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TWO-HIGGS DOUBLET MODEL FROM QUASILOCAL QUARK INTERACTIONS

Abstract. Quark models with four-fermion interaction including derivatives of fields in the strong coupling regime are used to implement composite-Higgs extensions of the Standard model. In this approach the dynamical breaking of chiral symmetry occurs in two (or more) channels (near polycritical values for coupling constants), giving rise to two (or more) composite Higgs doublets. Two types of models are built for which Flavour Changing Neutral Currents (FCNC) are naturally suppressed. In the first Model I the second Higgs doublet is regarded as a radial excitation of the first one. In the second Model II the quasilocal Yukawa interaction with Higgs doublets reduces at low energies to a conventional local one where each Higgs doublet couples to a definite charge current and its vacuum expectation value brings the mass either to up- or to down-components of fermion doublets. For the special configuration of four-fermion coupling constants the dynamical CP-violation in the Higgs sector appears as a result of complexity of v.e.v. for Higgs doublets.

1. Introduction

The fundamental particles of the Standard Model (SM) of electroweak interactions, leptons, quarks and gauge bosons, acquire masses through the interaction with a scalar field (Higgs boson). The mass generation is mediated by the Higgs mechanism which rests on the Electroweak Symmetry Breaking (EWSB). To accommodate the well-established electromagnetic and weak phenomena, the Higgs mechanism requires the existence of at least one iso-doublet, complex scalar field. After absorbing three Goldstone modes to build massive states of $W^{\pm}$, Z bosons, one degree of freedom remains, corresponding to a real scalar particle (Higgs boson). If SM holds valid as a weak-coupling theory till very high energies then this particle cannot have a mass heavier than few hundreds of GeV. Thus the search for the Higgs boson is one of the fundamental quests for testing the minimal SM. Current estimations based on the different theoretical requirements and experimental implications [1, 2] give the SM Higgs mass in the "intermediate mass" window $65 < M_H < 200\,GeV$

for a top quark mass value of about 175 GeV [3]. Despite of the recent successes of the SM in its excellent agreement with the precision measurements at present energies [4], it is generally believed that the SM is not the final theory of elementary particle interactions.

There are many extensions of the SM which lead to the enlargement of the Higgs sector of the SM. For instance, the Minimal Supersymmetric Standard Model (MSSM) [5] entails two elementary Higgs doublets at low energies, the Two-Higgs-Doublet Model (2HDM) contains two complex $SU(2)_L$-doublet scalar fields with hypercharge $Y = \pm 1$ to couple the up-type / down-type right-handed quarks to its Higgs doublet. The search for relations between the many Higgs-field dynamics and the masses of t-quark and Higgs boson give the selection rule for a particular model beyond the SM as well as for its acceptable parameters [6–9]. In more complicated theories (see [1] and references therein) such as SUSY SM ones, E_6 ones, or Left-Right symmetric ones [10], several neutral scalars, charged scalars and even double-charged scalars are required in order to give all amplitudes acceptable high-energy behavior [1, 2].

However there exists an alternative possibility [11] to restrict the number of elementary particles to the observable fermion and vector-boson sector with generation of scalar Higgs particles due to attractive self-fermion interaction. Namely, the quark self-interaction may be responsible for the production of quark-antiquark bound states which are identified as composite Higgs particles. The idea that the Higgs boson could be a bound state of heavy quark pairs has been developed and worked out in a series of papers [11], being motivated by the earlier work of Nambu and Jona–Lasinio (NJL) [12]. In particular, for t- quarks, it is provided by the Top-Mode Standard Model (TSM) Lagrangian, known also as the Bardeen–Hill–Lindner (BHL) Lagrangian [11, 13, 14]. The possibility that multiple four-fermion interactions (for three and a heavy fourth generations) are important in EWDSB, leading to an effective 2HDM at low energies, has been investigated in [15]. In this Model Higgs boson induced Flavour Changing Neutral Currents (FCNC's) are naturally suppressed [16]. Some recent theoretical aspects and questions of $\bar{t}t$-condensation frameworks one can find in the review of [17]. In these scenarios the heavy top mass is explained by the "top-condensation" where new strong forces lead to the formation of $\bar{t}t$ bound states and the EWSB. In a minimal version of quark models the top-condensation was triggered by a local four-fermion interaction.

The main goal of this paper is to give the description of the design of the Quasilocal Quark Models of type I and type II which provide two composite Higgs doublets and in principle satisfy phenomenological restrictions on FCNC suppression. The particular, sample choice of form-factors and bare Yukawa coupling constants is made to obtain estimations for typical mass spectra in Model I and Model II.

We propose the quark models with *Quasilocal* four-fermion interaction [18] where the derivatives of fermion fields are included into vertices to influence on the formation of the second Higgs doublet. Such extensions of the Higgs sector lead to a broad spectrum of excited bound states, moreover they may be viewed as more natural than other, above mentioned extensions since the particles involved in EWSB form only a ground state spectrum generic for SM. In these Quasilocal NJL-like quark models (QNJLM) the symmetries do not forbid further higher dimensional vertices and one should expect that the ground states could be accompanied by (radial) excitations with identical quantum numbers but much higher masses [19–22].

Thus, from the viewpoint of the 2HDM SM, the QNJLM are attractive because: i) it is an extension of the minimal TSM which adds new phenomena (e.g. a broad mass spectrum of bound states including charged Higgs bosons); ii) it is a minimal extension in that it adds the fewest new arbitrary constants; iii) it easily satisfies theoretical constraints on $\rho \simeq 1$ and the tree-level FCNC's suppression [16] in accordance with the experimental evidence; iv) such a Higgs structure is required in order to build a model with the CP-violation [23] because the one-Higgs doublet interaction does not provide any effect of dynamical CP-violation. We shall show in a toy model with quasilocal four-fermion interaction how P-parity breaks down dynamically for the special choice of coupling constants [24].

This article is organized as follows: Section 2 contains the simplest Gross-Neveu Model which reminds how the Dynamical Chiral Symmetry Breaking (DCSB) arises in the scalar channel due to strong interaction. In Sec. 3, we formulate the main rules for the construction of the QNJLM which admits the polycritical regime. Here the effective potential and the mass spectrum for composite scalar and pseudoscalar states are derived for them. For more evidence, in Subsec. 3.3, we investigate the two-channel QNJLM in the large-log approximation. In the vicinity of the tricritical point all possible solutions are analyzed. It turns out that there exist three phases with different correlation lengths in the scalar channel. Moreover the special phase of dynamical P-parity breaking is found. In Secs. 4 and 5 two types of models are built for which FCNC suppression may be naturally implemented. In Sec. 4, the first extension of the SM composite two-Higgs bosons for QNJLM (2HQM) is proposed where the second Higgs doublet is regarded as a radial excitation of the first one. The second model is constructed in Sec. 5 so that the quasilocal Yukawa interaction with Higgs doublets reduces at low energies to a conventional local Yukawa vertex where each Higgs doublet couples to a definite charge current and its v.e.v. brings the mass either to up- or to down-components of fermion doublets. In this version the top and bottom masses are explained by "top-, bottom- condensations." On the base of the effective potential for the Model II and the positivity of its variation the mass spectrum for composite states is investigated. It is interesting that for the special configuration of coupling constants the dynamical CP-violation appears in the Higgs sector. In the Summary we discuss the obtained results and a possibility to use them in different aspects of high energy physics. The Appendix contains the calculation of the matrices of the second variations for composite two-Higgs bosons in Model I, II and the effective potential of Model II for the special choice of quasilocal formfactors.

2. DCSB in Models with Four-Fermion Interaction and The Critical Regime

Let us remind how the Dynamical Chiral Symmetry Breaking (DCSB) arises in a model with local 4-fermion interaction due to strong attraction in the scalar channel. The simplest, Gross-Neveu (GN) Model retaining the scalar channel only can be presented by the Lagrangian density, in two forms (the Euclidean-space formulation is taken here),

$$\mathcal{L} = \bar{q}\, \not{D} q + \frac{g^2}{4N_c\Lambda^2}\,(\bar{q}q)^2 = \bar{q}\big(\not{D} + i\phi(x)\big)\, q + \frac{N_c\Lambda^2}{g^2}\,\phi^2(x), \qquad (1)$$

where $\not{D} = i\gamma_\mu\partial_\mu$ and $q \equiv (q_i)$ stands for color fermion fields with N_c components. For the time being we take the number of flavours $N_F = 1$ and the current quark mass $m_q = 0$. In Eq. (1) the scalar auxiliary field $\phi(x)$ (a prototype of the Higgs field) is introduced in order to describe the dynamical symmetry breaking phenomenon in the large-N_c limit.

This model is implemented by an $O(4)$-symmetric momentum cutoff Λ for the fermion energy spectrum. For a quark model the cutoff Λ can be thought of as a separation scale which appears when evaluating the SM low-energy effective action from a more fundamental theory. The regularized effective action $S_{\rm eff}$ for auxiliary field,

$$Z^{\Lambda}(\phi) = \exp(-S_{\rm eff}) \, = \, \left\langle \exp\left(- \int d^4x\, \mathcal{L}(\phi(x)) \right) \right\rangle_{\bar{q}q}, \qquad (2)$$

possesses the mean-field extremum on constant configurations $\phi = <\phi> \equiv m_d = \text{const}$.

The relevant effective potential $V_{\rm eff}$ can be obtained by integration over fermions,

$$V_{eff}(\phi) = \frac{S_{\rm eff}}{(\text{vol.})} = \frac{N_c}{8\pi^2} \left\{ \frac{\Lambda^4}{2} \left(\frac{1}{2} - \ln \frac{\Lambda^2 + \phi^2}{\mu^2} \right) - \right.$$

$$\left. - \frac{\phi^2 \Lambda^2}{2} + \frac{\phi^4}{2} \ln \frac{\Lambda^2 + \phi^2}{\phi^2} + \frac{8\pi^2 \Lambda^2\, \phi^2}{g^2} \right\}, \qquad (3)$$

where the constant μ is a normalization scale for quark fields. Its extrema can be derived from the mass-gap equation,

$$R(\phi) \equiv \frac{4\pi^2}{N_c} \cdot \frac{\partial V_{eff}}{\partial \phi} = \phi \left(\left(\frac{8\pi^2}{g^2} - 1 \right) \Lambda^2 + \phi^2 \ln \frac{\Lambda^2 + \phi^2}{\phi^2} \right) = 0. \qquad (4)$$

The main contribution into Eq. (4) is given by a tadpole term in the fermion loop which is related to (v.e.v.) of the scalar fermion density,

$$R(\phi) = \phi \frac{8\pi^2 \Lambda^2}{g^2} + i \frac{4\pi^2}{N_c} \langle\, \bar{q}q \,\rangle. \qquad (5)$$

The cutoff independence is realized with aid of fine-tuning, $8\pi^2/g^2 \simeq 1 - O(1/\Lambda^2)$. In the language of the theory of critical phenomena it is equivalent to developing our model around a critical or scaling point where the quantum system undergoes the second-order phase transition. By definition the critical coupling constant is $g^2_{\rm crit} = 8\pi^2$. When $g^2 < g^2_{\rm crit}$ the only solution of mass-gap Eq. (4) is $\phi = 0$, while for $g^2 > g^2_{\rm crit}$ there exists another nontrivial solution for dynamical mass $m_d \neq 0$ which brings the true minimum for $V_{\rm eff}$. Meanwhile the symmetric solution $\phi = 0$ does not provide then a minimum anymore but realizes a maximum.

The fine-tuning states that the strong Λ^2-dependence should be compensated by the corresponding term in the coupling constant,

$$\frac{8\pi^2}{g^2} = 1 - \frac{m_0^2}{\Lambda^2}. \qquad (6)$$

Its practical meaning is evident, namely, one produces a mass scale for physical states which is much less than the cutoff scale governing large radiative corrections. The deviation scale $m_0^2 << \Lambda^2$ determines the physical mass of scalar meson. Indeed its kinetic term can be obtained from the second variation of $S_{\rm eff}$ by calculating the one-fermion loop diagram (see App. Fig. 1),

$$\begin{aligned} S_{\rm eff} &\simeq S_{\rm eff}(\phi = m_d) + \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\,\tilde{\phi}(-p)\Gamma(p)\,\tilde{\phi}(p), \\ \phi &= m_d + \tilde{\phi}, \end{aligned} \tag{7}$$

where the inverse propagator of scalar field reads:

$$\begin{aligned} \Gamma(p) &= \frac{2N_c\Lambda^2}{g^2} - N_c \int\limits_{k<\Lambda} \frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\,\left[(\not k + \not p/2 + im_d)^{-1}(\not k - \not p/2 + im_d)^{-1}\right] \\ &= (m_\phi^2 + p^2)I(p^2) + O\left(\frac{1}{\Lambda^2}\right), \end{aligned} \tag{8}$$

in the chirally invariant regularization of the fermion loop. The scalar meson mass is given by the remarkable Nambu relation $m_\phi \simeq 2m_d$ and the formfactor $I(p)$ is determined by the relation,

$$I(p) = 2N_c \int\limits_{k<\Lambda} \frac{d^4k}{(2\pi)^4}\frac{1}{(k+\frac{1}{2}p)^2 + m_d^2}\,\frac{1}{(k-\frac{1}{2}p)^2 + m_d^2}. \tag{9}$$

In order that the physical mass parameters were insensitive to Λ, i.e. $\partial_\Lambda m_d = 0$, the scale m_0 should be weakly dependent, $m_0^2 \sim m_d^2 \ln(\Lambda^2/m_d^2)$, on the cutoff Λ.

What have we learned from the above model?

(i) The cut-off theory can be used for processes involving momenta p much less than Λ for the purposes of discarding high-energy states from the theory.

(ii) The mass scale of meson states is assumed to be much less than Λ which is implemented in the vicinity of critical values of coupling constants.

(iii) As a result of DCSB in these models only one type of scalar mesons (i.e. eventually one Higgs doublet) is created in the large-N_c approach.

(iv) In such models the (radial) excitations of composite meson states are not present in the large-N_c approach.

Meantime the conventional quark models with local four-fermion interaction may not represent a consistent part of the Beyond-Standard Model (BSM) effective action and conceivably they shall be extended with inclusion of higher dimensional vertices which are not forbidden by symmetries and induce the appearance of a reach spectrum of excited composite meson states.

3. Quasilocal Quark Models and Polycritical Regime

3.1. Dominant higher-dimensional vertices in DCSB phase

In order to involve in the theory the effects of the discarded states at scales of order Λ it is needed to adjust the existing couplings constants in the Lagrangian and to add new, quasilocal, non-renormalizable interactions (vertices). These vertices are polynomial in the fields and derivatives of the fields and only a finite number of interactions is required when working to a particular order in $\frac{p}{\Lambda}$, where p is a typical momentum in whatever process is under study.

