{2} b_{kl} \left[\bar{q}\Gamma^i_{V,k}q \cdot \bar{q}\Gamma^i_{V,l}q + \bar{q}\Gamma^i_{A,k}q \cdot \bar{q}\Gamma^i_{A,l}q\right] , \tag{5}$$
$$\Gamma^i_{V,k} \equiv i\gamma_\mu f_k(s)\tau^i , \qquad \Gamma^i_{A,k} \equiv i\gamma_\mu\gamma_5 f_k(s)\tau^i , \qquad i = 1, 2, 3 ,$$

where $q \equiv q_j$ ($j$ is the number of flavor $N_f$) are color fermionic fields with $N_c$ components, $b_{kl}$ represents the symmetric nonsingular matrix of real coupling constants, and $\tau^i$ denote Pauli matrices. The quantities $f_k(s)$, $s \to -\partial^2/\Lambda^2$ are the form factors specifying the quasilocal interaction. We accept the following sequence of action of the derivatives for the Hermitian fermion currents:

$$\bar{q}\frac{\partial^2}{\Lambda^2}q = \frac{1}{4}\bar{q}\left(\frac{\overrightarrow{\partial} - \overleftarrow{\partial}}{\Lambda}\right)^2 q . \tag{6}$$

In addition, let us regularize the interaction vertices by introducing the momentum cutoff

$$\bar{q}q \longrightarrow \bar{q}\theta(\Lambda^2 + \partial^2)q , \tag{7}$$

and choose the polynomial form factors as being orthogonal on the unit interval,

$$\int_0^1 f_k(s)f_l(s)ds = \delta_{kl} . \tag{8}$$

We select out here:

$$f_1(s) = 2 - 3s , \qquad f_2(s) = -\sqrt{3}s . \tag{9}$$

As this model interpolates the low-energy QCD action it is supplied with a cutoff $\Lambda$ (of order of the CSB scale) for virtual quark momenta in quark loops.

It is convenient to pass to the auxiliary vector ($\rho^i_\mu$) and axial-vector ($a^i_\mu$) fields,

$$\begin{aligned}\mathcal{L}_{\text{aux}} &= \bar{q}i\not{\partial}q + \sum_{k=1}^{2} i\bar{q}\left(\Gamma^i_{V,k}\rho^i_{k,\mu} + \Gamma^i_{A,k}a^i_{k,\mu}\right)q + \\ &+ N_f N_c \Lambda^2 \sum_{k,l=1}^{2}\left(\rho^i_{k,\mu}b^{-1}_{kl}\rho^i_{l,\mu} + a^i_{k,\mu}b^{-1}_{kl}a^i_{l,\mu}\right) .\end{aligned} \tag{10}$$

After integrating out the quark fields

$$\left\langle \exp\left( -\int d^4x\, \mathcal{L} \right) \right\rangle_{\bar{q}q} \equiv \exp(-S_{\text{eff}}),$$

one comes to the bosonic effective action:

$$\begin{aligned} S_{\text{eff}}(\rho^i_{\mu,k}, a^i_{\mu,k}) = N_f N_c \Lambda^2 \sum_{k,l=1}^{2} \{\rho^i_{k,\mu} b^{-1}_{kl} \rho^i_{l,\mu} + a^i_{k,\mu} b^{-1}_{kl} a^i_{l,\mu}\} \\ - N_f N_c \mathrm{Tr}\, \ln \not{D}|_{\text{reg}}\,, \end{aligned}$$

$$\not{D} \equiv i(\not{\partial} + M) + i \sum_{k=1}^{2} \left( \Gamma^i_{V,k} \rho^i_{k,\mu} + \Gamma^i_{A,k} a^i_{k,\mu} \right), \tag{11}$$

where we have introduced the dynamic mass function $M \equiv \sum_k \sigma_k f_k(s)$, with $\sigma_k$ being the vacuum expectation values of scalar fields [26]. We use the chirally invariant regularization of fermionic determinant:

$$\ln \det \not{D} = \mathrm{Tr}^{\,\text{all}} \ln \not{D} \longrightarrow N_f N_c \mathrm{Tr}\, \ln \not{D}|_{\text{reg}} \equiv \frac{1}{2} N_f N_c \mathrm{Tr}\, \ln \frac{\not{D}\,\not{D}^\dagger}{\mu^2},$$

where the constant $\mu$ is a normalization scale for quark fields and the trace "Tr" is assumed over all degrees of freedom except color and flavor ones. We will carry out further analysis in the mean field approximation ($N_c \gg 1$). Expanding the Eq. (11) in boson fields and retaining the quadratic in fields part only, one obtains:

$$S^{(2)}_{\text{eff}}(\rho^i_{\mu,k}, a^i_{\mu,k}) = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \sum_{k,l=1}^{2} [\rho^i_{k,\mu} C^{\rho,\mu\nu}_{kl} \rho^i_{l,\nu} + a^i_{k,\mu} C^{a,\mu\nu}_{kl} a^i_{l,\nu}]. \tag{12}$$

The inverse propagators are defined by the corresponding second variation of $S_{\text{eff}}(\rho^i_{\mu,k}, a^i_{\mu,k})$:

$$C^{(\rho,a)\mu\nu}_{kl} = 2 N_f N_c \Lambda^2 b^{-1}_{kl} \delta_{\mu\nu} - N_f N_c \int \frac{d^4q}{(2\pi)^4} \times$$

$$\mathrm{tr} \left\{ (i\gamma_\mu, i\gamma_\mu\gamma_5) \frac{f_k\left(\left(\frac{q+p/2}{\Lambda}\right)^2\right)}{\not{q} + \frac{1}{2}\not{p} + iM} (i\gamma_\nu, i\gamma_\nu\gamma_5) \frac{f_l\left(\left(\frac{q-p/2}{\Lambda}\right)^2\right)}{\not{q} - \frac{1}{2}\not{p} + iM} \right\}, \tag{13}$$

where the trace "tr" spans the Dirac indices only.

