Recognising Mathematical Creativity in Schoolchildren

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Abstract: Examples of tasks designed to recognise creative thinking within mathematics, used with 11–12-year-old pupils, are described. The first construct employed in the design of these tasks is the ability to overcome fixation. Sometimes pupils demonstrate content-universe fixation, by restricting their thinking about a problem to an insufficient or inappropriate range of elements. Other times they show algorithmic fixation by continuing to adhere to a routine procedure or stereotype response even when this becomes inefficient or inappropriate. The second construct employed is that of divergent production, indicated by flexibility and originality in mathematical tasks to which a large number of appropriate responses are possible. Examples of three categories of such tasks are described: (1) problem-solving, (2) problem-posing, and (3) redefinition. Examples of pupils' responses to various tasks are used to argue that they do indeed reveal thinking that can justifiably be described as creative. The relationship to conventional mathematics attainment is discussed – mathematics attainment is seen to limit but not to determine mathematical creativity.

Kurzreferat: *Mathematische Kreativität bei Schulkindern erkennen.* Es werden Beispielaufgaben beschrieben, die dem Erkennen kreativen Denkens in Mathematik bei 11–12 jährigen Schülern dienen sollen. Die erste Aufgabengruppe dient der Fähigkeit, Fixierungen zu überwinden. Manche Schüler zeigen eine Fixierung in der Gesamtheit eines Inhaltsbereichs, die dazu führt, daß sie ihr Problemdenken auf einen unzureichenden oder ungeeigneten Teilbereich von Möglichkeiten beschränken. Andere Schüler wiederum zeigen eine algorithmische Fixierung, indem sie Routinemethoden oder stereotype Antworten auch dann noch verwenden, wenn sich diese als uneffizient oder ungeeignet herausstellen. Die zweite Aufgabengruppe soll divergentes Denken fördern; sie ist gekennzeichnet durch Flexibilität und Originalität der mathematischen Aufgaben, zu denen es eine Vielzahl möglicher Ergebnisse gibt. Drei Kategorien solcher Aufgaben werden beispielhaft beschrieben: (1) Problemlösen, (2) Problemstellen und (3) Neudefinition. Beispielhafte Schülerantworten zu verschiedenen Aufgaben werden benutzt, um zu zeigen, daß sie tatsächlich ein Denken enthüllen, das kreativ genannt werden kann. Die Beziehung zu konventionellen mathematischen Leistungen wird diskutiert – diese scheinen mathematische Kreativität eher zu hemmen.

ZDM-Classification: C40

1. Introduction

In this paper consideration is given to identifying the kinds of responses in school mathematics that might justifiably be identified as "creative". The author has developed a battery of tasks for use with pupils around the age of 11–12 years, based on a number of key constructs for recognising mathematical creativity. The original collection of tasks (Haylock, 1987a) has been developed and supplemented by further tasks based on the same constructs. Examples of responses to these tasks by 11–12-year-old pupils in Britain are used to illustrate the constructs proposed and to argue that they do indeed identify aspects of mathematical ability that are significant in terms of pupils' responses to mathematical problems. The approach used is to start with ideas associated with creativity in general and to identify those aspects of creative thinking that would seem to be of most relevance to pupils doing mathematics in school.

1.1 What is creativity?

There is no single definition of *creativity* that is generally accepted or used in research. Creativity in general is a notion that embraces a wide range of cognitive styles, categories of performance, and kinds of outcomes. As Cropley (1992) points out, there is considerable confusion about the nature of creativity and there are at least two major ways in which the term is used. On the one hand, it refers to a special kind of thinking or mental functioning, often called *divergent thinking.* On the other hand, creativity is used to refer to the generation of products that are perceived to be creative, such as works of arts, architecture or music. In terms of teaching children in schools, Cropley leans towards the first of these and adopts the stance that creativity is "the capacity to get ideas, especially original, inventive and novel ideas."