We examine the DCSB patterns in the mean-field approach (large-N_c limit) and estimate the vertices with any number of fermion legs and derivatives. The main rule to select out relevant vertices is derived from the requirement of insensitivity in respect to the separation scale Λ, following the conception of low-energy effective action [18].

We assume that:

(i) Λ^2-order contributions from different vertices are dominant in creating the DCSB-critical surface that is provided by cancellation of all contributions of Λ^2-order and defines the polycritical regime;

(ii) Λ^0-order contributions from vertices assemble in the mean-field action to supply fermions with dynamical mass $m_d << \Lambda$ which establishes the low-energy physical scale;

(iii) respectively Λ^{-2} (etc.)-order contributions are irrelevant at energies much lower than Λ and so may be dropped from the theory if such accuracy is unnecessary.

In the large-N_c approach the following approximation for v.e.v. of fermion operators is valid,

$$\langle (\bar{q}q)^n \rangle = \Big(\langle \bar{q}q \rangle\Big)^n \Big(1 + O(1/N_c)\Big), \qquad (10)$$

where any number of derivatives can be inserted between antifermion and fermion operators.

V.e.v. of a bilinear operator is estimated in the assumption that quarks obtain a dynamical mass. Namely,

$$\left\langle \bar{q}\left(\frac{\partial^2}{\Lambda^2}\right)^n q \right\rangle \sim \frac{1}{\Lambda^{2n}} \int\limits_{|p|<\Lambda} \frac{d^4p}{(2\pi)^4} \operatorname{tr} \frac{p^{2n}}{\not{p} + im_d} \sim N_c m_d \Lambda^2. \qquad (11)$$

One can see that the vertices with derivatives in many-fermion interaction are not suppressed and play equal role in the mass-gap equation.

We omit the full classification of effective vertices relevant in the mass-gap Eq. (see [18]) and report only the minimal structure of the QNJLM which admits the polycritical regime,

$$\mathcal{L} = \bar{q}\, \not{D} q + \frac{1}{N_c\Lambda^2} \sum_{m,n=0}^{l} a_{mn}\, \bar{q}_R f_n\left(\frac{-\partial^2}{\Lambda^2}\right) q_L \cdot \bar{q}_L f_m\left(\frac{-\partial^2}{\Lambda^2}\right) q_R, \quad (12)$$

where a_{mn} is a hermitian matrix of coupling constants without zero eigenvalues and it is taken to be real symmetric one in order that the interaction did not break the CP-parity explicitely. Chiral fermion fields are given by $q_{L(R)} = 1/2(1 \pm \gamma_5)q$. We define the vertex formfactors to be polynomials of derivatives,

$$f_m(\tau) = \sum_{i=0}^{K_m} f_m^{(i)} \tau^i, \qquad (13)$$

to have quasilocal interactions. The variable τ is related to derivatives, $\tau \to -\partial^2/\Lambda^2$. We adopt the following rule for derivative action which provide the hermiticity of fermion currents:

$$\bar{q}\frac{\partial^2}{\Lambda^2} q \equiv \frac{1}{4}\bar{q}\left(\frac{\vec{\partial} - \overleftarrow{\partial}}{\Lambda}\right)^2 q. \qquad (14)$$

Besides, let us regularize the interaction vertices with the help of a momentum cutoff,

$$\bar{q}q \longrightarrow \bar{q}\theta(\Lambda^2 + \partial^2)q. \qquad (15)$$

Without loss of generality one can choose formfactors $f_i(\tau)$ being orthogonal polynomials on the unit interval,

$$\int\limits_0^1 d\tau f_m(\tau) f_n(\tau) = \delta_{mn}. \qquad (16)$$

Let us now introduce the appropriate set of auxiliary fields $\phi_n(x) \sim$ const and develop the mean-field approach,

$$\mathcal{L}(\phi) = \bar{q}\left(\not{D} + iM(\phi)P_L + iM^{+}(\phi)P_R\right)q + N_c\Lambda^2 \sum_{m,n=1}^{l} \phi_m^{*}\, a_{mn}^{-1}\, \phi_n . \quad (17)$$

The dynamical mass functional is a linear combination of formfactors,

$$M(\phi) \equiv \sum_{n=1}^{l} \phi_n(x) f_n\left(\frac{-\partial^2}{\Lambda^2}\right). \quad (18)$$

In accordance with Eq. (14) the differential operator $M(\phi)$ is understood as a Weyl ordered or fully antisymmetrized product of functions ϕ_n and derivatives. Thereby we come to a model with l channels. When integrating out the fermion fields one obtains the effective action of ϕ^*, ϕ - fields. The effective potential $V_{\rm eff}$ is proved to be a functional depending on the dynamical mass functional $M(\phi^*, \phi)$ and proportional to N_c that allows us to use the saddle point approximation for $N_c >> 1$.

3.2. Effective potential and equations on the mass spectrum for QNJL model

The effective potential for auxiliary fields can be derived with the momentum cutoff regularization by averaging over quark fields:

$$V_{\rm eff}(\phi) = N_c\Lambda^2 \sum_{m,n=1}^{l} \phi_m^{\star} a_{mn}^{-1}\phi_n - \frac{N_c\Lambda^4}{8\pi^2}\int_0^1 d\tau\tau \ln\left(1 + \frac{|M(\tau)|^2}{\Lambda^2\tau}\right)$$

$$= \frac{N_c}{8\pi^2}\Bigg[\Lambda^2 \sum_{m,n=1}^{l} \phi_m^{\star}(8\pi^2 a_{mn}^{-1} - \delta_{mn})\phi_n + \frac{1}{2}|M_0|^4\left(\ln\frac{\Lambda^2}{|M_0|^2} + \frac{1}{2}\right)$$

$$+\frac{1}{2}\int_0^1 \frac{d\tau}{\tau}(|M(\tau)|^4 - |M_0|^4) + O\left(\frac{1}{\Lambda^2}\right)\Bigg], \quad (19)$$

herein $M_0 \equiv M(0)$. The last approximation is valid in such a strong coupling regime where the dynamical mass $M_0 << \Lambda$. This regime is of our main interest and it is realized in the vicinity of a (poly)critical surface. The critical values of coupling constants, $a_{mn}^c = \delta_{mn}/8\pi^2$, are found from the cancellation of quadratic divergences. In this paper we study the critical regime in all l channels. The vicinity of this polycritical point is described by the following parametrization:

$$8\pi^2 a_{mn}^{-1} \sim \delta_{mn} + \frac{\Delta_{mn}}{\Lambda^2}, \qquad |\Delta_{mn}| \ll \Lambda^2. \tag{20}$$

The generalized mass gap equations,

$$\frac{\delta V_{\text{eff}}(\phi,\phi^*)}{\delta\phi_m^*} = 0 = \frac{\delta V_{\text{eff}}(\phi,\phi^*)}{\delta\phi_m}, \tag{21}$$

deliver the extremum to the effective potential which may cause the DCSB if it is an absolute minimum. They read:

$$\begin{aligned}\sum_{n=1}^{l}\Delta_{mn}\phi_n &= \int_0^1 \frac{d\tau}{\tau + \frac{|M(\tau)|^2}{\Lambda^2}}|M(\tau)|^2 M(\tau) f_m(\tau)\\ &\simeq f_m(0)|M_0|^2 M_0 \ln\frac{\Lambda^2}{M_0^2}\\ &+\int_0^1 \frac{d\tau}{\tau}(|M(\tau)|^2 M(\tau) f_m(\tau) - |M_0|^2 M_0 f_m(0)).\end{aligned} \tag{22}$$

It can be seen from the first relation that,

$$\sum_{m,n=1}^{l}\phi_m^\star\Delta_{mn}\phi_n = \int_0^1 d\tau\frac{|M(\tau)|^4}{\tau + \frac{|M(\tau)|^2}{\Lambda^2}} \geq 0, \tag{23}$$

which means that for the existence of a non-trivial dynamical mass it is necessary to have at least one positive eigenvalue of the matrix Δ_{mn}. However not all the solutions provide a minimum (see, the analysis of two-channel models in [24, 19]).

The true minimum is derived from the positivity of the second variation of the effective action around a solution of the mass-gap equation,

$$\phi_m =< \phi_m > + \sigma_m(x) + i\pi_m(x). \tag{24}$$

This variation reads:

$$\begin{aligned}&\frac{4\pi^2}{N_c}\delta^2 S_{\text{eff}} \equiv \left(\sigma, (A^{\sigma\sigma}p^2 + B^{\sigma\sigma})\sigma\right)\\ &+2\left(\pi, (A^{\pi\sigma}p^2 + B^{\pi\sigma})\sigma\right) + \left(\pi, (A^{\pi\pi}p^2 + B^{\pi\pi})\pi\right),\end{aligned} \tag{25}$$

where two symmetric matrices - for the kinetic term $\hat{A} = \left(A^{ij}_{mn}\right)$, $i, j = (\sigma, \pi)$ and for the constant, momentum independent part, $\hat{B} = \left(B^{ij}_{mn}\right)$ – have been introduced.

The positivity of the second variation corresponds to the formation of physical mass spectrum for composite scalar and pseudoscalar states which can be found from zeroes of the second variation determinant at the Minkovski momenta ($p^2 < 0$),

$$\det(\hat{A}p^2 + \hat{B}) = 0. \tag{26}$$

Matrix elements of $\hat{B}$ are given by the following relations:

$$\begin{aligned} B^{\sigma\sigma}_{mn} = 6 \int\limits_0^1 \frac{d\tau}{\tau} \Big[(\mathrm{Re}\, M)^2 f_m(\tau) f_n(\tau) - M_0^2 f_m(0) f_n(0) \Big] \\ + M_0^2 f_m(0) f_n(0) \left(6 \ln \frac{\Lambda^2}{M_0^2} - 4 \right) - 2\Delta_{mn} \\ + 2 \int\limits_0^1 \frac{d\tau}{\tau} (\mathrm{Im}\, M)^2 f_m(\tau) f_n(\tau), \end{aligned} \tag{27}$$

$$\begin{aligned} B^{\pi\pi}_{mn} = 2 \int\limits_0^1 \frac{d\tau}{\tau} \Big[(\mathrm{Re}\, M)^2 f_m(\tau) f_n(\tau) - M_0^2 f_m(0) f_n(0) \Big] \\ + 2 M_0^2 f_m(0) f_n(0) \ln \frac{\Lambda^2}{M_0^2} - 2\Delta_{mn} \\ + 6 \int\limits_0^1 \frac{d\tau}{\tau} (\mathrm{Im}\, M)^2 f_m(\tau) f_n(\tau), \end{aligned} \tag{28}$$

$$B^{\sigma\pi}_{mn} = 4 \int\limits_0^1 \frac{d\tau}{\tau} (\mathrm{Re}\, M)(\mathrm{Im}\, M) f_m(\tau) f_n(\tau), \tag{29}$$

where the terms of $1/\Lambda^2$-order are neglected.

When exploiting the mass-gap equation (22) one can prove that the matrix $\hat{B}$ has always a zero eigenvalue related to the eigenvector $\phi^0_m = < \pi_m > - i \cdot < \sigma_m >$. It corresponds to the arising of the Goldstone mode (the massless Goldstone bosons).

The kinetic energy matrix $\hat{A}$ turns out to be block-diagonal [24],

$$A^{\sigma\pi}_{mn} = 0, \quad A^{\pi\pi}_{mn} = A^{\sigma\sigma}_{mn}\left[1 + \left(\frac{1}{\ln\frac{\Lambda^2}{\mu^2}}\right)\right], \tag{30}$$

$$A^{\sigma\sigma}_{mn} = \frac{1}{2}\left[f_m(0)f_n(0)\left(\ln\frac{\Lambda^2}{M_0^2} + O(1)\right)\right.$$

$$\left. + \int\limits_0^1 [f_m(\tau)f_n(\tau) - f_m(0)f_n(0)]\,\frac{d\tau}{\tau}\right] + O\left(\frac{1}{\Lambda^2}\right), \tag{31}$$

herein we have displayed the leading terms only in the large-log approximation. The more detailed expression can be found in [24, 19].

3.3. Quasilocal Two-Channel quark models and possibility of dynamical breaking of P-parity

For the further investigation of composite Higgs extensions of the Standard Model let us consider the Quasilocal Two-Channel quark model in a tricritical point [24, 19]. We set $m, n = 1, 2$ in (12)–(20) and retain only the lowest derivatives in the potential, with $f_1 = 1$, $f_2 = \sqrt{3}(1-2\tau)$. The dynamical mass function is thereby, $M(\phi) = \bar{\phi}_1 + \bar{\phi}_2\sqrt{3}(1-2\tau)$. As $\bar{\phi}_j$ are complex functions, $M(\phi)$ is complex too. However, with the global chiral rotation $M(\phi) \to M(\phi)e^{i\omega}$, $\omega = \mathrm{const}$ it is always possible to implement $\mathrm{Im} < M_0 >_\phi = 0$ and we can choose the following parameterization:

$$\bar{\phi}_1 = \phi_1 + i\rho, \quad \bar{\phi}_2 = \phi_2 - i\frac{\rho}{\sqrt{3}}, \quad \phi_i \equiv \mathrm{Re}\,\bar{\phi}_i. \tag{32}$$

The equations (21) for the Two-Channel Model read:

$$\Delta_{11}\phi_1 + \Delta_{12}\phi_2$$

$$= M_0^3 \ln\frac{\Lambda^2}{M_0^2} - 6\sqrt{3}\phi_1^2\phi_2 - 18\phi_1\phi_2^2 - 8\sqrt{3}\phi_2^3 + 2\rho^2\left(\phi_1 - \frac{1}{\sqrt{3}}\phi_2\right),$$

$$d_1\phi_1 - d_2\phi_2 = 2\sqrt{3}\phi_1(\phi_1^2 + 3\phi_2^2) + 2\rho^2\left(\frac{4}{\sqrt{3}}\phi_1 - 2\phi_2\right),$$

$$\rho(\sqrt{3}\Delta_{11} - \Delta_{12}) = 2\rho\sqrt{3}\left(\phi_1^2 + \phi_2^2 + \frac{4}{3}\rho^2\right), \tag{33}$$

where

$$d_1 = \sqrt{3}\Delta_{11} - \Delta_{12}, \qquad d_2 = -\sqrt{3}\Delta_{21} + \Delta_{22} \tag{34}$$

We analyze the equations (33) near a polycritical point, $|\Delta_{ij}| \sim \mu^2 \ll \Lambda^2$, in the large-log approximation ($\ln \frac{\Lambda^2}{\mu^2} \gg \ln\ln \frac{\Lambda^2}{\mu^2}$). It gives rise to a set of solutions.