To obtain mass spectrum we expand (13) in a small external momentum $p$ ($p^2/\Lambda^2 \ll 1$) up to terms $\sim p^2$ and calculate the corresponding loop integral using the momentum cutoff regularization. To compensate the quadratic divergences in this integral we parametrize the matrix of coupling constants as follows:

$$16\pi^2 b_{kl}^{-1} = \delta_{kl} - \frac{4}{3}\frac{\bar{\Delta}_{kl}}{\Lambda^2}\,; \qquad \bar{\Delta}_{kl} \ll \Lambda^2. \tag{14}$$

The general structure of (13) is:

$$C_{kl}^{(\rho,a)\mu\nu} = \frac{N_f N_c}{12\pi^2}\left[\left(\hat{A}_{kl}^{(\rho,a)} p^2 + \hat{B}_{kl}^{(\rho,a)}\right)\delta_{\mu\nu} - \hat{A}_{kl}^{(\rho,a)} p_\mu p_\nu\right] + O\left(\frac{1}{\Lambda^2}\right), \tag{15}$$

where the two symmetric matrices - the kinetic term $\hat{A}$ and the momentum independent part $\hat{B}$ - have been introduced:

$$\hat{A}^{(\rho,a)} \equiv \begin{pmatrix} 4\ln\frac{\Lambda^2}{M_0^2} - \frac{15}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{pmatrix}, \tag{16}$$

$$\hat{B}^{\rho} \equiv \begin{pmatrix} -2\bar{\Delta}_{11} & -2\bar{\Delta}_{12} \\ -2\bar{\Delta}_{12} & -2\bar{\Delta}_{22} \end{pmatrix}, \tag{17}$$

$$\hat{B}^{a} \equiv \begin{pmatrix} -2\bar{\Delta}_{11} + \sigma_{11} & -2\bar{\Delta}_{12} + \sigma_{12} \\ -2\bar{\Delta}_{12} + \sigma_{12} & -2\bar{\Delta}_{22} + \sigma_{22} \end{pmatrix}, \tag{18}$$

$$\sigma_{11} \equiv 24M_0^2 \ln\frac{\Lambda^2}{M_0^2} - \frac{477}{2}\sigma_1^2 - 15\sqrt{3}\sigma_1\sigma_2 + \frac{9}{2}\sigma_2^2\,,$$

$$\sigma_{12} \equiv -\frac{15\sqrt{3}}{2}\sigma_1^2 + 9\sigma_1\sigma_2 + \frac{3\sqrt{3}}{2}\sigma_2^2\,,$$

$$\sigma_{22} \equiv \frac{9}{2}\sigma_1^2 + 3\sqrt{3}\sigma_1\sigma_2 + \frac{27}{2}\sigma_2^2\,.$$

Here $M_0 \equiv M(0) = 2\sigma_1$ is the dynamic quark mass at zero external momentum. The remaining logarithmic divergences will be absorbed later by meson masses and renormalization of meson fields.

The physical mass spectrum is defined by the secular equation

$$\det(\hat{A}p^2 + \hat{B})^{(\rho,a)} = 0\,, \qquad m_{\text{phys}}^2 = -p_0^2\,. \tag{19}$$

As it will be seen further the consistency with CSR sum rules imposes the following scale condition:

$$\bar{\Delta}_{kl} = O\left(\Lambda^2\right). \tag{20}$$

Using (16), (17) one has for the Eq. (19):

$$6\left(\ln\frac{\Lambda^2}{M_0^2}-2\right)p^4-\left(8\ln\frac{\Lambda^2}{M_0^2}\,\bar\Delta_{22}-15\bar\Delta_{22}+3\bar\Delta_{11}+2\sqrt{3}\bar\Delta_{12}\right)p^2+ \\ +4\,\det\hat{\bar\Delta}=0, \tag{21}$$

The solution of Eq. (21) in the large-log approximation $\ln\frac{\Lambda^2}{M_0^2}\gg 1$ is as follows:

$$m_\rho^2=-\frac{\det\hat{\bar\Delta}}{2\ln\frac{\Lambda^2}{M_0^2}\,\bar\Delta_{22}}+c_1+O\Big(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\Big), \tag{22}$$

$$m_{\rho'}^2=-\frac{4}{3}\bar\Delta_{22}+\delta+c_2+O\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right). \tag{23}$$

To obtain the A-meson mass spectrum it is enough to make the replacement (see (17), (18): $\bar\Delta_{kl}\to\bar\Delta_{kl}-1/2\sigma_{kl}$. The result is:

$$m_{a_1}^2=-\frac{\det\hat{\bar\Delta}}{2\ln\frac{\Lambda^2}{M_0^2}\,\bar\Delta_{22}}+6M_0^2+c_1+O\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right), \tag{24}$$

$$m_{a_1'}^2=-\frac{4}{3}\bar\Delta_{22}+3\bar\sigma+\delta+c_2+O\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right). \tag{25}$$