1.2 What is creativity in school mathematics?

Given the lack of an agreed definition for creativity in general, it is not surprising that there is not a single, clear definition of mathematical creativity. However the approach to creativity suggested by Cropley (quoted above) is most prevalent in discussions about creativity in school mathematics. The focus is on identifying the kinds of thinking in mathematical tasks that qualify for the description "creative". Krutetskii (1976) seems to equate mathematical creativity in schoolchildren with mathematical giftedness, using the two terms synonymously. He argues that mere mastery of mathematical material needs to be extended to an "independent *creative* mastery of mathematics under the conditions of school instruction." He then asserts that mathematical creativity will be recognised in "the independent formulation of uncomplicated mathematical problems, finding ways and means of solving these problems, the invention of proofs and theorems, the independent deduction of formulas, and finding original methods of solving nonstandard problems." Krutetskii's conception of mathematical creativity is clearly set within a problem-solving framework for mathematical ability and suggests that creativity in problem-solving in mathematics will be characterised by such features as problem-formulation, invention, independence and originality. Ideas such as these – along with others such as flexibility, fluency, forming new associations, and divergent production – that are associated with discussion and research about creativity in general, have been seen by many mathematics educators to have relevance to children doing mathematics in school (see, for example: Aiken, 1973; Barbeau, 1985; Ediger, 1992; Haylock, 1987a; Singh, 1990; Tammadge, 1979; Tuli, 1985; Whitcombe, 1988).

Two main approaches to the recognition of creative thinking can be identified. The first is to consider the responses of subjects to problem-solving tasks where a particular cognitive *process* that is understood to be characteristic of creative thinking might be required for success. This consideration leads to the recognition that one of the key cognitive processes in creative problem-solving in mathematics is the *overcoming of fixation*, the breaking of a mental set. The second approach is to determine the criteria for a *product* to be indicative of creative thinking having taken place. Various kinds of *divergent production* tasks can be devised in mathematics that generate responses that can be judged by such criteria as flexibility, originality and appropriateness. These two approaches provide the basis for the framework that has been adopted by the author for the recognition of creativity within school mathematics (Haylock, 1987b).

2. Overcoming fixation

Creative thinking is almost always seen as involving flexibility. Helson & Crutchfield (1970), for example, found that those research mathematicians who had been rated as more creative by other professional mathematicians scored significantly higher for flexibility than their peers. The opposite of flexibility is rigidity of thinking. One aspect of creativity that has clear relevance to mathematical problem-solving, therefore, might be the ability to *overcome fixations* or rigidity in thinking, to break from mental sets. Balka (1974) includes in a list of criteria for creative ability in mathematics "the ability to break from established mind sets to obtain solutions in a mathematical situation." A basic issue in mathematical problem-solving is why a person who knows all the mathematics they need in order to solve a particular problem still fails to solve it. Sometimes, a possible explanation is that their mind is set in an inappropriate direction, that they are adhering rigidly to an approach that does not lead to the solution. The classic accounts of mathematical invention and creation in mathematics by Poincaré (1952) and Hadamard (1954) discuss creative problem-solving in terms of the four stages of preparation (becoming familiar with the problem), incubation (allowing the mind to work on the problem), illumination (when the insight that leads to a solution is obtained) and verification (confirming that the insight is correct). In these accounts, the key to the transition from the incubation stage to illumination appears often to be an unexpected or novel way of considering the problem. When this does not occur it is often because the problem-solver's thinking is fixated along inappropriate lines.

Mathematics teachers will recognise fixation as a characteristic behaviour of many students. For example, when required to calculate 20×10, some students will resort to the long multiplication algorithm, even though they know the answer to be 200. In this case it is likely that the student is bringing to the problem an expectation that an algorithmic approach will be required. Cunningham (1966) calls this a subjective set: a set of attitudes, intentions or presuppositions that the subject brings to the situation. He distinguishes this from an objective set, where the mental set is established by the materials or sequencing of events within the situation. This is an interesting distinction, but one that is difficult to maintain in terms of pupils doing mathematics in a school context. It seems likely that behaviour suggesting fixation in problem-solving will be a combination of both pre-determined attitudes and the content or presentation of the problem-solving situation.