For $\rho = 0$ all the solutions are divided into the following classes:

a) Gross-Neveu-like solutions ϕ_j^{GN} are:

$$\phi_1^2 = \frac{d_2^2 \det \Delta}{(\sqrt{3}d_1 + d_2)^3 \ln \frac{\Lambda^2}{\mu^2}} \left[1 + \left(\frac{1}{\ln \frac{\Lambda^2}{\mu^2}}\right)\right], \qquad \phi_2 \approx \frac{d_1}{d_2}\phi_1. \tag{35}$$

These solutions deliver minima to the potential when $\sqrt{3}d_1 + d_2 < 0$, with one eigenvalue of the matrix Δ being in the over-critical regime and the other one in the sub-critical.

b) Abnormal solutions are:

$$\phi_1^2 = \frac{\sqrt{3}d_1 + d_2}{12}\left[1 + O\left(\frac{1}{\ln^{1/3} \frac{\Lambda^2}{\mu^2}}\right)\right], \qquad \phi_2 \approx -\frac{\phi_1}{\sqrt{3}}, \tag{36}$$

they correspond to the suppression of the large log-terms in Eqs. (33) of motion and give minima to the potential, when $\sqrt{3}d_1 + d_2 > 0$, $\sqrt{3}d_1 - 2d_2 \neq 0$ (either both eigenvalues of Δ are positive, or one is positive and the other one is negative).

c) On the planes $\sqrt{3}d_1 + d_2 = 0$ and $\sqrt{3}d_1 - 2d_2 = 0$ there appear special solutions with different, peculiar asymptotics [24, 19].

d) In general, in the models with more than one channel complex solutions are allowed, and the imaginary parts of all the variables ϕ_j cannot be removed simultaneously by a global chiral rotation. However the complex solutions ($\rho \neq 0$) minimize the effective action only (!) for the narrow domain in the vicinity of the plane $\sqrt{3}d_1 - 2d_2 = 0$. Their asymptotic expressions are:

$$\phi_1^2 = \frac{d_1 + 4\Delta_{12}}{16\sqrt{3}\left(\ln \frac{\Lambda^2}{\mu^2} - 3\right)}, \qquad \phi_2 \approx -\sqrt{3}\phi_1, \tag{37}$$

and the dynamical mass is $m_c^2 = 4\phi_1^2$. The axial part of the mass function looks as follows:

$$\rho^2 = \frac{d_1\sqrt{3}}{8} - \frac{3}{4}(\phi_1^2 + \phi_2^2) = \frac{d_1\sqrt{3}}{8}\left[1 + O\left(\frac{1}{\ln \frac{\Lambda^2}{\mu^2}}\right)\right]. \tag{38}$$

In each of the phase space domains mentioned above one finds four common boson states – two scalar and two pseudoscalar – for real ϕ_j, and, in general, – for complex ϕ_j, three states with mixed P-parity and the pseudoscalar one with zero mass, the latest is in accordance to the Goldstone theorem.

The mass spectrum of related bosonic states (collective excitations) is determined by zero-modes of the matrix of second variations of the effective potential (25) and respectively by Eqs. (26)–(31). Taking into account the conditions necessary for a minimum of the potential, we find the solutions at $-m^2 = p^2 \leqslant 0$, giving physical values of particle masses.

In the case of $\rho = 0$:

a) NJL-like mass spectrum:

$$m_\pi^2 = 0 \quad m_{\pi'}^2 \approx m_{\sigma'}^2 \approx -\frac{\sqrt{3}d_1 + d_2}{3},$$
$$m_\sigma^2 \approx 4m_d^2, \tag{39}$$

in this domain the radial excitation states are heavier than the lightest scalar meson by a factor of logarithm.

b) For the Abnormal solutions we have:

$$m_\pi^2 = 0, \quad m_{\pi'}^2 \approx \frac{1}{9}\left(\frac{4}{3}\right)^{1/3} \frac{(\sqrt{3}d_1 - 2d_2)^{4/3}}{(\sqrt{3}d_1 + d_2)^{1/3}} \frac{1}{\ln^{1/3}\frac{\Lambda^2}{\mu^2}},$$
$$m_\sigma^2 \approx 6m_d^2, \quad m_{\sigma'} \approx \frac{2}{3}(\sqrt{3}d_1 + d_2). \tag{40}$$

When comparing (39) and (40) we find the scalar channel correlation length to be different for each phase, that corresponds to the tricritical point conditions.

c) For the special real solutions the relations between scalar and pseudoscalar meson masses are different from (39), (40) (see [24, 19]).

d) Mass Spectrum in the P-parity Breaking Phase ($\rho \neq 0$). One can see from (37), (38) that in the large-log approximation the axial dynamical mass (the imaginary part of $M(\phi)$) dominates. It leads to appearance of a massless boson in the scalar channel in accordance to the Goldstone theorem. Conventionally, the massless boson is related to be a pseudoscalar meson corresponding to the generation of a real dynamical mass. In order to fit it we make a global chiral rotation of fermionic fields $q \to \exp(i\gamma_5\pi/4)q$ accompanied by corresponding rotation of the bosonic variables $\bar{\phi}_j \to i\bar{\phi}_j$:

$$\bar{\phi}_1 = i\phi_1 - \rho, \quad \bar{\phi}_2 = i\phi_2 + \frac{\rho}{\sqrt{3}}. \tag{41}$$

The classification of states given by the P-parity quantum number is relevant only in the large-log approximation, when:

$$\frac{B^{\pi\sigma}}{B^{\sigma\sigma}} \approx \frac{B^{\pi\sigma}}{B^{\pi\pi}} = O\left(\frac{1}{\ln\frac{\Lambda^2}{\mu^2}}\right), \tag{42}$$

next-to-leading logarithmic effects are of no importance and one can neglect the mixing of states with different P-parity. Then the mass spectrum of mesons is:

$$m_1^2 = 0, \quad m_2^2 \approx \frac{d_1 + 4\Delta_{12}}{\sqrt{3}\ln\frac{\Lambda^2}{\mu^2}} \approx 16\phi_1^2 = 4m_d^2,$$

$$m_3^2 \approx \sqrt{3}d_1, \quad m_4^2 \approx \frac{4(d_1 + \Delta_{12})}{9\sqrt{3}\ln\frac{\Lambda^2}{\mu^2}}. \tag{43}$$

The ratio of m_2 and m_4 does not depend on the logarithm, so both the masses are comparable. On the other hand, in the models with a finite momentum cut-off, when the effects of order of $1/\ln\frac{\Lambda^2}{\mu^2}$ make sense, the dynamical P-parity breaking is induced, since $B_{\pi\sigma} \neq 0$. This phenomenon of dynamical P-parity breaking can be used in extensions of the standard model [11] where several Higgs bosons are composite ones.

Thus we conclude that the models with polycritical (tricritical) points are drastically different from the local NJL models in the variety of the physical phenomena in the DCSB. Explorations of such QNJLM in extensions of the SM are pretty well motivated as the underlying dynamics responsible for the top quark condensate should most likely lead to a broad spectrum of excited states, just like the hadron dynamics with QCD as an underlying force. Moreover the QNJLM may even be viewed as more natural than the extensions to more generations, more elementary Higgses, or to SUSY in the SM since the particles involved in DCSB (with masses of order of the electroweak scale) belong in this context only to the ground state spectrum. In the following sections we shall present an extension of the SM with two-Higgs bosons of the QNJLM where one of the Higgses is a radial excitation of another one.

4. Higgs Bosons as Radial Excitations – Model I

4.1. Effective potential in Model I

Let us construct now the two-flavor quark models with quasilocal interaction in which the t- and b-quarks are involved in the DCSB. In accordance with the SM, the left components of both quarks form a doublet:

$$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \tag{44}$$

which transforms under $SU(2)_L$ group as a fundamental representation while the right components t_R, b_R are singlets.

The Model I which is to satisfy the FCNC suppression has the following Lagrangian:

$$\mathcal{L}_J = \bar{q}_L \not{D} q_L + \bar{t}_R \not{D} t_R + \bar{b}_R \not{D} b_R +$$
$$\frac{8\pi^2}{N_c \Lambda^2} \sum_{k,l=1}^{2} a_{kl} \left(g_{t,k} J_{t,k}^T + g_{b,k} \widetilde{J}_{b,k}^T \right) i\tau_2 \left(g_{b,l} J_{b,l} - g_{t,l} \widetilde{J}_{t,l} \right). \tag{45}$$

Here we have introduced the denotations for doublets of fermion currents:

$$J_{t,k} \equiv \bar{t}_R f_{t,k} \left(-\frac{\partial^2}{\Lambda^2} \right) q_L, \qquad J_{b,k} \equiv \bar{b}_R f_{b,k} \left(-\frac{\partial^2}{\Lambda^2} \right) q_L \tag{46}$$

and the tilde in $\widetilde{J}_{t,k}$ and $\widetilde{J}_{b,k}$ marks charge conjugated quark currents, rotated with τ_2 Pauli matrix

$$\tilde{J}_{t,k} = i\tau_2 J_{t,k}^\star, \qquad \widetilde{J}_{b,k} = i\tau_2 J_{b,k}^\star. \tag{47}$$

The subscripts t, b indicate right components of t and b quarks in the currents, the index k enumerates the formfactors:

$$f_{t,1} = 2 - 3 \left(-\frac{\partial^2}{\Lambda^2} \right), \quad f_{t,2} = -\sqrt{3} \left(-\frac{\partial^2}{\Lambda^2} \right),$$
$$f_{b,1} = 2 - 3 \left(-\frac{\partial^2}{\Lambda^2} \right), \quad f_{b,2} = -\sqrt{3} \left(-\frac{\partial^2}{\Lambda^2} \right). \tag{48}$$

As the spinor indices are contracted to each other in (46), $J_{t,k}$ transforms as a doublet under $SU(2)_L$. τ_2 is a Pauli matrix in the adjoint representation of the group $SU(2)_L$. Coupling constants of the four-fermion interaction are represented by 2×2 matrix a_{kl} and contributed also from the Yukawa constants $g_{k,l}$, $g_{b,k}$.

The Lagrangian density of the Model I (45) to describe the dynamics of composite Higgs bosons can be obtained by means of introduction of auxiliary bosonic variables and by integrating out fermionic degrees of freedom. According to this scheme, we define two scalar $SU(2)_L$-isodoublets:

$$\Phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix} \tag{49}$$

and their charge conjugates:

$$\widetilde{\Phi}_1 = \begin{pmatrix} \phi^\star_{12} \\ -\phi^\star_{11} \end{pmatrix}, \qquad \widetilde{\Phi}_2 = \begin{pmatrix} \phi^\star_{22} \\ -\phi^\star_{21} \end{pmatrix}. \tag{50}$$

In terms of auxiliary fields, the Lagrangian (45) can be rewritten in the following way:

$$\mathcal{L}_J = L_{kin} + \frac{N_c\Lambda^2}{8\pi^2} \sum_{k,l=1}^{2} \Phi^\dagger_k (a^{-1})_{kl} \Phi_l$$

$$+ i \sum_{k=1}^{2} \left[g_{t,k} \widetilde{\Phi}^\dagger_k J_{t,k} + g_{b,k} \Phi^\dagger_k J_{b,k} \right] + \text{ h.c.} \tag{51}$$

The integrating out of fermionic degrees of freedom will produce the effective action for Higgs bosons of which we shall keep only the kinetic term and the effective potential consisting of two- and four-particles vertices. The omitted terms are supposedly small, being proportional to inverse powers of a large scale factor Λ. The Yukawa constants are chosen of the form

$$g_{t,k} = 1; \qquad g_{b,k} = g \tag{52}$$

for $k = (1,2)$. The first constant is set to unit because the fields Φ_1 and Φ_2 can always be multiplied by an arbitrary factor which is absorbed by four fermion coupling constants through redefinition of polycritical coupling constants. The remaining constant g induces thereby the quark mass ratio m_b/m_t. The effective potential for the Model I has the following form:

$$V_{\text{eff}} = \frac{N_c}{8\pi^2} \Bigg(- \sum_{k,l=1}^{2} (\Phi^\dagger_k \Phi_l)\Delta_{kl} + 8(\Phi^\dagger_1\Phi_1)^2 \left(\ln \frac{\Lambda^2}{4(\Phi^\dagger_1\Phi_1)} + \frac{1}{2} \right)$$

$$+ 8g^4 (\Phi^\dagger_1\Phi_1)^2 \left(\ln \frac{\Lambda^2}{4g^2(\Phi^\dagger_1\Phi_1)} + \frac{1}{2} \right)$$

$$- \frac{159(1+g^4)}{8} (\Phi^\dagger_1\Phi_1)^2 + \frac{9(1+g^4)}{8} (\Phi^\dagger_2\Phi_2)^2 + \frac{3(1+g^4)}{4} (\Phi^\dagger_1\Phi_1)(\Phi^\dagger_2\Phi_2) +$$

$$+ \frac{3(1+g^4)}{4} (\Phi^\dagger_1\Phi_2)(\Phi^\dagger_2\Phi_1) + \frac{3(1+g^4)}{8} (\Phi^\dagger_1\Phi_2)^2 + \frac{3(1+g^4)}{8} (\Phi^\dagger_2\Phi_1)^2$$

$$- \frac{5\sqrt{3}(1+g^4)}{4} (\Phi^\dagger_1\Phi_1) \left[(\Phi^\dagger_1\Phi_2) + (\Phi^\dagger_2\Phi_1) \right]$$

$$+\frac{\sqrt{3}(1+g^4)}{4}(\Phi_2^\dagger\Phi_2)\left[(\Phi_1^\dagger\Phi_2)+(\Phi_2^\dagger\Phi_1)\right]\Bigg)+O\left(\frac{\ln\Lambda}{\Lambda^2}\right), \qquad (53)$$

where the bilinear "mass" term is in general non-diagonal and represented by the real, symmetric 2×2 matrix Δ_{kl}.

We assume the electric charge stability of vacuum or, in other words, that only neutral components of both Higgs doublets may have nonzero v.e.v. Hence, one can deal with only neutral components of the Higgs doublets in the effective action for studying DCSB. This part of the Higgs sector can be investigated separately as a model where two singlets (not doublets) appear as composite Higgs bosons. For this purpose, we use the Quasilocal Two-Channel Model which we have already developed in Sec. 3 for the case of one-flavour [29].