The prime labels everywhere the corresponding excited meson state and we have introduced the notations:

$$\delta\equiv-\frac{6m_\rho^2\ln\frac{\Lambda^1}{M_0^2}+d}{6\ln\frac{\Lambda^1}{M_0^2}},\quad c_1\sim c_2=O\left(\frac{\Lambda^2}{\ln^2\frac{\Lambda^2}{M_0^2}}\right), \tag{26}$$

$$d\equiv 3\bar\Delta_{11}+2\sqrt{3}\bar\Delta_{12}+\bar\Delta_{22}\,,\quad \bar\sigma\equiv\sigma_1^2+\frac{2\sqrt{3}}{3}\sigma_1\sigma_2+3\sigma_2^2>0\,. \tag{27}$$

As it is seen from the Eqs. (22)–(25) the scale of mass squared for ground VA states is $O(\Lambda^2/\ln\frac{\Lambda^2}{M_0^2})$ and for excited ones is $O(\Lambda^2)$. Thus, the excited states turn out to be logarithmically heavier than ground ones as it was for the scalar (S) and pseudoscalar (P) case [26]. . This qualitative property is independent of any concrete choice of form factors. Combining the Eqs. (22)–(25) with the corresponding results in [26] one obtains:

$$m_{a_1}^2-m_\rho^2=6M_0^2+O\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right)=\frac{3}{2}m_\sigma^2+O\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right), \tag{28}$$

$$m_{a_1'}^2 - m_{\rho'}^2 = 3\bar{\sigma} + O\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right) = \frac{3}{2}(m_{\sigma'}^2 - m_{\pi'}^2) + O\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right). \quad (29)$$

The last equalities in Eqs. (28), (29) do not depend on model parameters. Note also, that differences of masses squared both in the Eq. (28) and in the Eq. (29) are of order $O(1)$, which indicates the chiral symmetry restoration at a scale over 1 GeV.

It can be shown also by corresponding fits. Having as input $\Lambda = 1000\,\mathrm{MeV}$ and

$$M_0 = 2\sigma_1 = 320\mathrm{MeV}, \; <\bar{q}q> \simeq -\frac{N_c\Lambda^2}{8\pi^2}(\sigma_1 - \sqrt{3}\sigma_2) = -(250\mathrm{MeV})^3,$$

one can fix

$$\sigma_1 = 160\mathrm{MeV}, \quad \sigma_2 = -145\mathrm{MeV}.$$

From Eq. (29) one obtains the mass splittings

$$m_{\sigma'} - m_{\pi'} \approx 45\mathrm{MeV}, \quad m_{a_1'} - m_{\rho'} \approx 60\mathrm{MeV},$$

which point out a fast restoration of chiral symmetry. We confront the $\pi'$ and $\rho'$ mesons with the states [12] $\pi(1300)$ ($m_{\pi'} = 1300 \pm 100\mathrm{MeV}$) and $\rho(1450)$ ($m_{\rho'} = 1465 \pm 25\mathrm{MeV}$), respectively. Then the QQM predicts the mass of scalar state $m_{\sigma'} = 1250 \div 1450\mathrm{MeV}$, which may be identified with $f_0(1370)$ [12], and for the axial-vector one $m_{a_1'} = 1500 \div 1550\mathrm{MeV}$. The former particle is confronted with the scalar state $f_0(1500)$ [12]. Since the unitarization effects are not included into such models one should compare, generally speaking, the predictions with bare masses if they are calculated somehow. For example, the bare mass of $f_0(1500)$ is approximately 1250MeV [30].

Let us comment the approximations used to derive the meson mass spectrum: namely, the large $N_c$ and leading-log approximations. The first one is equivalent [19] to the neglect of meson loops. The second one fits well the quarks confinement as quark-antiquark threshold contributions are suppressed in two-point functions in the leading log approximation. The accuracy of this approximation is controlled also by the magnitudes of higher dimensional operators neglected in QQM, i.e. by contributions of heavy mass resonances not included into QQM. All these approximations are mutually consistent. In particular, in the effective action without gluons the quark confinement should be realized with the help of an infinite number of quasilocal vertices with higher-order derivatives. Then the imaginary part of quark loops can be compensated and their momentum dependence can eventually reproduce the infinite sum of meson resonances in the large-$N_c$ limit. If the effective action is truncated with a finite number of vertices and thereby deals with only a few resonances one has to retain only a finite number of terms in the low-momentum expansion of quark loops in the CSB phase, with a non-zero dynamic mass.