However, a useful distinction between two kinds of fixation in mathematics can be made. Krutetskii (1976) identified "flexibility of mental processes" as a key component of creative mathematical ability in school-children. He illustrated this with problems involving "self-restriction" and examples of pupils "leaving the patterned stereotyped means of solving a problem and finding different ways ..." This suggests, therefore, that there are at least these two kinds of fixation that are especially significant in mathematical problem-solving: self-restriction and adherence to stereotype approaches. These are referred to in this paper as *content-universe fixation* and algorithmic *fixation.*

2.1 Content-universe fixation

This construct is derived from Krutetskii's notion of self-restriction, where the pupil's thinking about a mathematical problem is restricted unnecessarily to an insufficient range of elements that may be used or related to the problem. Two examples of tasks – one numerical and one geometric – that have been used with 11–12-year-olds, are described below. These tasks target very specifically the pupil's ability to overcome fixations. The argument is that the ability to overcome these kinds of fixation – to allow the mind to range over a wider set of possibilities than might at first come into the conscious awareness of the problem-solver – is a significant aspect of problem-solving in mathematics and warrants the description "creative thinking". Readers must judge for themselves whether it is justified to assert that the minority of pupils who show the ability to overcome such fixations stand out as being more creative in their mathematical thinking than their peers.

2.1.1 Sum and difference

Pupils are given a series of questions in which they are asked to find two numbers that have a given sum and a given difference. The early examples reinforce the pupils' tendency to restrict their thinking to a content-universe of positive integers. For example, they might find two numbers with sum 10 and difference 4. Later in the sequence they are asked to find, for example, two numbers with sum 10 and difference 10. A surprising number of pupils fail on this because they appear to exclude the possibility that one of the numbers might be zero. Then, when asked for two numbers with sum 9 and difference 2, the vast majority of pupils assert that this cannot be done. By trying all combinations of pairs of positive integers summing to 9 they believe they have exhausted all the possibilities. To obtain the solution (3.5 and 5.5) it is necessary to overcome this self-restriction and to adopt an interpretation of "number" that includes other than whole numbers.

2.1.2 Isosceles triangles

This is a geometric task requiring the pupil to overcome the tendency to restrict their thinking to an inappropriate content-universe. In Fig. 1 pupils are asked to draw and shade isosceles triangles inside the given shapes with the following constraints: (1) they must use XY as one of the sides of the triangle; (2) they must make the area of the triangle as large as possible. They may use a ruler.

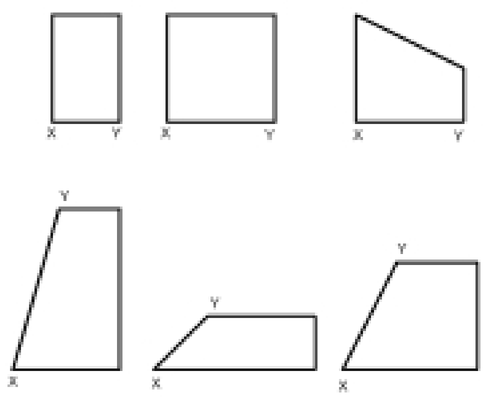

Fig. 1: Diagrams for "Isosceles triangles" task

The first two examples in each line confirm the tendency to conceive of isosceles triangles always with the "base" horizontal. The correct solution for the third one in each line requires the pupil to break from this fixation and to use an isosceles triangle in a different orientation. However, nearly all the pupils who have sufficient geometric skills and knowledge to tackle this task come up with the incorrect responses for these that are shown in Fig. 2. Again the argument here is that the pupil who overcomes the tendency to conceive of isosceles triangles in other than the stereotype orientation is showing more-creative thinking.