Following the definitions made in [24], we relate the fields ϕ_1, ϕ_2 and ρ to the neutral components of Higgs doublets:

$$\phi_{11}=\phi_1; \qquad \phi_{22}=\phi_2+i\rho. \qquad (54)$$

The condition of minimum of the potential (53) with the charged components of Higgs doublets put to zero values: $\phi_{12}=\phi_{21}=0$, brings the mass-gap equations for them:

$$\Delta_{11}\phi_1+\Delta_{12}\phi_2=16\phi_1^3\ln\frac{\Lambda^2}{4\phi_1^2}+16g^4\phi_1^3\ln\frac{\Lambda^2}{4g^2\phi_1^2}$$

$$-(1+g^4)\Big[\frac{159}{4}\phi_1^3-\frac{15\sqrt{3}}{4}\phi_1^2\phi_2+\frac{9}{4}\phi_1\phi_2^2+\frac{\sqrt{3}}{4}\phi_2^3+\frac{\rho^2}{4}(3\phi_1+\sqrt{3}\phi_2)\Big], \qquad (55)$$

$$\Delta_{12}\phi_1+\Delta_{22}\phi_2=(1+g^4)\Big[-\frac{5\sqrt{3}}{4}\phi_1^3+\frac{9}{4}\phi_1^2\phi_2$$

$$+\frac{3\sqrt{3}}{4}\phi_1\phi_2^2+\frac{9}{4}\phi_2^3+\frac{\rho^2}{4}(\sqrt{3}\phi_1+9\phi_2)\Big], \qquad (56)$$

$$4\rho\Delta_{22}=\rho(1+g^4)\left(3\phi_1^2+2\sqrt{3}\phi_1\phi_2+9\phi_2^2+9\rho^2\right). \qquad (57)$$

Let us consider the equations (55)–(57) for two cases: 1) $\rho=0$ and 2) $\rho\neq 0$.

When $\rho=0$, assuming that $\phi_1\neq 0$ and $\phi_2\neq 0$, we rewrite the equations (55)–(57) in the following way:

$$\Delta_{11}=16\phi_1^2\ln\frac{\Lambda^2}{4\phi_1^2}+16\phi_1^2g^4\ln\frac{\Lambda^2}{4g^2\phi_1^2}+(1+g^4)\Big[-\frac{159}{4}\phi_1^2$$

$$-\frac{15\sqrt{3}}{4}\phi_1\phi_2 + +\frac{9}{4}\phi_2^2 + \frac{\sqrt{3}}{4}\frac{\phi_2^3}{\phi_1}\Big] - \Delta_{12}\frac{\phi_2}{\phi_1}, \tag{58}$$

$$\Delta_{22} = (1+g^4)\Big[\frac{9}{4}\phi_1^2 + \frac{3\sqrt{3}}{4}\phi_1\phi_2 + \frac{9}{4}\phi_2^2 - \frac{5\sqrt{3}}{4}\frac{\phi_1^3}{\phi_2}\Big] - \Delta_{12}\frac{\phi_1}{\phi_2}. \tag{59}$$

The solution of the mass-gap equation of Gross–Neveu-type is:

$$\phi_1^2 \approx \frac{\det\Delta}{16(1+g^4)\Delta_{22}\ln\left(\frac{\Lambda^2}{\mu^2}\right)}, \quad \phi_2 \approx -\frac{\Delta_{12}}{\Delta_{22}}\phi_1. \tag{60}$$

The solution of the mass-gap equation of the Abnormal-type is:

$$\phi_2^2 \approx \frac{4}{9(1+g^4)}\Delta_{22}, \quad \phi_1^2 \approx \frac{\Delta_{22}^{1/3}(3\sqrt{3}\Delta_{12}-\Delta_{22})^{2/3}}{4\cdot 3^{5/3}(1+g^4)\ln^{2/3}\left(\frac{\Lambda^2}{\mu^2}\right)}, \tag{61}$$

For the case 2, of a non-zero ρ , the mass-gap equations reads:

$$\Delta_{11} = 16\phi_1^2\ln\frac{\Lambda^2}{4\phi_1^2} + 16g^4\phi_1^2\ln\frac{\Lambda^2}{4g^2\phi_1^2}$$

$$+(1+g^4)\Big[-\frac{159}{4}\phi_1^2 - \frac{5\sqrt{3}}{2}\phi_1\phi_2 + \frac{3}{4}\left(\phi_2^2+\rho^2\right)\Big], \tag{62}$$

$$\Delta_{12} = (1+g^4)\Big[-\frac{5\sqrt{3}}{4}\phi_1^2 + \frac{3}{2}\phi_1\phi_2 + \frac{\sqrt{3}}{4}\left(\phi_2^2+\rho^2\right)\Big], \tag{63}$$

$$\Delta_{22} = (1+g^4)\Big[\frac{3}{4}\phi_1^2 + \frac{\sqrt{3}}{2}\phi_1\phi_2 + \frac{9}{4}\left(\phi_2^2+\rho^2\right)\Big] \tag{64}$$

The mass-gap equations (62)–(64) can be rewritten in an equivalent form:

$$3\Delta_{11} - 3\Delta_{22} + 9\sqrt{3}\Delta_{12}$$

$$= 48\phi_1^2\left(\ln\frac{\Lambda^2}{4\phi_1^2} - 3\right) + 48g^4\phi_1^2\left(\ln\frac{\Lambda^2}{4g^2\phi_1^2} - 3\right), \tag{65}$$

$$\Delta_{22} - 3\sqrt{3}\Delta_{12} = (1+g^4)(12\phi_1^2 - 4\sqrt{3}\phi_1\phi_2), \tag{66}$$

$$\Delta_{22} = (1+g^4)\left[\frac{3}{4}\phi_1^2 + \frac{\sqrt{3}}{2}\phi_1\phi_2 + \frac{9}{4}\left(\phi_2^2+\rho^2\right)\right]. \tag{67}$$

From (65)–(67) it is clear that for fixed Δ_{kl} while Λ grows larger, the solution exists if Δ_{kl} parameters are chosen close to a particular plane in the parametric space. This plane is defined by the equation:

$$\Delta_{22} = 3\sqrt{3}\Delta_{12}. \tag{68}$$

When Δ_{kl} satisfy the equation (68) exactly, the solution is found to be as follows (in the large-log approximation):

$$\phi_1^2 = \frac{\Delta_{11}}{16(1+g^4)\ln\frac{\Lambda^2}{\mu^2}}\left[1 + O\left(\frac{1}{\ln\frac{\Lambda^2}{\mu^2}}\right)\right], \tag{69}$$

$$\phi_2 = \sqrt{3}\phi_1, \tag{70}$$

$$\rho^2 = \frac{4}{9}\frac{\Delta_{22}}{(1+g^4)}\left[1 + O\left(\frac{1}{\ln\frac{\Lambda^2}{\mu^2}}\right)\right]. \tag{71}$$

4.2. Mass spectrum in Model I

The mass spectrum of related bosonic states is determined by the Eqs. (26)–(31) and taking into account the conditions necessary for a minimum of the potential (52, 53). The solutions at $-m^2 = p^2 < 0$ one can obtain from the Eqs:

$$\det(Ap^2 + B) = 0. \tag{72}$$

The "kinetic" matrix $\hat{A}$ as being proportional to p^2 is derived in the soft-momentum expansion in powers of p^2 and in a large-logΛ approximation. Because the expressions for $\hat{A}$ and $\hat{B}$ are cumbersome we give their explicit form in the Appendix A and B correspondingly. After substituting expressions for the matrix $\widehat{A}$, $\widehat{B}$ into (72) one can get the mass spectrum for the neutral Higgs bosons in Model I. For the case, $\rho = 0$, the mass spectrum is resembling ones in Two-Channel Model, in particular, the Gross–Neveu-type solution brings the spectrum for scalars:

$$m_{\sigma'}^2 \approx -\frac{4\Delta_{22}}{3(1+g^2)}, \tag{73}$$

$$m_\sigma^2 \approx -\frac{\det\Delta}{(1+g^2)\Delta_{22}\ln\left(\frac{\Lambda^2}{\mu^2}\right)} = 4m_d^2 \tag{74}$$

and for pseudoscalars:

$$m_{\pi'}^2 \approx -\frac{4\Delta_{22}}{3(1+g^2)}, \tag{75}$$

$$m_\pi^2 = 0. \tag{76}$$

The Abnormal solution induces the following mass spectrum for scalars:

$$m_{\sigma'}^2 \approx \frac{8\Delta_{22}}{3(1+g^2)}, \tag{77}$$

$$m_\sigma^2 \approx \frac{2\Delta_{22}^{1/3}(3\sqrt{3}\Delta_{12}-\Delta_{22})^{2/3}}{3^{2/3}(1+g^2)\ln^{2/3}\left(\frac{\Lambda^2}{\mu^2}\right)} = 6m_d^2 \tag{78}$$

and for pseudoscalars:

$$m_{\pi'}^2 \approx \frac{3^{2/3}(3\sqrt{3}\Delta_{12}-\Delta_{22})^{4/3}}{27(1+g^2)\Delta_{22}^{1/3}\ln^{1/3}\left(\frac{\Lambda^2}{\mu^2}\right)}, \tag{79}$$

$$m_\pi^2 = 0. \tag{80}$$

Remark that dynamical mass m_d is in fact the mass of t quark in the Model-I, because the v.e.v. of ϕ_{11}, which is parametrized as $\phi_1 \equiv \langle\, \phi_{11} \,\rangle$, gives the value of mass of t-quark.

The mass spectrum in the P-parity Breaking Phase, for a non-zero ρ is:

$$m_1^2 = 0, \tag{81}$$

$$m_2^2 \approx \frac{3\Delta_{11}-\Delta_{22}}{3(1+g^2)\ln\frac{\Lambda^2}{\mu^2}} \approx 16\phi_1^2 = 4m_d^2, \tag{82}$$

$$m_3^2 \approx \frac{8\Delta_{22}}{3(1+g^2)}, \tag{83}$$

$$m_4^2 \approx \frac{3\Delta_{11}+7\Delta_{22}}{27(1+g^2)\ln\frac{\Lambda^2}{\mu^2}}, \tag{84}$$

Thus, we have constructed the Model I where:

a) Two composite Higgs doublets are created dynamically as a consequence of DCSB in two channels.
b) In 2HQ Model I Higgs bosons are rather radial, ground and excited states in the scalar-pseudoscalar channels.
c) The appropriate fine tuning leads also to spontaneous breaking of P-parity and, therefore, of CP-parity in the Higgs sector.

5. Top-Bottom Condensation for 2HQM Model- II

5.1. Effective potential in Model II

The Lagrangian density of the Model II to describe the dynamic of two composite Higgs bosons which consist of bound states (condensates $\bar{t}t$, $\bar{b}b$) and satisfy the FCNC suppression [16] can be written as:

$$\mathcal{L}_J = L_{\rm kin} + \frac{N_c \Lambda^2}{8\pi^2} \sum_{k,l=1}^{2} \Phi_k^\dagger (a^{-1})_{kl} \Phi_l$$

$$+ i\bar{q}\,(\widetilde{\mathbf{M}}\, P_L + \widetilde{\mathbf{M}}^\dagger P_R) q + \text{ h.c.}, \qquad (85)$$

where $P_{L(R)} = 1/2(1 \pm \gamma_5)$ – the left and right projectors, and $\widetilde{\mathbf{M}}$ is the two-by-two flavour matrix:

$$\widetilde{\mathbf{M}} = \sum_{m=1}^{2} \begin{pmatrix} \phi_{m2}\, f_{t,m}\left(\frac{p^2}{\Lambda^2}\right) & -\phi_{m1}\, f_{t,m}\left(\frac{p^2}{\Lambda^2}\right) \\ \phi^*_{m1}\, f_{b,m}\left(\frac{p^2}{\Lambda^2}\right) & \phi^*_{m2}\, f_{b,m}\left(\frac{p^2}{\Lambda^2}\right) \end{pmatrix}, \qquad (86)$$

where we set for the Yukawa coupling constants $g_{t,k}, g_{b,k} = 1$ (two Yukawa constants are due to the renormalization of Higgs fields and other ones we choose equal one). In this Model II Φ_1, Φ_2 give masses to *up-, down*-type quarks. The structure of quark interaction is specified in four formfactors:

$$f_{t,1} = 1 - c_{t,1}\frac{\partial^2}{\Lambda^2},$$

$$f_{t,2} = -c_{t,2}\frac{\partial^2}{\Lambda^2},$$

$$f_{b,1} = -c_{b,1}\frac{\partial^2}{\Lambda^2},$$

$$f_{b,2} = 1 - c_{b,2}\frac{\partial^2}{\Lambda^2}. \qquad (87)$$

When the chiral symmetry is broken, the v.e.v. of neutral Higgs fields are non-zero and the true Yukawa vertices should be obtained by subtracting from $\mathbf{M}$ its v.e.v.

$$\mathbf{M} = \widetilde{\mathbf{M}} - \langle\, \widetilde{\mathbf{M}}\, \rangle, \qquad (88)$$

where $\langle \dots \rangle$ means a v.e.v.:

$$\langle \mathbf{M} \rangle = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix}. \tag{89}$$

The elements of the matrix (89) are the quark mass functions:

$$m_t(\tau) = \phi_1(1 + c_{t,1}\tau) + \phi_2\, e^{i\delta_0}\, c_{t,2}\tau,$$

$$m_b(\tau) = \phi_1 c_{b,1}\tau + \phi_2\, e^{i\delta_0}(1 + c_{b,2}\tau), \quad \tau \equiv -\frac{\partial^2}{\Lambda^2}, \tag{90}$$

defined to be real and $\phi_1 =< \phi_{12} >$, $\phi_2 =< \phi_{22} >$. The non-zero phase at m_b, which is displayed explicitly in (89), may appear if the v.e.v. of ϕ_{22} acquires irremovable phase factor when the chiral symmetry is broken.

As the vacuum charge stability is assumed, $\langle \widetilde{\mathbf{M}} \rangle$ is diagonal, so $\langle \widetilde{\mathbf{M}} \rangle$ and $\langle \widetilde{\mathbf{M}} \rangle^\dagger$ commute and can be placed in any order in products of themselves.