## 3. VA, $U(3)$ extension of Quasilocal Quark Model

In this section we build $U(3)$ version of the VA, $SU(2)$ QQM [27], with current quark masses being taken into account. The corresponding extension of the VA, $SU(2)$ QQM Lagrangian (5) is:

$$\mathcal{L}_{VA} = \bar{q} i(\hat{\partial} + \hat{m})q + \frac{1}{4N_f N_c \Lambda^2} \sum_{k,l=1}^{2} b^i_{kl} \left[\bar{q}\Gamma^i_{V,k} q \cdot \bar{q}\Gamma^i_{V,l} q + \bar{q}\Gamma^i_{A,k} q \cdot \bar{q}\Gamma^i_{A,l} q\right],$$
$$\Gamma^i_{V,k} \equiv i\gamma_\mu f_k(s)\lambda^i, \qquad \Gamma^i_{A,k} \equiv i\gamma_\mu\gamma_5 f_k(s)\lambda^i, \qquad i = 0, ..., 8, \tag{30}$$

where $\lambda^i$ represent Gell-Mann matrices. The current quark mass matrix is $\hat{m} = \mathrm{diag}\,(m_u, m_d, m_s)$. In the sequel we adopt the exact isospin symmetry $m_u = m_d$. The symbol $\tilde{u}$ will stand everywhere for the $u$, $d$, $\bar{u}$, $\bar{d}$ quarks. The symbol $\tilde{s}$ will denote $s$ or $\bar{s}$ quarks. The generalization of fine-tuning condition (14) takes the form:

$$16\pi^2 (b^i_{kl})^{-1} = \delta^i_{kl} - \frac{4}{3}\frac{\bar{\Delta}^i_{kl}}{\Lambda^2}; \qquad \bar{\Delta}^i_{kl} \ll \Lambda^2. \tag{31}$$

The couplings $\bar{\Delta}^i_{kl}$ satisfy the relations

$$\bar{\Delta}^m_{kl} = \bar{\Delta}^0_{kl}, \quad \bar{\Delta}^n_{kl} = \frac{1}{2}\left(\bar{\Delta}^0_{kl} + \bar{\Delta}^8_{kl}\right); \quad m = 1, 2, 3, \quad n = 4, 5, 6, 7, \tag{32}$$

which provide the $U(3)$ Gell-Mann-Okubo relations

$$m^2_{\alpha,\tilde{u}\tilde{u}} + m^2_{\alpha,\tilde{s}\tilde{s}} = 2m^2_{\alpha,\tilde{s}\tilde{u}}; \qquad m_{\alpha,\tilde{u}\tilde{u}} = m_{\alpha,\mathrm{singlet}}. \tag{33}$$

Here $\alpha \equiv V, A, V', A'$. The scheme of calculation of the mass-spectrum for VA, $U(3)$ QQM is the same as in the Sec. 2 and the details can be found in [27]. We point out only some features. First, the self-consistency of mass-spectrum imposes the self-consistency condition (20) for all $i$. Second, in the effective action we did not take into account so far both the $P - A$ mixing and different mixing terms, which are proportional to the current quark masses, since these contributions do not influence on the meson mass spectrum.

The mass relations, which are independent of model parameters in the large-log approach are:

$$m^2_{a_1,\bar{u}\bar{u}} - m^2_\rho \simeq \frac{3}{2}\left(m^2_{\sigma,\bar{u}\bar{u}} - m^2_\pi\right), \quad m^2_{a_1,\bar{s}\bar{u}} - m^2_{K^*} \simeq \frac{3}{2}\left(m^2_{\sigma,\bar{s}\bar{u}} - m^2_K\right), \tag{34}$$

$$m^2_{a_1,\bar{s}\bar{s}} - m^2_\varphi \simeq \frac{3}{2}\left[m^2_{\sigma,\bar{s}\bar{s}} - \left(2m^2_K - m^2_\pi\right)\right]. \tag{35}$$

$$m^2_{a_1',\bar{u}\bar{u}} - m^2_{\rho'} \simeq \frac{3}{2}\left(m^2_{\sigma',\bar{u}\bar{u}} - m^2_{\pi'}\right),$$

$$m^2_{a_1',\bar{s}\bar{u}} - m^2_{K^{*\prime}} \simeq \frac{3}{2}\left(m^2_{\sigma',\bar{s}\bar{u}} - m^2_{K'}\right), \tag{36}$$

$$m^2_{a_1',\bar{s}\bar{s}} - m^2_{\varphi'} \simeq \frac{3}{2}\left(m^2_{\sigma',\bar{s}\bar{s}} - m^2_{(\eta)'}\right). \tag{37}$$

The $(\eta)'$ in the Eq. (37) is the radial excitation of $\eta$ meson and throughout the paper the sign $\simeq$ denotes the large-log approximation. The corresponding fits and comparisons with experiment are carried out in [27, 28]. The agreement with experimental data is within 10% for V-case and 15% for the A-one. In such a way, the VA, $U(3)$ extension of QQM provides the certain relations between meson masses of multiplets which are independent of model parameters.

The axial anomaly was not yet considered in framework of QQM and, thus, we did not include the $\eta'$ meson. Since the ground P-meson masses are subject to rather $SU(3)$ Gell-Mann-Okubo relation one has no $\eta$ meson mass in the Eq. (35). On the other hand, we do not expect a strong $U_A(1)$ anomaly effect for the case of excited $\eta'$ meson. As a result, the relations (33) hold for excited P-multiplet, which is reflected in the Eq. (37).