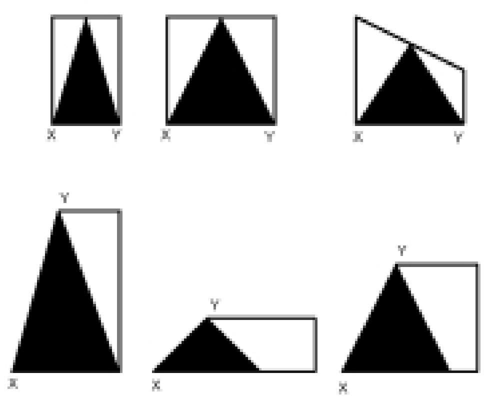

Fig. 2: Typical responses to "Isosceles triangles" task

2.2 Algorithmic fixation

The second kind of fixation that would seem to be significant in mathematics is derived from the notion of *Einstellung*, as used, for example, in the classic studies in psychology by Duncker (1945) and Luchins (1942, 1951). This kind of fixation is shown where a pupil shows continued adherence to an initially successful algorithm, even when this becomes inappropriate or less than optimal. This might be an algorithm learnt beforehand, such as those used for various calculations – or it might be one that is developed through the sequence of questions in the task itself. Even in the latter case it seems likely that pupils will have been conditioned by their previous experience of what leads to success in school mathematics to look for an algorithm or some such process that can be applied repeatedly to a sequence of similar-looking questions. The argument here is that it is justified to describe as more creative the thinking of the pupil, who, having established a routine procedure that works for a sequence of problems, is nevertheless able to hold in their mind the possibility that there might be a more elegant or more efficient alternative to the stereotype. Much mathematics learning necessarily contributes to the formation of standard procedures, algorithms and stereotype methods – but creative problem-solving sometimes requires the student to break away from the stereotypes in order to achieve an insight. The following are examples of one numerical and one geometric task that – following Luchins – have been designed specifically to encourage pupils to establish an algorithmic procedure and to continue to apply it unnecessarily or when it is no longer the optimal or most elegant solution.

2.2.1 Weights

In this task pupils are required to determine how to measure out a given mass of sand in a sequence of problems, where in each case a balance and three masses are provided. As an example, they are shown that, given masses of 20 g, 9 g and 5 g, it is possible to measure out 24 g of sand in pan B by placing the 20 g and 9 g masses in pan A, the 5 g mass in pan B and then pouring sand into pan B until it balances. Table 1 shows the sequence of problems used – and the solutions provided by the majority of pupils, showing algorithmic fixation. In each case it is possible to measure out the sand by the same procedure as in the given example. So a routine is established in the early examples: find the sum of two masses in pan A and subtract the third mass to give the required amount of sand. Only a small number of pupils, having established this procedure, then deviate from it in problems 7, 8 and 10, where a more elegant or more efficient solution is possible. Even in the last problem in the sequence, where a 20 g mass is provided, about 90% of pupils propose that the 20 g of sand be obtained by placing the 32 g and 8 g masses in pan A and the 20 g mass in pan B! Those pupils, who, having established a routine that works, continue to hold in their minds the possibility that a more desirable alternative may be available, are judged to show more creativity in their thinking.

Table 1:
Sequence of problems in "Weights"
and solutions showing algorithmic fixation

	masses available	mass of sand	how to do it	
			pan A	pan B
1.	20,9,5	24	20,9	5
2.	16,7,3	12	16,3	7
3.	2,50,40	12	50,2	40
4.	5,55,50	10	5,55	50
5.	14,11,3	6	14,3	11
6.	81,7,8	80	81,7	8
7.	55,10,5	60	55,10	5
8.	7,6,10	3	7,6	10
9.	30,20,8	18	30,8	20
10.	32,20,8	20	32,8	20

2.2.2 Cuts

Similar patterns of thinking in a geometric context are shown in this task. In a sequence of problems pupils are asked to draw lines on a rectangle to divide it up into a given number of parts of the same size and shape. In an example they are shown that 1 line is required to divide the rectangle into 2 equal parts. Then the subsequent problems ask for: 3 parts, 5 parts, 7 parts, 9 parts and 6 parts. The majority of pupils establish a procedure in the first few examples and continue to use this: 2 lines, 4 lines, 6 lines ... with the number of lines one less than the number of parts. Only the occasional pupil shows what might be recognised as creative thinking by breaking from this algorithmic fixation, obtaining 9 equal parts with 4 lines (2 horizontal and 2 vertical) and 6 equal parts with 3 lines.