The effective potential of composite two-Higgs Model II in which the interaction of quarks and Higgs bosons is described by formfactors (87) reads:

$$V_{\text{eff}} = -\Delta_{11}\phi_1^2 - 2\Delta_{12}\phi_1\phi_2\cos\delta_0 - \Delta_{22}\phi_2^2 + \frac{1}{2}\phi_1^4\left(\ln\frac{\Lambda^2}{\phi_1^2} + \frac{1}{2}\right)$$
$$+\frac{1}{2}\phi_2^4\left(\ln\frac{\Lambda^2}{\phi_2^2} + \frac{1}{2}\right) + \frac{1}{2}(J_{1111}\phi_1^4 + 4J_{1112}\phi_1^3\phi_2\cos\delta_0$$
$$+2(J_{1122} + J_{1221} + J_{1212}\cos 2\delta_0)\phi_1^2\phi_2^2 + 4J_{1222}\phi_1\phi_2^3\cos\delta_0$$
$$+J_{2222}\phi_2^4) + O\left(\frac{1}{\Lambda^2}\right), \tag{91}$$

where v.e.v. of fields are: $\langle\phi_{12}\rangle = \phi_1$, $\langle\phi_{22}\rangle = \phi_2$. (For the concrete choice of formfactors in Model II the form of the effective potential is displayed in the Appendix C.) Its minimum is described by solutions of the mass-gap equations (21), (22) which for the Model II are:

$$2\Delta_{11}\phi_1 + 2\Delta_{12}\phi_2\cos\delta_0$$
$$= 2\phi_1^3\ln\left(\frac{\Lambda^2}{\phi_1^2}\right) + 2J_{1111}\phi_1^3 + 6J_{1112}\phi_1^2\phi_2\cos\delta_0 + 2J_{1222}\phi_2^3\cos\delta_0$$
$$+2\left(J_{1122} + J_{1221} + J_{1212}\cos 2\delta_0\right)\phi_1\phi_2^2, \tag{92}$$

$$2\Delta_{12}\phi_1\phi_2\sin\delta_0 = 2J_{1112}\phi_1^3\phi_2\sin\delta_0$$

$$+2J_{1212}\phi_1^2\phi_2^2\sin 2\delta_0 + J_{1222}\phi_1\phi_2^3\sin\delta_0, \tag{93}$$

$$2\Delta_{22}\phi_2 + 2\Delta_{12}\phi_1\cos\delta_0 = 2\phi_2^3\ln\left(\frac{\Lambda^2}{\phi_2^2}\right) + 2J_{2222}\phi_2^3 + 6J_{1222}\phi_1\phi_2^2\cos\delta_0$$

$$+2J_{1112}\phi_1^3\cos\delta_0 + 2\left(J_{1122} + J_{1221} + J_{1212}\cos 2\delta_0\right)\phi_1^2\phi_2, \tag{94}$$

where J_{klmn} $(k,l,m,n = 1,2)$ are the integrals:

$$J_{klmn} = \int_0^1 \left(f_{t,k}(\tau)f_{t,l}(\tau)f_{t,m}(\tau)f_{t,n}(\tau) + f_{b,k}(\tau)f_{b,l}(\tau)f_{b,m}(\tau)f_{b,n}(\tau)\right.$$

$$+f_{t,k}(\tau)f_{b,l}(\tau)f_{b,m}(\tau)f_{t,n}(\tau) - f_{t,k}(\tau)f_{t,l}(\tau)f_{b,m}(\tau)f_{b,n}(\tau)$$

$$+f_{b,k}(\tau)f_{b,l}(\tau)f_{t,m}(\tau)f_{t,n}(\tau) - f_{b,k}(\tau)f_{t,l}(\tau)f_{t,m}(\tau)f_{b,n}(\tau)$$

$$-f_{t,k}(0)f_{t,l}(0)f_{t,m}(0)f_{t,n}(0) - f_{b,k}(0)f_{b,l}(0)f_{b,m}(0)f_{b,n}(0)$$

$$-f_{t,k}(0)f_{b,l}(0)f_{b,m}(0)f_{t,n}(0) + f_{t,k}(0)f_{t,l}(0)f_{b,m}(0)f_{b,n}(0)$$

$$\left.-f_{b,k}(0)f_{b,l}(0)f_{t,m}(0)f_{t,n}(0) + f_{b,k}(0)f_{t,l}(0)f_{t,m}(0)f_{b,n}(0)\right)\frac{d\tau}{\tau}. \tag{95}$$

It is more convenient to solve the equations (92)–(94) for the variables Δ_{lm} rather than ϕ_1, ϕ_2, δ_0. The variables ϕ_1, ϕ_2, δ_0 will be treated as input parameters while Δ_{lm} as the unknowns. The reason for this is that we do not know Δ_{lm} from any global theory, we rather fit them so that ϕ_1, ϕ_2, δ_0 conform to experiment. The equations (92)–(94) are linear for Δ_{lm} and can easily be solved; one just expresses Δ_{lm} through ϕ_1, ϕ_2, δ_0 and substitutes them in every place they appear. As usual two cases must be considered separately.

1) For $\delta_0 = 0$:

$$\Delta_{11} = \phi_1^2\ln\left(\frac{\Lambda^2}{\phi_1^2}\right) + J_{1111}\phi_1^2 + 3J_{1112}\phi_1\phi_2$$

$$+\left(J_{1122} + J_{1221} + J_{1212}\right)\phi_2^2 + J_{1222}\frac{\phi_2^3}{\phi_1} - \Delta_{12}\frac{\phi_2}{\phi_1}, \tag{96}$$

$$\Delta_{22} = \phi_2^2\ln\left(\frac{\Lambda^2}{\phi_2^2}\right) + J_{2222}\phi_2^2 + 3J_{1222}\phi_1\phi_2$$

$$+ \left(J_{1122} + J_{1221} + J_{1212}\right) \phi_1^2 + J_{1112}\frac{\phi_1^3}{\phi_2} - \Delta_{12}\frac{\phi_1}{\phi_2}, \tag{97}$$

where Δ_{12}, ϕ_1, ϕ_2 are treated as input parameters;
2) and for $\delta_0 \neq 0$:

$$\Delta_{11} = \phi_1^2 \ln\left(\frac{\Lambda^2}{\phi_1^2}\right) + J_{1111}\phi_1^2 + 2J_{1112}\phi_1\phi_2\cos\delta_0$$

$$+ \left(J_{1122} + J_{1221} - J_{1212}\right) \phi_2^2, \tag{98}$$

$$\Delta_{22} = \phi_2^2 \ln\left(\frac{\Lambda^2}{\phi_2^2}\right) + J_{2222}\phi_2^2 + 2J_{1222}\phi_1\phi_2\cos\delta_0$$

$$+ \left(J_{1122} + J_{1221} - J_{1212}\right) \phi_1^2, \tag{99}$$

$$\Delta_{12} = J_{1112}\phi_1^2 + J_{1222}\phi_2^2 + 2J_{1212}\phi_1\phi_2\cos\delta_0. \tag{100}$$

The mass spectrum of related bosonic states is determined by the matrices $\widehat{A}$ and $\widehat{B}$ of the second variations of the effective potential (91) (see Appendix A, B).

5.2. Mass spectrum in Model II

After substituting explicit forms for the $\widehat{A}$, $\widehat{B}$ into (72), one can obtain the mass-spectrum for the composite neutral Higgses in Model II.
1) For $\delta_0 = 0$:

$$m_{\sigma'} \approx 2m_t,$$

$$m_\pi = 0,$$

$$m_\sigma \approx \sqrt{2\frac{(\Delta_{12} - m_t^2 J_{1112})}{r}},$$

$$m_{\pi'} \approx m_{\sigma'}, \tag{101}$$

where

$$r = \frac{m_b}{m_t}\ln\frac{\Lambda^2}{m_t^2} \sim 1. \tag{102}$$

If the ratio were (say, for the fourth generation):

$$\frac{m_b}{m_t} = 0(1) \quad (\Lambda \to \infty), \tag{103}$$

than one would get:

$$m_\sigma = 2m_b,$$

$$m_{\pi'}^2 = \frac{2(m_t^2+m_b^2)}{m_t\, m_b \ln\frac{\Lambda^2}{m_t^2}} \left(\Delta_{12} - m_t^2 J_{1112} - 2m_t\, m_b\, J_{1212} - m_b^2 J_{1222}\right). \quad (104)$$

2) For $\delta_0 \neq 0$:

$$m_1 = 0, \quad (105)$$

$$m_2 \approx 2m_t, \quad (106)$$

$$m_3 \approx m_t \sqrt{\frac{J_{1212}}{\ln\frac{\Lambda^2}{m_t^2}}} \quad (107)$$

$$m_4 \approx 2rm_b|\sin\delta_0|\sqrt{J_{1212}}. \quad (108)$$

For the case of $\delta_0 \neq 0$, the model predicts a low value for m_4.

If one considered (that may take place for the fourth generation):

$$m_t \sim m_b \sim 1 \quad (\Lambda \to \infty), \quad (109)$$

the mass-spectrum would turn out to be as follows:

$$m_1 = 0 \quad (110)$$

$$m_2 \approx 2m_t \quad (111)$$

$$m_3 \approx 2m_b \quad (112)$$

$$m_4 \approx 2|\sin\delta_0| \sqrt{\frac{J_{1212}(m_t^2+m_b^2)}{\ln\frac{\Lambda^2}{m_t^2}}}. \quad (113)$$

We notice that when $\delta_0 \neq 0$ we have neither scalars no pseudoscalars any longer because the particles which are eigenstates of the energy operator, are mixed of both P-even parity and P-odd parity fields, hence the former classification by parity does not hold for this particular case.

5.3. Mass spectrum in Model II for the special choice of Yukawa couplings with FCNC suppression

The suppression of the flavor changing neutral currents in a composite two-Higgs model can be implemented when Yukawa couplings are chosen as follows:

$$g_{t,1} = g_{b,2}; \qquad g_{t,2} = g_{b,1} = \sqrt{5}. \quad (114)$$

It can be easily seen from the potential in Appendix C that with the above choice of $g_{t,k}$, $g_{b,k}$ the last two terms in (191) come to zero. This prevents the appearence of the Yukawa coupling inducing FCNC from radiative corrections. In this case the potential of Model type II reads:

$$
\begin{aligned}
V_{\text{eff}} = \frac{N_c}{8\pi^2}\Bigg\{ & -\Delta_{11}(\Phi_1^\dagger\Phi_1) - \Delta_{12}(\Phi_1^\dagger\Phi_2) - \Delta_{21}(\Phi_2^\dagger\Phi_1) - \Delta_{22}(\Phi_2^\dagger\Phi_2) \\
& + \frac{\nu_+^2(0)}{2}\left(\ln\frac{\Lambda^2}{\nu_+(0)} + \frac{1}{2}\right) + \frac{\nu_-^2(0)}{2}\left(\ln\frac{\Lambda^2}{\nu_-(0)} + \frac{1}{2}\right) \\
& + \frac{33}{4}(\Phi_1^\dagger\Phi_1)^2 + \frac{33}{4}(\Phi_2^\dagger\Phi_2)^2 + \frac{33}{2}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\
& - \frac{3}{2}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{15}{4}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_1^\dagger\Phi_2)^2\right]\Bigg\},
\end{aligned}
\tag{115}
$$

where for $\nu_\pm(0)$ we have

$$
\begin{aligned}
\nu_\pm = {} & 2(\Phi_1^\dagger\Phi_1) + 2(\Phi_2^\dagger\Phi_2) \pm 2\Big[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2 \\
& + 2(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - 4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\Big]^{1/2}.
\end{aligned}
\tag{116}
$$

To be short we skip the calculations and display only the mass spectrum. When the mass ratio of t- and b-quarks m_t/m_b holds unchanged while Λ goes to infinity, in the large-log approach we get the following estimations for Higgs' masses:

1 for the case of concerved P-parity ($\delta_0 = 0$):

$$
\begin{aligned}
m_\sigma &= 2m_b, \\
m_{\sigma'} &\approx 2m_t, \\
m_\pi &= 0, \\
m_{\pi'} &= \sqrt{\frac{(m_t^2 + m_b^2)}{2m_t m_b \ln\frac{\Lambda^2}{m_t^2}}\left(\Delta_{12} - \frac{15}{4}m_t m_b\right)};
\end{aligned}
\tag{117}
$$

2 and for the P-parity breaking phase:

$$m_1 = 0 \tag{118}$$

$$m_2 \approx 2m_t \tag{119}$$

$$m_3 \approx 2m_b \tag{120}$$

$$m_4 \approx |\sin\delta_0| \sqrt{\frac{15(m_t^2+m_b^2)}{8\ln\frac{\Lambda^2}{m_t^2}}}. \tag{121}$$

Thus in the Model II the Quasilocal Yukawa interaction with Higgs doublets reduces at low energies to a conventional local one where each Higgs doublet couples to a definite charge current and its v.e.v. brings the mass either to *up*- or to *down*- components of fermion doublets. Based on the FCNC suppression, the Model II leads to the relation $m_t >> m_b$ and so to an enhanced coupling of the light scalar (pseudoscalar) boson to the *down*-type quarks while suppressing the coupling to the *up*-type quarks. The Model II has a broad spectrum of excited bound states which is to parametrize the data, in particular, obtained from the Next Linear Collider.

6. Summary

In our paper we have proposed a set of Quasilocal NJL-type quark models (QNJLM) which lead to a larger spectrum of ground and excited states in the polycritical regime. From the viewpoint of the SM, these models are considered as more natural than common extensions of the SM, since they do not enlarge the number of elementary particles in fermionic sector and preserve the symmetries of the SM. For the toy Two-Channel Quasilocal quark model, near tricritical point we have found three major phases: a symmetrical one and two phases with DCSB, different in correlation lengths in scalar channels. On a particular plane in the space of coupling constants we discovered the special P-parity breaking phase. It means that in such a phase there exist heavy scalar states which can decay into two or three light pseudoscalars. This phenomenon of dynamical P-parity breaking can be used in the extensions of the SM where several Higgs bosons are composite ones. In the framework of the QNJLM we have presented two models which provide at low energies two composite Higgs doublets, as minimal extensions of the Top-Mode Standard Model [11, 14]. In the 2HQ Model I Higgs bosons are rather radial, ground and excited states in the scalar -pseudoscalar channels. In the 2HQ Model II, which consistent with the requirement of natural flavour conservation 16], strong forces lead to the formation of top and bottom bound states (and corresponding condensates) and generate masses of t, b-quarks. In Model II we have concentrated on the scenario where each of the neutral components of the two doublets $\phi_{1,2}$ (with v.e.v. $v_{1,2}$) couple at low energies respectively to the $I_3 = \pm\frac{1}{2}$ fermion fields The FCNC suppression leads to the relation $m_t >> m_b$ and to an enhanced coupling of the light scalar (pseudoscalar) boson to the *down*-type quarks and the charged leptons while suppressing the coupling to the *up*-type quarks. The existence of light neutral Higgs (pseudo)scalar bosons in the framework of 2HDM is not excluded by existing data ($< 40\,GeV$). The chance that it can be seen at the Next Linear Collider in the $\gamma\gamma$ processes has been pointed out in [26, 27]. As a result of complexity of two v.e.v.'s for two composite Higgs doublets the dynamical CP-violation may appear in the Higgs sector. At high energies these channels are strongly coupled and one could say that two-composite Higgs doublets partially represent the mixture with excited states. If such excited states exist then they will modify the Higgs mass predictions. In addition, we remark that low values for the Higgs masses of the additional states could actually change the window for M_H since these states could give a significant contribution to the ρ-parameter [28]. From our consideration we have seen that the appearance of dynamical CP-violation in the Higgs sector imposes strong bounds on Higgs masses, in particular, one light scalar Higgs boson is unavoidable. The experimental implications of such effects are expected to be rather small in the fermion sector of the SM [1, 26]. These effects are observable in decays of heavy Higgs particles (namely, pseudoscalar Higgses may decay into scalar ones, scalar Higgs may decay into pseudoscalar ones) and in decays of Higgses particles into two vector bosons where CP-even and CP-odd amplitudes appear. At high energies the appearance of the appreciable CP-violation could be important both as a source of electron and neutron electric dipole moments [29] and as a mechanism for EW scale baryogenesis [30, 31]. Besides one expect also that modifications of the SM Lagrangian (the Higgs and Top interactions) by higher dimensional vertices may enhance the Higgs production at hadron colliders [32].