## 4. Chiral symmetry restoration sum rules and constraints on parameters of VA, $SU(2)$ QQM

In this section we exploit the constraints based on chiral symmetry restoration in QCD at high energies for the $SU(2)$ QQM. Expanding the meson correlators (1) in powers of $p^2$ one arrives to the CSR sum rules:

$$\sum_n Z^V_n - \sum_n Z^A_n = 4F^2_\pi, \tag{38}$$

$$\sum_n Z_n^V m_{V,n}^2 - \sum_n Z_n^A m_{A,n}^2 = 0\,, \tag{39}$$

$$\sum_n Z_n^V m_{V,n}^4 - \sum_n Z_n^A m_{A,n}^4 = \Delta_{VA}\,, \tag{40}$$

$$\sum_n Z_n^V m_{V,n}^6 - \sum_n Z_n^A m_{A,n}^6 = -m_0^2 \Delta_{VA}\,. \tag{41}$$

The first two relations are the Weinberg sum rules. The quantity $F_\pi$ is the pion decay constant defined as

$$\langle 0|A_\mu|\pi\rangle = i\sqrt{2} f_\pi p_\mu ,$$

with $A_\mu$ being the axial-vector current. This quantity is equal in the QQM [26]:

$$F_\pi^2 = \frac{N_f N_c M_0^2}{4\pi^2} \ln \frac{\Lambda^2}{M_0^2} + O(1). \tag{42}$$

The residues in resonance pole contributions in the vector and axial-vector correlators have the structure,

$$Z_n^{(V,A)} = 4 f_{(V,A),n}^2 m_{(V,A),n}^2 ,$$

with $f_{(V,A),n}$ being defined as electromagnetic decay constants.

The corresponding two-point correlators for the $SU(2)$ QQM can be calculated by variation of external vector fields. Let us first consider the V-case. Taking into account the external vector sources $V_{k,\mu}^i$ the Lagrangian reads as follows:

$$\mathcal{L}_{\rm aux}^V = \bar q \left( \not{D} + i \sum_{k=1}^{2} \Gamma_{V,k}^i V_{k,\mu}^i \right) q + N_f N_c \Lambda^2 \sum_{k,l=1}^{2} \rho_{k,\mu}^i b_{kl}^{-1} \rho_{l,\mu}^i \,.$$

After shifting the bosonic fields

$$\rho_{k,\mu}^i \longrightarrow \rho_{k,\mu}^i - V_{k,\mu}^i \,,$$

and integrating over fermionic degrees of freedom one comes to the following effective action in external V-fields:

$$S_{\rm eff}^V(\rho_{k,\mu}^i, V_{k,\mu}^i) = -N_f N_c {\rm Tr}\, \ln \not{D}|_{\rm reg}$$

$$+ N_f N_c \Lambda^2 \int d^4x \sum_{k,l=1}^{2} b_{kl}^{-1} \left\{ \rho_{k,\mu}^i \rho_{l,\mu}^i - 2 V_{k,\mu}^i \rho_{l,\mu}^i + V_{k,\mu}^i V_{l,\mu}^i \right\} \,.$$

Expanding Tr ln $\not{D}|_{\rm reg}$ up to quadratic in fields terms, one has

$$S^{(2)}_{\rm eff}(\rho^i_{k,\mu}, V^i_{k,\mu}) \simeq \sum_{k,l=1}^{2} \Big\{ \frac{1}{2}\rho_{k,\mu} C^{(\rho)\mu\nu}_{kl} \rho_{l,\nu}$$
$$+ N_c \Lambda^2 b^{-1}_{kl} \left[ -2V_{k,\mu}\rho_{l,\mu} + V_{k,\mu}V_{l,\mu} \right] \Big\},$$

where $C^{(\rho)\mu\nu}_{kl}$ is given by Eq. (15). Introducing the vectors

$$\rho_\mu \equiv \begin{pmatrix} \rho^i_{1,\mu} \\ \rho^i_{2,\mu} \end{pmatrix}, \qquad V_\mu \equiv \begin{pmatrix} V^i_{1,\mu} \\ V^i_{2,\mu} \end{pmatrix}$$

and taking into account (14) (where we neglect the last term), one integrates over $\rho_\mu$ with the result:

$$\frac{12\pi^2}{N_f N_c} S_{\rm eff}(V_\mu) = -\frac{9}{8}\left(\Lambda^4 + O(\Lambda^2)\right) V^T_\mu \hat{H}^\rho_{\mu\nu} V_\nu + \frac{3\Lambda^2}{4} V^T_\mu V_\nu \delta_{\mu\nu}\,, \quad (44)$$

$$\hat{H}^\rho_{\mu\nu} \equiv \left(\hat{A}p^2 + \hat{B}^\rho\right)^{-1} \hat{A} \left(\hat{B}^\rho\right)^{-1} \left(-p^2\delta_{\mu\nu} + p_\mu p_\nu\right) + \left(\hat{B}^\rho\right)^{-1} \delta_{\mu\nu}\,. \quad (45)$$

The last term in the Eq. (45) together with that of in (44) form a local term which will be cancelled by the same term in the A-case. Substituting the identity

$$\bar{q}\gamma_\mu q = \frac{1}{2}\left(\bar{q} f_1(s)\gamma_\mu q - \sqrt{3}\,\bar{q} f_2(s)\gamma_\mu q\right)$$

into the vector correlator

$$\Pi^\rho_{\mu\nu}(p^2) = 4 \int d^4x \exp(ipx) \langle \bar{q}\gamma_\mu q(x) \bar{q}\gamma_\nu q(0) \rangle, \quad (46)$$

the latter can be rewritten through the second variational derivatives:

$$\Pi^{(\rho)}_{\mu\nu}(p^2) \simeq \frac{N_f N_c}{12\pi^2} \left[ \Pi^{(\rho)}_{11} + 3\Pi^{(\rho)}_{22} - 2\sqrt{3}\,\Pi^{(\rho)}_{12} \right] \left(-\delta_{\mu\nu} p^2 + p_\mu p_\nu\right), \quad (47)$$

$$\hat{\Pi}^{(\rho)} \equiv \left(\hat{A}p^2 + \hat{B}^\rho\right)^{-1} \hat{A} \left(\hat{B}^\rho\right)^{-1}. \quad (48)$$

