The Methods of Fostering Creativity through Mathematical Problem Solving

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Abstract: Which methods could be used to foster mathematical creativity in school situations? The following topics are treated with the respect to this question: 1. "Open-ended approach" and "From problem to problem", 2. Relation to mathematical creativity, 3. Teacher's belief and the mathematics textbook.

Kurzreferat: *Methoden, Kreativität durch mathematisches Problemlösen zu fördern.* Welche Methoden gibt es, mathematische Kreativität in der Schule zu fördern? Im Hinblick auf diese Frage werden folgende Punkte angesprochen: 1. Methode "Offener Zugang" und "Vom Problem zum Problem", 2. Beziehung zur mathematischen Kreativität, 3. Lehrereinstellung und Mathematikschulbuch.

ZDM-Classification: D50

E. Pehkonen, chief organizer in our topic group 7 showed us "the state of art in mathematical creativity". In this paper, the following aspect will be treated. Which methods could be used to foster mathematical creativity within school situations?

1. "Open-ended Approach" and "From Problem to Problem"
2. Relation To Mathematical Creativity, and
3. Teacher's Belief and the Mathematics Textbook

in terms of the aspect are described.

1. "Open-ended approach" and "From Problem To Problem"

Two methods of fostering mathematical creativity are as follows.

One method which was called an "open-ended approach" means that an incomplete problem is presented at first, and the lesson proceeds by utilizing a multiplicity of correct approaches to solving the given problem in order to provide experience in finding something new in the process through variously combining students' own knowledge, skills, or ways of thinking which have been previously learned. We found three types of open-ended problems:

– problem type "finding" (A-1)
– problem type "classifying" (B-1)
– problem type "measuring" (C-1)

A-1: The teacher asks the students: Find rules or properties from this table as many as possible.

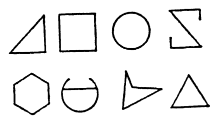
B-1: Classify the figures below according to some common properties which you identify.

C-1: Three students A, B, C throw five marbles that come to rest as in Fig. 1. In this game, the students with the smallest scatter of marbles is the winner. To determine the winner, we will need to have some numerical way of measuring the scatter of the marbles. Think about this situation from various points of view and write down different ways of indicating the degree of scattering. (Becker & Shimada, in press)

A-1

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

B-1



C-1

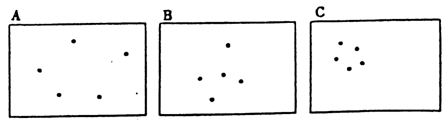


Fig. 1

Another method is "From Problem to Problem". For example, squares are made by using toothpicks as shown in Fig. 2.

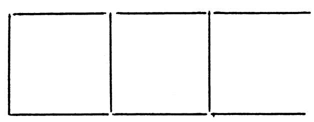


Fig. 2

When the number of squares is 5, how many toothpicks are used? Make up many similar problems by changing some parts of the given problem. Students can make up many problems by changing the number of square, the shape, and the object and so on.

This method focused on learning activities with students formulating new problems by using generalization, analogy, and the idea of converse, from a given problem, and then solving them.

2. Relation to mathematical creativity

I think that openness like "open-ended approach" and "from problem to problem" is one aspect of fostering mathematical creativity. Because, "open-ended approach" means endproducts are open, and "from problem to problem" means ways to develop are open.

It is important that students can combine different ways of thinking in one problem. We can often see "creativity" in mathematics appear by combining seemingly different aspects.

The next is one example related to Fermat's last theorem.

The Taniyama-Shimura conjecture links two seemingly disparate mathematical fields: the algebraic subject of elliptic curves and the analytic subject of modular forms. (Cipra, 1996).

3. Teacher's belief and the mathematics textbook

Generally speaking, most of classroom teachers, in particular, at elementary school level, think that there is only one correct answer in mathematics and *one way* of method to solve a mathematics problem. Excellent teachers know there are many correct approaches to solving a given mathematics problem.

Therefore, according to the first and second things which I described, I think the teacher's belief about mathematics is connected to foster mathematical creativity.

One more thing, a role of textbooks is important. "From Problem to Problem" can be seen in Japanese elementary textbooks since 1996 and in junior high school textbooks since 1993. Until then, we have never seen such a problem in the *official* mathematics textbooks in Japan. Classroom teachers who use this textbook have to treat "making up a new problem from a given problem". I think it is a short cut to foster mathematical creativity within class.

4. References

Becker, J. P.; Shimada, S. (Eds.) (in press): The open Ended Approach: A New Proposal for Teaching Mathematics. – Reston (VA): National council of Teachers of Mathematics

Cipra, B. (1996): Fermat's Theorem – At Last! – In: What's Happening in the Mathematical Sciences, Vol. 3, 1995–1996. American Mathematical Society

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