The theory of two composite Higgs bosons which we have discussed in our paper should be regarded as a viable alternative to other approaches to the BSM and perhaps the major progress in the alternative approaches will come when the first direct experimental results associated with the origin of EWSB begin to appear.

The purpose of this paper has been to elaborate the very design of quasilocal NJL-quark models with two-composite Higgs bosons. A more comprehensive analysis of low-energy particle characteristics in these models is postponed to the next paper in this series of. The numerical computation of bounds on mass spectra, Yukawa coupling constants and decay widths with taking into account the renormalization-group corrections will be presented elsewhere.

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Appendix A: Kinetic Matrix $\widehat{A}$ for Composite Two-Higgs Bosons

In this appendix we calculate the kinetic term for composite Two-Higgs Quasilocal Quark Models which is obtained by calculation of the one-loop diagram:

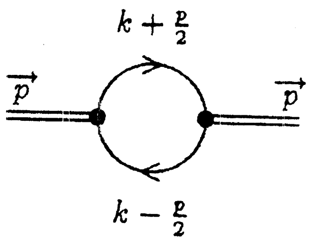

Fig. 1 One-loop diagram for calculation of kinetic term.

Here p is an incoming momentum, and k is a momentum running around the loop.

The loop diagram (Fig. 1) gives the following expression:

$$-\frac{1}{2}\int\limits_{|k|<\Lambda}\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\left(i\mathbf{M}\,P_L+i\mathbf{M}^{\dagger}P_R\right)\mathbf{D}^{-1}\left(\left(k+\frac{p}{2}\right)^2\right)\right.$$
$$\left.\times\left(i\mathbf{M}\,P_L+i\mathbf{M}^{\dagger}P_R\right)\mathbf{D}^{-1}\left(\left(k-\frac{p}{2}\right)^2\right)\right]. \qquad (122)$$

Here the coefficient 1/2 is due to symmetry of the diagram (Fig. 1). The full expression within the square brackets in (122) is an element of a direct product of three spaces: color, flavor and spinor and the trace operation is related to all them. We define the vertex:

$$= i(\mathbf{M}\, P_L + \mathbf{M}^\dagger P_R)$$

and the fermion propagator:

$$= \mathbf{D}^{-1}(q) = (\hat{q} + i\langle\, \widetilde{\mathbf{M}}\,\rangle P_L + i\langle\, \widetilde{\mathbf{M}}\,\rangle^\dagger P_R)^{-1}$$

$$= (\hat{q} - i\langle\, \widetilde{\mathbf{M}}\,\rangle P_L - i\langle\, \widetilde{\mathbf{M}}\,\rangle^\dagger P_R)\Delta(q^2),$$

where Δ is a matrix function:

$$\begin{aligned}\Delta(q^2) &= (q^2 + \langle\, \widetilde{\mathbf{M}}\,\rangle\langle\, \widetilde{\mathbf{M}}\,\rangle^\dagger)^{-1} \\ &= \begin{pmatrix} \frac{1}{q^2+m_t^2\left(\frac{k^2}{\Lambda^2}\right)} & 0 \\ 0 & \frac{1}{q^2+m_b^2\left(\frac{k^2}{\Lambda^2}\right)} \end{pmatrix}. \end{aligned} \tag{123}$$

First, we calculate the trace of the sum of all the products of γ-matrices displayed in the expression (122). After that we come to the following expression:

$$\begin{aligned} \frac{N_c}{2} \int\limits_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \Bigg\{ & 2\,\mathrm{tr}\left[\widetilde{\mathbf{M}}\Delta\left(\left(k+\frac{p}{2}\right)^2\right)\widetilde{\mathbf{M}}^\dagger\Delta\left(\left(k-\frac{p}{2}\right)^2\right)\right]\left(k^2-\frac{p^2}{4}\right) \\ & -2\,\mathrm{tr}\left[\widetilde{\mathbf{M}}^\dagger\langle\,\widetilde{\mathbf{M}}\,\rangle\Delta\left(\left(k+\frac{p}{2}\right)^2\right)\widetilde{\mathbf{M}}^\dagger\langle\,\widetilde{\mathbf{M}}\,\rangle\Delta\left(\left(k-\frac{p}{2}\right)^2\right)\right] \\ & +2\,\mathrm{tr}\left[\widetilde{\mathbf{M}}^\dagger\Delta\left(\left(k+\frac{p}{2}\right)^2\right)\widetilde{\mathbf{M}}^\dagger\Delta\left(\left(k-\frac{p}{2}\right)^2\right)\right]\left(k^2-\frac{p^2}{4}\right) \\ & -2\,\mathrm{tr}\left[\widetilde{\mathbf{M}}\langle\,\widetilde{\mathbf{M}}\,\rangle\Delta\left(\left(k+\frac{p}{2}\right)^2\right)\widetilde{\mathbf{M}}\langle\,\widetilde{\mathbf{M}}\,\rangle\Delta\left(\left(k-\frac{p}{2}\right)^2\right)\right]\Bigg\}. \end{aligned} \tag{124}$$

In this formula and further on the trace is applied only for the flavour two-by-two matrices.

The kinetic term is derived as being proportional to p^2 in the soft-momentum expansion of (124) in powers of p^2. We obtain this term by means of calculating the second derivative of (124) at zero external momentum p. First, let us rewrite the expression (124) in a form:

$$\frac{N_c}{32\pi^4}\int\limits_{|k|<\Lambda}\Big(f(x,y)-p^2\,\mathrm{tr}\,\Big[\widetilde{\mathbf{M}}\Delta(k^2)\widetilde{\mathbf{M}}^{\dagger}\Delta(k^2)\Big]\Big)\,d^4k, \tag{125}$$

where

$$x=\Big(k+\frac{p}{2}\Big)^2,\qquad y=\Big(k-\frac{p}{2}\Big)^2, \tag{126}$$

with the function f defined as:

$$f(x,y)=4\,\mathrm{tr}\,\Big[\widetilde{\mathbf{M}}\Delta(x)\widetilde{\mathbf{M}}^{\dagger}\Delta(y)\Big]\,k^2-2\,\mathrm{tr}\,\Big[\widetilde{\mathbf{M}}\langle\widetilde{\mathbf{M}}\rangle^{\dagger}\Delta(x)\widetilde{\mathbf{M}}\langle\widetilde{\mathbf{M}}\rangle^{\dagger}\Delta(y)\Big]$$
$$-2\,\mathrm{tr}\,\Big[\widetilde{\mathbf{M}}^{\dagger}\langle\widetilde{\mathbf{M}}\rangle\Delta(x)\widetilde{\mathbf{M}}^{\dagger}\langle\widetilde{\mathbf{M}}\rangle\Delta(y)\Big]\,. \tag{127}$$

Let us expand the expression (127) in series of p and extract the term proportional to p^2:

$$\frac{N_c p^2}{32\pi^2}\int\limits_0^{\Lambda^2}\Big\langle\frac{1}{2}\frac{\partial^2}{\partial p_0\partial p_0}\Big|_{p=0}f(x,y)$$
$$-\,\mathrm{tr}\,\Big[\widetilde{\mathbf{M}}\Delta(k^2)\widetilde{\mathbf{M}}^{\dagger}\Delta(k^2)\Big]\Big\rangle k^2\,dk^2, \tag{128}$$

where the angular brackets stand for angular average in 4-dimensional Euclidean space and p_0 is a component of 4-momentum p. The second derivative of the function f reads:

$$\left\langle\frac{\partial^2}{\partial p_0\partial p_0}\Big|_{p=0}f(x,y)\right\rangle=\frac{1}{2}\Big[\Big(f_{xx}(x_0,y_0)-2f_{xy}(x_0,y_0)$$
$$+f_{yy}(x_0,y_0)\Big)\frac{k^2}{2}+f_x(x_0,y_0)+f_y(x_0,y_0)\Big]. \tag{129}$$

The subscripts in f_{xx}, f_{xy}, f_{yy}, f_x, f_y stand for partial derivatives by variables x and y. The derivatives are calculated at $x_0=y_0=k^2$ (see (126)). The function Δ, defined in (123), is a flavour matrix:

$$\Delta(x)=\begin{pmatrix}\frac{1}{x+m_t^2(\frac{k^2}{\Lambda^2})} & 0\\ 0 & \frac{1}{x+m_b^2(\frac{k^2}{\Lambda^2})}\end{pmatrix}. \tag{130}$$

Using (86), (88), (89), (127) and (130), one gets $f(x,y)$:

$$
\begin{aligned}
f(x,y) = \sum_{l,m=1}^{2} \Bigg\{ \Bigg[4 \Bigg(& \frac{f_{t,l}(\frac{p^2}{\Lambda^2}) f_{t,m}(\frac{p^2}{\Lambda^2})}{(x+m_t(k^2/\Lambda^2))(y+m_b(k^2/\Lambda^2))} \\
& + \frac{f_{b,l}(\frac{p^2}{\Lambda^2}) f_{b,m}(\frac{p^2}{\Lambda^2})}{(x+m_b(k^2)/\Lambda^2))(y+m_t(k^2/\Lambda^2))} \Bigg) k^2 \\
& + 2 m_t\left(\frac{k^2}{\Lambda^2}\right) m_b\left(\frac{k^2}{\Lambda^2}\right) \left(f_{t,l}\left(\frac{p^2}{\Lambda^2}\right) f_{b,m}\left(\frac{p^2}{\Lambda^2}\right) e^{-i\delta_0} \right. \\
& \left. + f_{b,l}\left(\frac{p^2}{\Lambda^2}\right) f_{t,m}\left(\frac{p^2}{\Lambda^2}\right) e^{i\delta_0} \right) \\
& \times \left(\frac{1}{(x+m_t(k^2/\Lambda^2))(y+m_b(k^2/\Lambda^2))} \right. \\
& \left. + \frac{1}{(x+m_b(k^2/\Lambda^2))(y+m_t(k^2/\Lambda^2))} \right) \Bigg] \phi_{l1}^{*}\, \phi_{m1} \\
& + 4k^2 \Bigg[\frac{f_{b,l}(\frac{p^2}{\Lambda^2}) f_{b,m}(\frac{p^2}{\Lambda^2})}{(x+m_b(k^2/\Lambda^2))(y+m_b(k^2/\Lambda^2))} \\
& + \frac{f_{t,l}(\frac{p^2}{\Lambda^2}) f_{t,m}(\frac{p^2}{\Lambda^2})}{(x+m_t(k^2/\Lambda^2))(y+m_t(k^2/\Lambda^2))} \Bigg] \phi_{l2}^{*}\, \phi_{m2} \\
& - 2 \Bigg[\frac{f_{b,l}(\frac{p^2}{\Lambda^2}) f_{l,m}(\frac{p^2}{\Lambda^2}) m_b^2(\frac{k^2}{\Lambda^2}) e^{-2i\delta_0}}{(x+m_b(k^2/\Lambda^2))(y+m_b(k^2/\Lambda^2))} \\
& + \frac{f_{t,l}(\frac{p^2}{\Lambda^2}) f_{t,m}(\frac{p^2}{\Lambda^2}) m_t^2(\frac{k^2}{\Lambda^2})}{(x+m_t(k^2/\Lambda^2))(y+m_t(k^2/\Lambda^2))} \Bigg] \phi_{l2} \phi_{m2} \\
& - 2 \Bigg[\frac{f_{b,l}(\frac{p^2}{\Lambda^2}) f_{b,m}(\frac{p^2}{\Lambda^2}) m_b^2(\frac{k^2}{\Lambda^2}) e^{2i\delta_0}}{(x+m_b(k^2/\Lambda^2))(y+m_b(k^2/\Lambda^2))} \\
& + \frac{f_{t,l}(\frac{p^2}{\Lambda^2}) f_{t,m}(\frac{p^2}{\Lambda^2}) m_t^2(\frac{k^2}{\Lambda^2})}{(x+m_t(k^2/\Lambda^2))(y+m_t(k^2/\Lambda^2))} \Bigg] \phi_{l2}^{*} \phi_{m2}^{*} \Bigg\}. \qquad (131)
\end{aligned}
$$

After applying the derivative procedure displayed in (129) to the function f, one gets the kinetic term in the following form:

$$
\frac{N_c}{16\pi^2} \sum_{l,m=1}^{2} \Bigg(I_{lm}^{(1)} (\partial_\mu \phi_{l1}^{*})(\partial_\mu\, \phi_{m1}) + I_{lm}^{(2)} (\partial_\mu \phi_{l2}^{*})(\partial_\mu\, \phi_{m2})
$$

$$+I^{(3)}_{lm}(\partial_\mu\phi_{l2})(\partial_\mu\,\phi_{m2})+I^{(4)}_{lm}(\partial_\mu\phi^*_{l2})(\partial_\mu\,\phi^*_{m2})\Big). \qquad (132)$$