2.3 Creative thinking demonstrated

One further example serves to demonstrate that the thinking shown by the pupil who overcomes fixations of the kind discussed above and deviates from the stereotypical responses in mathematical tasks may justifiably be described as creative. In a task called “Areas” pupils are given a number of incomplete four-sided figures and asked to complete them in order to make the areas satisfy various conditions. For example, in Fig. 3 they are asked to draw the other two sides of the four-sided figure so that the area of the figure is less than $2\,cm^2$. The self-restrictions that are evident in the way most pupils fail to find a solution in this task include the tendency to draw only rectangles or only convex quadrilaterals.

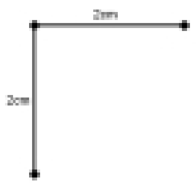

Fig. 3: Two sides of a four-sided figure with area less than $2\,cm^2$

The few pupils who manage to find the intended correct solution shown in Fig. 4 show that they are not imposing this kind of self-restriction on their thinking. One pupil surprised the author by the solution shown in Fig. 5. This surely justifies the description “creative”.

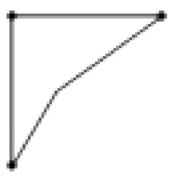

Fig. 4: The intended correct solution to the “Areas” task

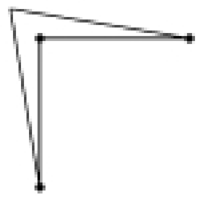

Fig. 5: An unexpected creative solution to the “Areas” task

3. Divergent production

The second major construct in creativity that can be applied readily to mathematics relates to the notion of *divergent production.* Divergent production tasks have their origin in the work of American researchers such as Torrance and Guilford in the fifties and sixties. In these tasks a pupil is given an open-ended situation to which many possible responses may be made. A number of researchers have developed examples of similar, open-ended mathematical tasks that allow pupils to show divergent thinking in their responses (for example: Haylock, 1987b, Pehkonen, 1992; Singh, 1990; Tuli, 1985, Zosa, 1978).

3.1 Criteria for judging divergent production

A typical Torrance test would be to think of many possible uses for a tin can. The subject’s responses are then assessed for creativity in terms of fluency (the number of acceptable responses), flexibility (the number of different kinds of response) and originality (the statistical infrequency of the responses in relation to the peer group). In a mathematical context the criterion of fluency often seems less useful for indicating creative thought than flexibility. For example, if asked to generate questions with the answer 4, a pupil might start with “5 – 1”, “6 – 2”, “7 – 3”, and continue this sequence indefinitely, thus scoring highly but not showing any creativity. Flexibility, on the other hand, focuses on the number of different ideas used. So generally it is found that the responses in divergent production tasks in mathematics can be rewarded using the criteria of flexibility and originality. Another highly-significant criterion is “appropriateness”: a mathematical response may be highly original but it is of little use if it is not appropriate within accepted mathematical criteria. For example, the response $\sqrt{8}$ as a question generating the answer 4 may be original, but it is also wrong! This last criterion raises one of a number of conflicts and paradoxes involved in the notion of mathematical creativity in the context of schooling. At times a teacher (or researcher) may find it a difficult matter of judgement in deciding how to respond to a pupil who shows imagination and originality mixed in with an irritating level of inaccuracy. There may be, therefore, something of a conflict between accuracy and creativity. Similar conflicts may occur between teaching mathematical routines and encouraging the willingness to break from stereotyped procedures, and between being systematic and being flexible (Haylock, 1985).