On the other hand, this correlator is parametrized as follows (see Eq. (1)):

$$\Pi^{(\rho)}_{\mu\nu}(p^2) = \left[ \frac{Z_\rho}{p^2 + m^2_\rho} + \frac{Z_{\rho'}}{p^2 + m^2_{\rho'}} \right] \left(-\delta_{\mu\nu} p^2 + p_\mu p_\nu\right). \quad (49)$$

Comparison of (47) and (49) allows to obtain the corresponding residues. (see Eqs. (54), (55)). In the mean-field approximation the vector correlator and residues can be calculated exactly which is displayed in the Appendix.

The A-mesons must be considered together with the P-ones due to the $P-A$ mixing. The relevant term in the effective action

$$S^{(2)}_{\text{eff}}(\pi^i_k, a^i_{k,\mu}) = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\sum_{k,l=1}^{2} 2\pi^i_k C^{\pi a,\mu}_{kl} a^i_{l,\mu}\,,$$

appears by virtue of non-zero value of the corresponding second variation of $S_{\text{eff}}(\pi^i_k, a^i_{k,\mu})$:

$$C^{\pi a,\mu}_{kl} = -N_f N_c \int \frac{d^4q}{(2\pi)^4}\text{tr}\left\{(i\gamma_5)\frac{f_k\left(\left(\frac{q+p/2}{\Lambda}\right)^2\right)}{\not q + \frac{1}{2}\not p + iM}(i\gamma_\mu\gamma_5)\frac{f_l\left(\left(\frac{q+p/2}{\Lambda}\right)^2\right)}{\not q - \frac{1}{2}\not p + iM}\right\}$$

$$= -\frac{4iN_f N_c}{(2\pi)^4}\int \frac{d^4q M f_k\left(\frac{q^2}{\Lambda^2}\right) f_l\left(\frac{q^2}{\Lambda^2}\right)}{\left[\left(q+\frac{1}{2}p\right)^2 + M^2\right]\left[\left(q-\frac{1}{2}p\right)^2 + M^2\right]} p_\mu + O\left(\frac{1}{\Lambda^2}\right). \tag{50}$$

In order to exclude the mixing terms one makes the shift of A-fields:

$$a^i_{k,\mu} \longrightarrow a^i_{k,\mu} + D^i_{kl}\pi^i_l p_\mu\,,$$

with the elements $D^i_{kl}$ being defined by the requirement of cancellation of $P-A$ terms. This leads to changing of the kinetic matrix $\hat{A}^\pi$ due to contribution of longitudinal A-part:

$$\hat{A}^\pi \longrightarrow \hat{A}^\pi_{\text{ren}} = \begin{pmatrix} 4\frac{m_\rho^2}{m_{a_1}^2}\ln\frac{\Lambda^2}{M_0^2} + O(1) & -\frac{\sqrt{3}}{2} + O\left(\frac{\ln\frac{\Lambda^2}{M_0^2}}{\Lambda^2}\right) \\ -\frac{\sqrt{3}}{2} + O\left(\frac{\ln\frac{\Lambda^2}{M_0^2}}{\Lambda^2}\right) & \frac{3}{2} + O\left(\frac{1}{\Lambda^2}\right) \end{pmatrix}, \tag{51}$$

where the Eq. (28) and the scale (20) have been exploited. The redefinition (51) does not influence on the mass spectrum of P-mesons, but renormalizes the pion decay constant (42) is:

$$F_\pi^2 = \frac{N_f N_c M_0^2 m_\rho^2}{4\pi^2 m_{a_1}^2}\ln\frac{\Lambda^2}{M_0^2} + O(1)\,. \tag{52}$$

For the model under consideration the relation (52) fixes the logarithm of the cutoff in terms of physical parameters.

Taking into account the point above the further calculations for the A-case are the same as for the V-one. As a result one finds the residues in the meson poles. In the large-log approach one has:

$$\tilde{Z}_\pi = 4F_\pi^2 \simeq \frac{(m_{a_1}^2 - m_\rho^2)\,\delta}{m_\rho^2 m_{a_1}^2 m_{a_1'}^2} Z_1\,, \qquad Z_1 \equiv \frac{3N_f N_c \Lambda^4}{16\pi^2}\,, \tag{53}$$

$$Z_\rho \simeq \frac{m_{a_1}^2}{m_{a_1}^2 - m_\rho^2} 4F_\pi^2\,, \qquad Z_{a_1} \simeq \frac{m_\rho^2}{m_{a_1}^2 - m_\rho^2} 4F_\pi^2\,, \tag{54}$$

$$Z_{\rho'} \simeq \frac{Z_1}{m_{\rho'}^2}\,, \qquad Z_{a_1'} \simeq \frac{Z_1}{m_{a_1'}^2}\,, \tag{55}$$

where $\delta$ is given by (26). Unlike the situation in the SP-case the residues in the VA-poles are of the same order of magnitude:

$$Z_\rho \sim Z_{a_1} \sim Z_{\rho'} \sim Z_{a_1'} = O\left(\Lambda^2\right).$$

The statement (20) follows from the comparison of Eqs. (52), (53). The relation for $\tilde{Z}_\pi$ is a constraint on effective coupling constants of the QQM $\bar{\Delta}_{kl}$.