$I^{(1)}_{lm}$ contributes to the kinetic term for charged components of Higgs doublets, $I^{(2)}_{lm}$, $I^{(3)}_{lm}$, and $I^{(4)}_{lm}$ do the same for the neutral components. The expressions for them are cumbersome and we have divided the total expression in three parts. For the charged components one has:

$$\begin{aligned}
I^{(1)}_{lm} = \frac{1}{2}\int\limits_0^{\Lambda^2}\Bigg[\Bigg(&\frac{m_t^2(k^2/\Lambda^2)k^2}{(k^2+m_t^2(k^2/\Lambda^2))^3(k^2+m_b^2(k^2/\Lambda^2))}\\
&+\frac{m_b^2(k^2/\Lambda^2)k^2}{(k^2+m_t^2(k^2/\Lambda^2))(k^2+m_b^2(k^2/\Lambda^2))^3}\\
&+\frac{k^4}{(k^2+m_t^2(k^2/\Lambda^2))^2(k^2+m_b^2(k^2/\Lambda^2))^2}\\
&+\frac{1}{(k^2+m_t^2(k^2/\Lambda^2))(k^2+m_b^2(k^2/\Lambda^2))}\Bigg)\\
&\times\left(f_{t,l}\left(\frac{p^2}{\Lambda^2}\right)f_{t,m}\left(\frac{p^2}{\Lambda^2}\right)+f_{b,l}\left(\frac{p^2}{\Lambda^2}\right)f_{b,m}\left(\frac{p^2}{\Lambda^2}\right)\right)\\
&+\Bigg(\frac{m_t^3(k^2/\Lambda^2)m_b(k^2/\Lambda^2)}{(k^2+m_t^2(k^2/\Lambda^2))^3(k^2+m_b^2(k^2/\Lambda^2))}\\
&+\frac{m_b^3(k^2/\Lambda^2)m_t(k^2/\Lambda^2)}{(k^2+m_t^2(k^2/\Lambda^2))(k^2+m_b^2(k^2/\Lambda^2))}\\
&+\frac{m_t^3(k^2/\Lambda^2)m_b(k^2/\Lambda^2)k^2}{(k^2+m_t^2(k^2/\Lambda^2))^2(k^2+m_b^2(k^2/\Lambda^2))^2}\Bigg)\\
\times\Bigg(f_{t,l}\left(\frac{p^2}{\Lambda^2}\right)&f_{b,m}\left(\frac{p^2}{\Lambda^2}\right)e^{-i\delta_0}+f_{b,l}\left(\frac{p^2}{\Lambda^2}\right)f_{t,m}\left(\frac{p^2}{\Lambda^2}\right)e^{i\delta_0}\Bigg)\Bigg]k^2\,dk^2, \qquad (133)
\end{aligned}$$

for the neutral ones:

$$\begin{aligned}
I^{(2)}_{lm} = \frac{1}{2}\int\limits_0^{\Lambda^2}\Bigg[&f_{t,l}\left(\frac{p^2}{\Lambda^2}\right)f_{t,m}\left(\frac{p^2}{\Lambda^2}\right)\Bigg(-\frac{k^4}{(k^2+m_t^2(k^2/\Lambda^2))^4}\\
&+\frac{2k^2}{(k^2+m_t^2(k^2/\Lambda^2))^3}+\frac{1}{(k^2+m_t^2(k^2/\Lambda^2))^2}\Bigg)
\end{aligned}$$

$$+f_{b,l}\left(\frac{p^2}{\Lambda^2}\right) f_{b,m}\left(\frac{p^2}{\Lambda^2}\right)\left(-\frac{k^4}{(k^2+m_b^2(k^2/\Lambda^2))^4}\right.$$

$$\left.\left.+\frac{2k^2}{(k^2+m_b^2(k^2/\Lambda^2))^3}+\frac{1}{(k^2+m_b^2(k^2/\Lambda^2))^2}\right)\right] k^2\, dk^2 \qquad (134)$$

and

$$I_{lm}^{(3)} = \frac{1}{2}\int_0^{\Lambda^2}\left[f_{t,l}\left(\frac{p^2}{\Lambda^2}\right) f_{t,m}\left(\frac{p^2}{\Lambda^2}\right)\left(\frac{k^2 m_t^2(k^2/\Lambda^2)}{2(k^2+m_t^2(k^2/\Lambda^2))^4}\right.\right.$$

$$\left.-\frac{m_t^2(k^2/\Lambda^2)}{(k^2+m_t^2(k^2/\Lambda^2))^3}\right)+f_{b,l}\left(\frac{p^2}{\Lambda^2}\right) f_{b,m}\left(\frac{p^2}{\Lambda^2}\right) e^{-2i\delta_0}$$

$$\left.\times\left(-\frac{k^2 m_b^2(k^2/\Lambda^2)}{2(k^2+m_b^2(k^2/\Lambda^2))^4}-\frac{m_b^2(k^2/\Lambda^2)}{(k^2+m_b^2(k^2/\Lambda^2))^3}\right)\right] k^2\, dk^2, \qquad (135)$$

$$I_{lm}^{(4)} = \frac{1}{2}\int_0^{\Lambda^2}\left[f_{t,l}\left(\frac{p^2}{\Lambda^2}\right) f_{t,m}\left(\frac{p^2}{\Lambda^2}\right)\left(\frac{k^2 m_t^2(k^2/\Lambda^2)}{2(k^2+m_t^2(k^2/\Lambda^2))^4}\right.\right.$$

$$\left.-\frac{m_t^2(k^2/\Lambda^2)}{(k^2+m_t^2(k^2/\Lambda^2))^3}\right)+f_{b,l}\left(\frac{p^2}{\Lambda^2}\right) f_{b,m}\left(\frac{p^2}{\Lambda^2}\right) e^{2i\delta_0}$$

$$\left.\times\left(-\frac{k^2 m_b^2(k^2/\Lambda^2)}{2(k^2+m_b^2(k^2/\Lambda^2))^4}-\frac{m_b^2(k^2/\Lambda^2)}{(k^2+m_b^2(k^2/\Lambda^2))^3}\right)\right] k^2\, dk^2. \qquad (136)$$

The next task to do is to calculate the integrals for the large value of Λ, ignoring all contributions which disappear in the $\Lambda \to \infty$ limit. Thus one obtains:

$$I_{lm}^{(1)} = (f_{t,l}(0)\, f_{t,m}(0) + f_{b,l}(0)\, f_{b,m}(0))$$

$$\times\left[\ln\frac{\Lambda^2}{m_t^2}+\frac{3m_b^4 m_t^2 - m_b^6}{(m_t^2-m_b^2)^3}\ln\frac{m_t^2}{m_b^2}-\frac{m_t^4+6m_t^2m_b^2+m_b^4}{4(m_t^2-m_b^2)^2}\right]$$

$$+\left(f_{t,l}(0)\, f_{b,m}(0)\, e^{-i\delta_0} + f_{t,l}(0)\, f_{b,m}(0)\, e^{i\delta_0}\right)$$

$$\times\left[-\frac{2m_t^3 m_b^3}{(m_t^2-m_b^2)^3}\ln\frac{m_t^2}{m_b^2}+\frac{m_t+m_b(m_t^2m_b^2}{4(m_t^2-m_b^2)^2}\right]$$

$$+\int_0^1 \big(f_{t,l}(\tau)\, f_{t,m}(\tau) + f_{b,l}(\tau)\, f_{b,m}(\tau)$$

$$-f_{t,l}(0)\, f_{t,m}(0) - f_{b,l}(0)\, f_{b,m}(0)\big)\frac{d\tau}{\tau} + O\left(\frac{\ln\frac{\Lambda^2}{\mu^2}}{\Lambda^2}\right), \tag{137}$$

$$I^{(2)}_{lm} = f_{t,l}(0) f_{t,m}(0)\left[\ln\frac{\Lambda^2}{m_t^2} - \frac{13}{12}\right] + f_{b,l}(0) f_{b,m}(0)\left[\ln\frac{\Lambda^2}{m_b^2} - \frac{13}{12}\right]$$

$$+\int_0^1 \big(f_{t,l}(\tau)\, f_{t,m}(\tau) + f_{b,l}(\tau)\, f_{b,m}(\tau)$$

$$-f_{t,l}(0)\, f_{t,m}(0) - f_{b,l}(0)\, f_{b,m}(0)\big)\frac{d\tau}{\tau} + O\left(\frac{\ln\frac{\Lambda^2}{\mu^2}}{\Lambda^2}\right), \tag{138}$$

$$I^{(3)}_{kl} = -\frac{1}{3}\big(f_{t,l}(0)\, f_{t,m}(0) + f_{b,l}\, f_{b,m}(0)\, e^{-2i\delta_0}\big) + O\left(\frac{\ln\frac{\Lambda^2}{\mu^2}}{\Lambda^2}\right), \tag{139}$$

$$I^{(4)}_{kl} = -\frac{1}{3}\big(f_{t,l}(0)\, f_{t,m}(0) + f_{b,l}\, f_{b,m}(0)\, e^{2i\delta_0}\big) + O\left(\frac{\ln\frac{\Lambda^2}{\mu^2}}{\Lambda^2}\right). \tag{140}$$

Herein and further on m_t and m_b stand for quark masses.

As we know from experiment, the mass of the top quark is much greater than that of the bottom quark. Regarding $m_t \gg m_b$, for the choice of formfactors (87) one gets:

$$I^{(1)}_{11} \approx \ln\frac{\Lambda^2}{m_t^2} - \frac{1}{4} + 2c_{t,1} + \frac{1}{2}(c_{t,1}^2 + c_{b,1}^2), \tag{141}$$

$$I^{(1)}_{12} \approx c_{t,2} + c_{b,1} + \frac{1}{2}\big(c_{t,1}\, c_{t,2} + c_{b,1}\, c_{b,2}\big), \tag{142}$$

$$I^{(1)}_{22} \approx \ln\frac{\Lambda^2}{m_b^2} - \frac{1}{4} + 2c_{b,2} + \frac{1}{2}(c_{t,2}^2 + c_{b,2}^2), \tag{143}$$

$$I^{(2)}_{11} \approx \ln\frac{\Lambda^2}{m_t^2} - \frac{13}{12} + 2c_{t,1} + \frac{1}{2}(c_{t,1}^2 + c_{b,1}^2), \tag{144}$$

$$I^{(2)}_{12} \approx c_{t,2} + c_{b,1} + \frac{1}{2}\big(c_{t,1}\, c_{t,2} + c_{b,1}\, c_{b,2}\big), \tag{145}$$

$$I_{22}^{(2)} \approx \ln\frac{\Lambda^2}{m_b^2} - \frac{13}{12} + 2c_{b,2} + \frac{1}{2}(c_{t,2}^2 + c_{b,2}^2), \tag{146}$$

$$I_{11}^{(3)} \approx -\frac{1}{3}, \tag{147}$$

$$I_{12}^{(3)} \approx 0, \tag{148}$$

$$I_{22}^{(3)} \approx -\frac{1}{3}\, e^{-2i\delta_0}. \tag{149}$$

$$I_{11}^{(4)} \approx -\frac{1}{3}, \tag{150}$$

$$I_{12}^{(4)} \approx 0, \tag{151}$$

$$I_{22}^{(4)} \approx -\frac{1}{3}\, e^{2i\delta_0}. \tag{152}$$

Next, we shall change variables and rewrite the total expression for kinetic term. For the neutral components we choose non-linear parametrization:

$$\phi_1 e^{i\alpha} \equiv \phi_{12}, \qquad \phi_2 e^{i(\alpha+\delta)} \equiv \phi_{22}. \tag{153}$$

Let us substitute (153) into (132) and expand it with derivatives, leaving only quadratic terms and omitting the rest of the expression. As the v.e.v. of ϕ_1 and ϕ_2 provide quarks with dynamical masses, we replace them with m_t and m_b, respectively. The variable α is regarded as a Goldstone boson, which is absent in the effective potential; it appears only in higher-derivative terms. The other phase, δ , is associated with the relative phase. The variables ϕ_1 and ϕ_2 parameterize radial excitations. For the fields α and δ we use different notations

$$\delta \equiv \phi_3, \qquad \alpha \equiv \phi_4, \tag{154}$$

so that one can rewrite the kinetic term for the neutral components of Higgs doublets uniformly:

$$\frac{N_c}{16\pi^2}\sum_{l,m=1}^{4} A_{lm}(\partial_\mu\, \phi_l)(\partial_\mu \phi_m), \tag{155}$$

where A is four-by-four matrix:

$$\begin{pmatrix} I_{11}^{(2)} - \frac{2}{3} & I_{12}^{(2)} \cos\delta_0 & -I_{12}^{(2)} m_b \sin\delta_0 & -I_{12}^{(2)} m_b \sin\delta_0 \\ I_{12}^{(2)} \cos\delta_0 & I_{22}^{(2)} - \frac{2}{3} & 0 & I_{12}^{(2)} m_t \sin\delta_0 \\ -I_{12}^{(2)} m_b \sin\delta_0 & 0 & (I_{12}^{(2)} + \frac{2}{3}) m_b^2 & \begin{array}{c} (I_{22}^{(2)} + \frac{2}{3}) m_b^2 + \\ + I_{12}^{(2)} m_b m_t \cos\delta_0 \end{array} \\ -I_{12}^{(2)} m_b \sin\delta_0 & I_{12}^{(2)} m_t \sin\delta_0 & \begin{array}{c} I_{12}^{(2)} m_b m_t \cos\delta_0 + \\ + (I_{22}^{(2)} + \frac{2}{3}) m_b^2 \end{array} & \begin{array}{c} (I_{11}^{(2)} + \frac{2}{3}) m_t^2 + \\ + 2 I_{12}^{(2)} m_t m_b \cos\delta_0 + \\ + (I_{22}^{(2)} + \frac{2}{3}) m_b^2 \end{array} \end{pmatrix}. \qquad (156)$$

Appendix B: Momentum Independent Matrix $\widehat{B}$

Let us define the matrix of second variations of the effective potential for the Model I in the following way:

$$B_{kl}^{\pi\sigma} = \frac{8\pi^2}{N_c} \frac{\partial^2}{\partial\phi_l \partial\phi_m} V_{\text{eff}}, \qquad (l, m = 1, 2). \qquad (157)$$

For the case, when $\rho = 0$, the $B_{kl}^{\sigma\sigma}$ matrix for scalars and $B_{kl}^{\pi\pi}$ matrix for pseudoscalars are represented:

$$B_{11}^{\sigma\sigma} = (1 + g^4)\left[64\phi_1^2 \ln\left(\frac{\Lambda^2}{4\phi_1^2}\right) - 223\phi_1^2 - \frac{15\sqrt{3}}{2}\phi_1\phi_2 - \frac{\sqrt{3}}{2}\frac{\phi_2^3}{\phi_1}\right] + 2\Delta_{12}\frac{\phi_2}{\phi_1} - 64\phi_1^2 g^4 \ln g^2, \qquad (158)$$

$$B_{12}^{\sigma\sigma} = -2\Delta_{12} + (1 + g^4)\left[9\phi_1\phi_2 - \frac{15\sqrt{3}}{2}\phi_1^2 + \frac{3\sqrt{3}}{2}\phi_2^2\right], \qquad (159)$$

$$B_{22}^{\sigma\sigma} = (1 + g^4)\left[9\phi_2^2 + \frac{3\sqrt{3}}{2}\phi_1\phi_2 + \frac{5\sqrt{3}}{2}\frac{\phi_1^3}{\phi_2}\right] + 2\Delta_{12}\frac{\phi_1}{\phi_2}, \qquad (160)$$

$$B_{11}^{\pi\pi} = (1 + g^4)\left[\frac{5\sqrt{3}}{2}\phi_1\phi_2 - \frac{\sqrt{3}}{2}\frac{\phi_2^3}{\phi_1} - 3\phi_2^2\right] + 2\Delta_{12}\frac{\phi_2}{\phi_1}, \qquad (161)$$

$$B_{12}^{\pi\pi} = -2\Delta_{12} + (1 + g^4)\left[\frac{5\sqrt{3}}{2}\phi_1^2 + \frac{\sqrt{3}}{2}\phi_2^2 + 3\phi_1\phi_2\right], \qquad (162)$$

$$B_{22}^{\pi\pi} = (1 + g^4)\left[-3\phi_1^2 - \frac{\sqrt{3}}{2}\phi_1\phi_2 + \frac{5\sqrt{3}}{2}\frac{\phi_1^3}{\phi_2}\right] + 2\Delta_{12}\frac{\phi_1}{\phi_2}, \qquad (163)$$

$$B_{kl}^{\sigma\pi} = 0 \qquad k, l = (1, 2). \qquad (164)$$

For the case, ρ non-zero, the corresponding matrix of second variations of the effective potental is a 4×4 matrix. One can arrange the neutral Higgses in a vector-column with 4 components:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \pi_1 \\ \pi_2 \end{pmatrix},$$

where σ_k - scalar fields, π_k - pseudoscalar.