3.2 Criteria for a good divergent production task

In the development of divergent production tasks the author has identified six criteria for a task to be effective in revealing creativity and in distinguishing between pupils in a particular population in terms of the creativity of their responses: (1) The pupils' responses must show that a range of mathematical ideas have been used. (2) At least twenty appropriate responses are possible for these pupils. (3) The pupils' responses should show a consistent interpretation of the instructions in the task. (4) There should be several obvious responses that can be obtained by most pupils. (5) There should be a number of appropriate responses that are obtained by relatively few pupils. (6) These original responses should have a degree of face validity for indicating creative ability in mathematics and they should not be mathematically trivial.

3.3 Categories of divergent production tasks in mathematics

Three categories of open-ended, divergent-production tasks, meeting the criteria above, that have been identified and used by the author are: (i) *problem-solving,* where the pupil is given a problem that has many solutions and invited to find as many different and interesting solutions as he can; (ii) *problem-posing,* where the pupil is given a situation and invited to make up as many interesting mathematical questions as possible that can be answered from the given data; (iii) *redefinition,* where the pupil is required repeatedly to redefine the elements of a situation in terms of their mathematical attributes. These are not presented as hard-and-fast categories, but as a framework for generating tasks that might reveal useful divergent-thinking in mathematics.

3.3.1 Divergent production in a problem-solving task

There is a well-established link between the processes of problem-solving and creative thinking (Weisberg, 1988). Given the significance of problem-solving in mathematics it is natural to start by constructing divergent production tasks in this field that require the pupil to solve a problem – but this will be a problem with many possible solutions. An example of a problem-solving task in mathematics that can reveal divergent thinking is: given a nine-dot centimetre-square grid draw as many shapes as possible with an area of $2\,\mathrm{cm}^2$, by joining up the dots with straight lines. About 98% of pupils produce the easiest solution: a rectangle 2 cm by 1 cm. Fig. 6 shows four of the other possible solutions, with the percentages of 11–12-year-olds that have been found to produce these responses. Solutions like the first one shown can be obtained by a straightforward combination of square units and half-squares (triangles). Most pupils come up with a number of these. More difficult appear to be those solutions like the second one shown, where one of the internal angles is 315°. Divergent thinking is shown in the third example, where one of the lines used joins two non-adjacent points on the grid. The solutions that require the most creative thinking would appear to be those like the fourth example, where these two ideas are combined.

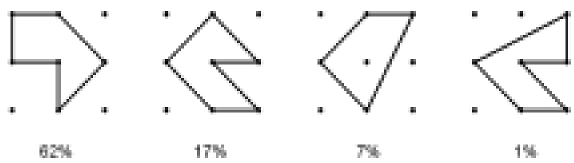

Fig. 6: Four shapes with area $2\,\mathrm{cm}^2$, with percentages of 11–12-year-olds obtaining them

Considerable ingenuity and imaginative thinking was shown by one pupil who devised the solution shown in Fig. 7. It is rare in conventional mathematics for pupils to have the opportunity provided by more open-ended problems of this kind to demonstrate such truly creative thinking in mathematics.

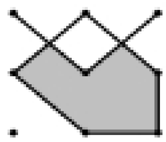

Fig. 7: A highly-creative idea for a shape with an area of $2\,\mathrm{cm}^2$

3.3.2 Divergent production in a problem-posing task

Problem-solving has always been central to mathematics education, but a number of researchers, such as Silver (1994, 1995) and Stoyanova & Ellerton (1996), have recently been exploring the significance of problem-posing in learning mathematics. Problem-posing situations can provide opportunities for pupils to demonstrate considerable creativity. For example, pupils might be given a scattergram showing the numbers of boys and girls in the families of the children in a class and asked to make up as many questions as they can that can be answered from the graph. Experience suggests that it is advisable sometimes for pupils to be asked to answer their own problems, in order to make their intentions clear. In one task with a problem-posing aspect, for example, pupils are asked to write down other results that can be deduced easily from a given result: i.e. $23 \times 35 = 805$. So they pose questions that can be answered easily from the given information, but also provide the answers. Some of the responses of two mathematically high-attaining pupils who had scored equally high marks on a standardised test of mathematical attainment are shown in Table 2. Pupil A used at most three ideas in generating new questions from the given result: that one number can be multiplied by 10; that the other number can be multiplied by 10; and that you can go on doing this repeatedly. Pupil B, by contrast, showed considerable flexibility and originality in generating a total of 26 responses, drawing on a wide range of mathematical ideas: multiplication by 10, commutativity, division as the inverse of multiplication, doubling, halving, decimals, brackets, and so on. The fact that two pupils such as these, with equal levels of attainment in conventional mathematics tests, demonstrate such markedly differing performances in a divergent production task lends support to the view that these tasks may reveal some aspects of creative ability in mathematics.