The first and the second sum rules are fulfilled identically. The third one takes the form:

$$Z_1\left(m_{a_1'}^2 - m_{\rho'}^2\right) \simeq 16\pi\alpha_s\langle\bar{q}q\rangle^2 \quad \text{or} \quad 3Z_1\bar{\sigma} = -\Delta_{VA}\,. \tag{56}$$

Notice that the analog of Eq. (56) in the scalar case [26] may be cast into the form

$$\frac{N_f N_c}{2\pi^2}\left(m_{\sigma'}^2 - m_{\pi'}^2\right) \simeq 24\pi\alpha_s\langle\bar{q}q\rangle^2 \quad \text{or} \quad 3Z_1\bar{\sigma} = -\frac{27}{32}\Delta_{VA}\,. \tag{57}$$

The minor discrepancy between relations (56) and (57) is about 16% and can be referred to the quality of two-resonance approximation. The fourth sum rule looks as follows (28):

$$m_{a_1'}^2 \simeq m_{\rho'}^2 \simeq \frac{m_0^2}{2} \quad \text{or} \quad -\frac{4}{3}\bar{\Delta}_{22}\cdot 3\bar{\sigma} \simeq -\frac{m_0^2\Delta_{VA}}{2Z_1}\,. \tag{58}$$

Numerical estimations (28) show that the last sum rule fails for QQM with the ground and first excited sets of $VA$ mesons. The structure of $Z_{\rho'}$

and $Z_{a_1'}$ shows that if $m_{a_1'} \simeq m_{\rho'}$ then $Z_{a_1'} \simeq Z_{\rho'}$ and therefore $f_{a_1'} \simeq f_{\rho'}$. As a consequence, these residues approximately cancel each other in sum rules and the one-channel results for $f_\rho$, $f_{a_1}$ hold [25] (see also [29]):

$$f_\rho \simeq \frac{F_\pi m_{a_1}}{m_\rho \sqrt{m_{a_1}^2 - m_\rho^2}}\,, \qquad f_{a_1} \simeq \frac{F_\pi m_\rho}{m_{a_1} \sqrt{m_{a_1}^2 - m_\rho^2}}\,. \tag{59}$$

After evaluating we get $f_\rho \approx 0.15$ and $f_{a_1} \approx 0.06$ to be compared with the experimental values $f_\rho = 0.20 \pm 0.01$, $f_{a_1} = 0.10 \pm 0.02$ [25].

It should be mentioned that introducing VA-meson fields influences on some CSR constraints for SP-case obtained in [26]. Due to the redefinition (51) the first SP sum rule is not fulfilled now identically but up to terms of order $O(1/\Lambda^2)$ because of the scaling (20). The second SP sum rule is not changed in the large-$\Lambda$ approximation. The relation $Z_\sigma \simeq Z_\pi$ is not now valid. Instead one has

$$Z_\sigma \simeq \frac{m_\rho^2}{m_{a_1}^2} Z_\pi\,.$$

The relevant physical discussions and fits for the SPVA QQM can be traced in [25, 28]. From the estimations [28], in particular, follows that $m_{a_1'} - m_{\rho'} \approx 60$ MeV which proves a fast restoration of chiral symmetry.

## 5. Summary and conclusions

1. We showed that VA, $SU(2)$ and $U(3)$ versions of the Quasilocal Quark Model with chirally invariant four-fermion vector and axial-vector vertices including higher-dimensional quark operators with derivatives can serve to describe the physics of vector and axial-vector meson resonances at intermediate energies. The corresponding mass spectrum for the ground and first excited VA boson states was derived in the mean field and large-log approximations. The qualitative features of the mass spectrum obtained turned out to be the same as in the scalar-pseudoscalar case [26]: the excited states are logarithmically heavier than ground ones and a fast restoration of chiral symmetry over the scale 1 GeV is predicted. Comparison with the SP, $SU(2)$ QQM in the chiral limit permitted to obtain two remarkable relations between boson masses independent of model parameters:

$$m_{a_1}^2 - m_\rho^2 \simeq \frac{3}{2} m_\sigma^2\,,$$

$$m^2_{a'_1} - m^2_{\rho'} \simeq \frac{3}{2}(m^2_{\sigma'} - m^2_{\pi'}).$$

The latter relation predict the mass of the first radial excitation of axial-vector meson in the energy range $m'_{a_1} = 1500 \div 1550$ MeV which confronts the phenomenologically accepted estimation [31]. From estimations [28] for mass splittings follows $m_{\sigma'} - m_{\pi'} = 45$ MeV and $m_{a'_1} - m_{\rho'} = 60$ MeV that point out fast restoration of the chiral symmetry at intermediate energies.