Let us display the non-vanishing components of the matrix of second variations, calculated in the minimum of the potential, in the following form:

$$B_{11}^{\sigma\sigma} = (1+g^4)\Big[64\phi_1^2 \ln\left(\frac{\Lambda^2}{4\phi_1^2}\right) - 223\phi_1^2$$

$$-10\sqrt{3}\phi_1\phi_2 + 3\phi_2^2\Big] - 64\phi_1^2 g^4 \ln\, g^2, \tag{165}$$

$$B_{12}^{\sigma\sigma} = (1+g^4)\Big[-5\sqrt{3}\phi_1^2 + 6\phi_1\phi_2 + \sqrt{3}\phi_2^2\Big], \tag{166}$$

$$B_{22}^{\sigma\sigma} = (1+g^4)\Big[3\phi_1^2 + 2\sqrt{3}\phi_1\phi_2 + 9\phi_2^2\Big], \tag{167}$$

$$B_{11}^{\pi\pi} = 3(1+g^4)\rho^2, \tag{168}$$

$$B_{12}^{\pi\pi} = \sqrt{3}(1+g^4)\rho^2, \tag{169}$$

$$B_{22}^{\pi\pi} = 9(1+g^4)\rho^2, \tag{170}$$

$$B_{11}^{\sigma\pi} = \rho(1+g^4)\Big(-5\sqrt{3}\phi_1 + 3\phi_2\Big), \tag{171}$$

$$B_{12}^{\sigma\pi} = \rho(1+g^4)\Big(3\phi_1 + \sqrt{3}\phi_2\Big), \tag{172}$$

$$B_{21}^{\sigma\pi} = \rho(1+g^4)\Big(3\phi_1 + \sqrt{3}\phi_2\Big), \tag{173}$$

$$B_{22}^{\sigma\pi} = \rho(1+g^4)\Big(\sqrt{3}\phi_1 + 9\phi_2\Big). \tag{174}$$

The matrix $\hat{B}$ for the effective potential in the Model II is:

$$B_{11} = -2\Delta_{11} + 6\phi_1^2 \ln\left(\frac{\Lambda^2}{\phi_1^2}\right) - 4\phi_1^2 + 6J_{1111}\phi_1^2$$

$$+12J_{1112}\phi_1\phi_2\cos\delta_0 + 2\left(J_{1122} + J_{1221} + J_{1212}\cos 2\delta_0\right)\phi_2^2, \qquad (175)$$

$$B_{12} = -2\Delta_{12}\cos\delta_0 + 6J_{1112}\phi_1^2\cos\delta_0 + 6J_{1222}\phi_2^2\cos\delta_0$$

$$+4\big(J_{1122} + J_{1221} + J_{1212}\cos 2\delta_0\big)\phi_1\phi_2, \qquad (176)$$

$$B_{13} = \phi_2\sin\delta_0\big(2\Delta_{12} - 6J_{1112}\phi_1^2 - 8J_{1212}\phi_1\phi_2\cos\delta_0$$

$$-2J_{1222}\phi_2^2\big), \qquad (177)$$

$$B_{14} = 0, \qquad (178)$$

$$B_{21} = B_{12}, \qquad (179)$$

$$B_{22} = -2\Delta_{22} + 6\phi_2^2 \ln\left(\frac{\Lambda^2}{\phi_2^2}\right) - 4\phi_2^2 + 12J_{1222}\phi_1\phi_2\cos\delta_0$$

$$+6J_{2222}\phi_2^2 + 2\left(J_{1122} + J_{1221} + J_{1212}\cos 2\delta_0\right)\phi_1^2, \qquad (180)$$

$$B_{23} = \phi_1\sin\delta_0\big(2\Delta_{12} - 6J_{1222}\phi_2^2 - 8J_{1212}\phi_1\phi_2\cos\delta_0$$

$$-2J_{1112}\phi_1^2\big), \qquad (181)$$

$$B_{24} = 0, \qquad (182)$$

$$B_{31} = B_{13}. \qquad (183)$$

$$B_{32} = B_{23}, \qquad (184)$$

$$B_{33} = 2\phi_1\phi_2\big(\Delta_{12}\cos\delta_0 - J_{1112}\phi_1^2\cos\delta_0$$

$$-2J_{1212}\phi_1\phi_2\cos 2\delta_0 - J_{1222}\phi_2^2\cos\delta_0\big), \qquad (185)$$

$$B_{41} = 0, \qquad (186)$$

$$B_{42} = 0, \qquad (187)$$

$$B_{43} = 0, \qquad (188)$$

$$B_{44} = 0. \qquad (189)$$

Appendix C: The Effective Potential of Model II for the Special Choice of Formfactors

For purposes of further calculations of realistic mass spectra, Yukawa coupling constants and decay widths with taking into account the renormalization-group corrections we present in this appendix the effective potential of Model II for the following set of formfactors:

$$f_{t,1} = 2 - 3\tau; \qquad f_{t,2} = -\sqrt{3}\tau;$$
$$f_{b,1} = -\sqrt{3}\tau; \qquad f_{b,2} = 2 - 3\tau; \tag{190}$$

the constants J_{klmn} are evaluated to definite numbers. Seven of then are defined as follows:

$$J_{1111} = \frac{3}{4}\left(3g_{b,1}^4 - 53g_{t,1}^4\right), \tag{191}$$

$$J_{1112} = \frac{\sqrt{3}}{4}\left(g_{b,1}^3 g_{b,2} - 5g_{t,1}^3 g_{t,2}\right), \tag{192}$$

$$J_{1122} = \frac{3}{4}\big(g_{b,1}^2 g_{b,2}^2 - 53g_{b,2}^2 g_{t,1}^2 - 2g_{b,1}g_{b,2}g_{t,1}g_{t,2}$$
$$+3g_{b,1}^2 g_{t,2}^2 + g_{t,1}^2 g_{t,2}^2\big), \tag{193}$$

$$J_{1212} = \frac{3}{4}\left(g_{b,1}^2 g_{b,2}^2 + g_{t,1}^2 g_{t,2}^2\right), \tag{194}$$

$$J_{1221} = \frac{3}{4}\big(g_{b,1}^2 g_{b,2}^2 + 53g_{b,2}^2 g_{t,1}^2 + 2g_{b,1}g_{b,2}g_{t,1}g_{t,2}$$
$$-3g_{b,1}^2 g_{t,2}^2 + g_{t,1}^2 g_{t,2}^2\big), \tag{195}$$

$$J_{1222} = \frac{\sqrt{3}}{4}\left(g_{t,1} g_{t,2}^3 - 5g_{b,1} g_{b,2}^3\right), \tag{196}$$

$$J_{2222} = \frac{3}{4}\left(3g_{t,2}^4 - 53g_{b,2}^4\right). \tag{197}$$

The rest of them is found from their symmetry property:

$$J_{klmn} = J_{mnkl} = J_{lknm}.$$

With (191)–(197) the potential for the Model II reads:

$$V_{\rm eff} = \frac{N_c}{8\pi^2}\Bigg\{ -\Delta_{11}(\Phi_1^\dagger\Phi_1) - \Delta_{12}(\Phi_1^\dagger\Phi_2) - \Delta_{21}(\Phi_2^\dagger\Phi_1) - \Delta_{22}(\Phi_2^\dagger\Phi_2)$$

$$+ \frac{\nu_+^2(0)}{2}\left(\ln\frac{\Lambda^2}{\nu_+(0)} + \frac{1}{2}\right) + \frac{\nu_-^2(0)}{2}\left(\ln\frac{\Lambda^2}{\nu_-(0)} + \frac{1}{2}\right)$$

$$+ \frac{3}{8}\left(3g_{b,1}^4 - 53g_{t,1}^4\right)(\Phi_1^\dagger\Phi_1)^2 + \frac{3}{8}\left(3g_{t,2}^4 - 53g_{b,2}^4\right)(\Phi_2^\dagger\Phi_2)^2$$

$$+ \frac{3}{4}(g_{b,1}^2 g_{b,2}^2 - 53g_{b,2}^2 g_{t,1}^2 - 2g_{b,1}g_{b,2}g_{t,1}g_{t,2}$$

$$+ 3g_{b,1}^2 g_{t,2}^2 + g_{t,1}^2 g_{t,2}^2)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$

$$+ \frac{3}{4}(g_{b,1}^2 g_{b,2}^2 + 53g_{b,2}^2 g_{t,1}^2 + 2g_{b,1}g_{b,2}g_{t,1}g_{t,2}$$

$$- 3g_{b,1}^2 g_{t,2}^2 + g_{t,1}^2 g_{t,2}^2)(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$+ \frac{3}{8}\left(g_{b,1}^2 g_{b,2}^2 + g_{t,1}^2 g_{t,2}^2\right)\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_1^\dagger\Phi_2)^2\right]$$

$$+ \frac{\sqrt{3}}{4}\left(g_{b,1}^3 g_{b,2} - 5g_{t,1}^3 g_{t,2}\right)(\Phi_1^\dagger\Phi_1)\left[(\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1)\right]$$

$$+ \frac{\sqrt{3}}{4}\left(g_{t,1}g_{t,2}^3 - 5g_{b,1}g_{b,2}^3\right)(\Phi_2^\dagger\Phi_2)\left[(\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1)\right]\Bigg\}, \tag{198}$$

where we adopt the definition for $\nu_\pm(0)$:

$$\nu_\pm = 2g_{t,1}^2(\Phi_1^\dagger\Phi_1) + 2g_{b,2}^2(\Phi_2^\dagger\Phi_2) \pm 2\Big[g_{t,1}^4(\Phi_1^\dagger\Phi_1)^2 + g_{b,2}^4(\Phi_2^\dagger\Phi_2)^2$$
$$+2g_{t,1}^2 g_{b,2}^2(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - 4g_{t,1}^2 g_{b,2}^2(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\Big]^{1/2}. \tag{199}$$

References

1. J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide*. Addison-Wesley, (1990).
2. R. Rodenberg, Int. J. Mod. Phys. **A11** (1996), 4779 and references therein.
3. CDF Collaboration, F. Abe et. al., Phys. Rev. Lett. **74** (1995), 2626.
   D0 Collaboration, S. Abachi et. al., ibid. **74** (1995), 2632.
4. The LEP EW Group, Data presented at the 1996 Summer Conferences, LEP-EWWG/96-02.
5. N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. **B158** (1979), 295.
   H. P. Nilles, — Phys. Rep. **110** (1984), 1.
6. G. Kreyerhoff, and R. Rodenberg, — Phys. Lett. **B226** (1989), 323.
   J. Freund, G. Kreyerhoff, and R. Rodenberg, — Phys. Lett. **B280** (1992), 267.
7. N. V. Krasnikov, S. Pokorski, — Phys. Lett. **B288** (1992), 184.
8. A. A. Andrianov, R. Rodenberg, and N. V. Romanenko,, — Nuovo Cim. **108A (5)** (1995), 577.
9. N. V. Krasnikov, R. Rodenberg, — Nuovo Cim. **A105** (1992), 1601.
10. J. Pati, A. Salam, — Phys. Rev. **D10** (1974), 275.
    R. Mohapatra, J. Pati, — Phys. Rev. **D11** (1975), 566,1558.
    G. Senjanovich, — Phys. Rev. **D12** (1975), 1502.
    H. Fritsch, P. Minkowski, — Nucl. Phys. **B103** (1976), 61.
    R. Mohapatra, G. Senjanovich, — Phys. Rev. Lett. **44** (1980), 912.
11. V. A. Miransky, M. Tanabashi, and K. Yamawaki, Mod. Phys. Lett., **A4** (1989), 1043; Phys. Lett., **B221** (1989), 177.
    W. J. Marciano, Phys. Rev. Lett., **62** (1989), 2793; Phys. Rev., **D41** (1990), 219.
    W. A. Bardeen, C. T. Hill, and M. Lindner, — Phys. Rev. **D41** (1990), 1647.
12. Y. Nambu, G. Jona-Lasinio, — Phys. Rev. **122** (1961), 345.
13. M. Lindner, D. Lüst, — Phys. Lett. **B272** (1991), 91.
14. M. Lindner, — Int. J. Mod. Phys. **A8** (1993), 2167.
15. M. Luty, — Phys. Rev. **D 41** (1990), 2893.
16. S. L. Glashow, S. Weinberg, — Phys. Rev. **D15** (1958).
17. G. Cvetic, Preprint DO-TH-97-03 (1997); hep-ph/9702381.
18. A. A. Andrianov, V. A. Andrianov, Int. J. Mod. Phys., **A8**, No. 11 (1993), 1981; Russ. J. Theor. Math. Phys., **93** (1992), 67.
19. A. A. Andrianov, V. A. Andrianov, Zap. Nauchn. Semin. POMI, **209**, 12 (1994) 3; Nucl. Phys B (Proc. Suppl.) **39B, C** (1995), 257.
20. E. Pallante, R. Petronzio, — Z.-Phys. **C 65** (1995), 487.
21. M. K. Volkov, C. Weiss, — Phys. Rev. **D 56** (1997), 221.
22. M. K. Volkov, D. Ebert, and M. Nagy, Preprint HEP-PH/9705334.
23. T. D. Lee, — Phys. Rev. **D8** (1973), 1226.
24. A. A. Andrianov, V. A. Andrianov, and V. L. Yudichev, Proc. X Internat. Workshop on High Energy Physics and Quantum Field Theory (Zvenigorod, Russia, Sept. 1995), p. 211, hep-ph/9512256; J. Theor. Math. Phys., **108** (1996), 276.
25. A. A. Andrianov, V. A. Andrianov, and V. L. Yudichev, Preprint HEP-PH/9610376.

26. A. Skjold, P. Osland, — Phys. Lett. (ibid.) **B311** (1993), 261, ibid. **B329** (1994), 305.
27. D. Choudhury, M. Krawczyk, Preprint hep-ph/9607271.
28. P. Langacker, Preprint UPR-0435T (1990).
29. S. M. Barr, A. Zee, — Phys. Rev. Lett. **65** (1990), 860.
J. F. Gunion, F. Vega, — Phys. Lett. **B251** (1991), 222.
30. L. McLerran, M. Shaposhnikov, N. Turok, and M. Voloshin, — Phys. Lett. **B256** (1991), 451.
31. J. M. Cline, K. Kainulainen, and Axel P. Vischer, — Phys. Rev. **D54** (1996), 2451.
32. J. L. Diaz-Cruz, M. A. Perez, and J. J. Toscano, — Phys. Lett. **B398** (1997), 347.

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