Table 2:
Some responses of two pupils of equal mathematical attainment to a problem-posing task

Pupil A	Pupil B
$230 \times 35 = 8050$	$230 \times 35 = 8050$
$2300 \times 35 = 8500$	$35 \times 23 = 805$
$230 \times 350 = 80500$	$17.5 \times 23 = 402.5$
$23000 \times 35 = 805000$	$35 \times 11.5 = 402.5$
$230000 \times 35 = 8050000$	$70 \times 23 = 1610$
$2300000 \times 35 = 80500000$	$46 \times 35 = 1610$
$2300000 \times 35 = 805000000$	$1610 \div 70 = 23$
	$805 \div 23 = 35$
	$(20 + 3) \times 35 = 805$
	$(70 \div 2) \times 23 = 805$

3.3.3 Divergent production in a redefinition task

The term "redefinition" was coined by Guilford (1959) as a trait of creativity, referring to the ability to give up old interpretations of familiar objects to use them in a novel way. In mathematics it is often useful to be able to reinterpret the component parts of a problem situation. So, for example, a particular line segment in a geometric problem might be seen first as the side of a triangle, then as the radius of a circle, then as half of the diameter, and so on. Wallach and Kogan (1965) in their classic study of creativity and intelligence, used a task in which pupils were asked to state all the ways in which a carrot and a potato are alike. To produce many and varied responses this requires continual redefinition of the potato and the carrot in terms of their attributes and functions. Similar tasks using this notion of redefinition suggest themselves in the content of mathematics. For example, in one such task pupils are asked to state all the things that are the same about the two numbers 16 and 36. Such tasks as this again reveal considerable variation between pupils in terms of flexibility and originality. One pupil scoring 140 on a standardised mathematics attainment test produced only these three responses to this task: both have 6s in them; both are multiples of 2; both are multiples of 4. By contrast, another pupil with the same score for mathematics attainment produced these responses: both are even; both divide by 2; both divide by 4; both have a 6 in them; both less than 40; both above 15; both whole numbers; both not prime; both factors of 576.

4. Relationship of mathematical creativity to mathematical attainment

It is clear from the examples quoted above that pupils of equal mathematical attainment can show vastly-different performances on tasks designed to reveal mathematical creativity. The author's research (Haylock, 1987a) suggests that mathematical attainment *limits* the pupil's performance on both overcoming-fixation and divergent-production tasks, but does not *determine* it. Low-attaining pupils do not have sufficient mathematical knowledge and skills to demonstrate creative thinking on the kinds of tasks described in this report. The higher the level of attainment the more possible it becomes to discriminate between pupils in terms of these indicators of mathematical creativity. The pupils with the greatest facility for overcoming fixation and for thinking divergently are usually in the very highest attaining group – but even in this group there are significant numbers of pupils who show very low levels of these kinds of creative thinking in mathematics. Significant differences are identified between these mathematically high-attaining, low-creative pupils and their high-attaining, high-creative peers, in that the first group tend to be more anxious about mathematics, to have low self-concept, to be narrow coders and to be less willing to take reasonable risks in mathematics. A challenge for further research in this area is to identify teaching approaches that are effective in moving these pupils who have good mathematical knowledge and skills away from their over-reliance on routines and stereotypes and their rigidity in thinking about mathematical situations towards the kinds of thinking that have been identified above as representing creativity in mathematics in the school context.

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