2. The VA, $U(3)$ generalization of QQM allowed to derive much more relations between the masses of meson states which do not depend on any model parameters (see Eqs. (34)–(37)). The agreement with experimental data is within 10% for the V-case and 15% for the A-one [28].
3. From the expansion of QCD two-point colour-singlet current correlators for VA-fields in inverse powers of large momentum and the comparison with the OPE one obtains the set of sum rules for the differences of VA-correlators, which show a rapid decrease at large momenta. Therefore, the chiral symmetry is restored at high energies and one can perform the QCD matching by means of few lowest lying VA-resonances that gives a set of constraints on parameters for the VA, $SU(2)$ QQM. In particular, the residues for the ground and excited VA states turned out to be of the same order of magnitude unlike the situation in the SP case. The inclusion of excited states did not change the electromagnetic decay constants of the ground VA states compared with the one-channel results: $f_\rho \approx 0.15$, $f_{a_1} \approx 0.06$.
4. The main results of our work have been obtained by means of the method of effective action and on the basis of the idea of chiral symmetry restoration at high energies and the OPE of two-point correlators for vector and axial-vector quark densities. All calculations have been performed in large-$N_c$ and log-approximations.
5. We conclude that the VA Quasilocal Quark Model reflects the phenomenology of low and intermediate energy meson physics and the matching to non-perturbative QCD based on the chiral symmetry restoration at high energies improves the predictability of the models for the spectral characteristics of vector and axial-vector mesons.

## Acknowledgements

We are grateful to A. A. Andrianov for the numerous fruitful discussions and attention to our work.

## Appendix

In this Appendix we calculate in the mean field approach the two-point vector correlator for the two-channel QQM. After introducing external vector fields $V^i_{k,\mu}$ and integrating over $\rho^i_{k,\mu}$ one has for the quadratic in fields part of the effective action:

$$\frac{12\pi^2}{N_f N_c} S^{(2)}_{\text{eff}}(V_\mu) = -\frac{9}{8}\Lambda^4 V^T_\mu \hat{K}\left(\hat{A}p^2 + \hat{B}^\rho\right)^{-1}\hat{A}\left(\hat{B}^\rho\right)^{-1}\hat{K}V_\nu(-p^2\delta_{\mu\nu}$$

$$+p_\mu p_\nu) + \frac{3}{4}\Lambda^2 V^T_\mu\left(\hat{K} - \frac{3}{2}\Lambda^2\hat{K}\left(\hat{B}^\rho\right)^{-1}\hat{K}\right)V_\nu\delta_{\mu\nu}\,, \tag{60}$$

where

$$\hat{K} = \begin{pmatrix} 1 - \frac{4\bar{\Delta}_{11}}{3\Lambda^2} & -\frac{4\bar{\Delta}_{12}}{3\Lambda^2} \\ -\frac{4\bar{\Delta}_{12}}{3\Lambda^2} & 1 - \frac{4\bar{\Delta}_{22}}{3\Lambda^2} \end{pmatrix}.$$

In order to obtain the correlator of local currents (1) one needs to project the expression (60) on the local current:

$$\Pi^V_{\mu\nu}(p^2) = \text{tr}\left\{\hat{P}\frac{\delta^2 S^{(2)}_{\text{eff}}}{\delta V_\mu \delta V_\nu}\right\}, \qquad \hat{P} = \begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{pmatrix},$$

where $\hat{P}$ is the projection operator. It is a formal way to obtain the relation (47).

The result is:

$$\Pi^V_{\mu\nu}(p^2) = \frac{Z_1\left(ap^2 + b + c\right)\left(-p^2\delta_{\mu\nu} + p_\mu p_\nu\right)}{24\left(\ln\frac{\Lambda^2}{M_0^2} - 2\right)\det\hat{\bar{\Delta}}\left(p^2 + m_\rho^2\right)\left(p^2 + m^2_{\rho'}\right)} +$$

$$+\frac{3}{4}\Lambda^2\left(\frac{3d\Lambda^2}{4\det\hat{\bar{\Delta}}} - 4\right)\delta_{\mu\nu}\,. \tag{61}$$

In the Eq. (61) we introduced the following notations:

$$a \equiv -6\left(\ln\frac{\Lambda^2}{M_0^2} - 2\right)\left[d + 8\varepsilon\det\hat{\bar{\Delta}} + \varepsilon^2\det\hat{\bar{\Delta}}(\bar{\Delta}_{11} + 3\bar{\Delta}_{22} - 2\sqrt{3}\bar{\Delta}_{12})\right],$$

$$b \equiv 8\left(\ln\frac{\Lambda^2}{M_0^2} - 2\right)\left(\bar{\Delta}_{22} + \sqrt{3}\bar{\Delta}_{12} + \varepsilon\det\hat{\bar{\Delta}}\right)^2,$$

$$c \equiv \left(d + 4\varepsilon\det\hat{\bar{\Delta}}\right)^2, \qquad \varepsilon \equiv -\frac{4}{3\Lambda^2},$$

and $d$, $Z_1$ are given by (27), (53). The residues in poles of the transverse part of the correlator (61) are:

$$Z_\rho = \frac{Z_1\left(-am_\rho^2 + b + c\right)}{24\left(\ln\frac{\Lambda^2}{M_0^2} - 2\right)\det\hat{\hat{\Delta}}\left(m_{\rho'}^2 - m_\rho^2\right)} \tag{62}$$

$$Z_{\rho'} = \frac{Z_1\left(-am_{\rho'}^2 + b + c\right)}{24\left(\ln\frac{\Lambda^2}{M_0^2} - 2\right)\det\hat{\hat{\Delta}}\left(m_\rho^2 - m_{\rho'}^2\right)} \tag{63}$$

One can check that the expressions (62), 63) in the large-log approximation turn into corresponding quantities (54), (55).

The exact calculation of axial-vector correlator can be also fulfilled after taking into account the $P - A$ mixing effect.

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Поступило 28 ноября 2002 г.