Introduction:

The State-of-Art in Mathematical Creativity

Erkki Pehkonen, Helsinki

Abstract: Creativity is a topic which is often neglected within mathematics teaching. Usually teachers think that it is logic that is needed in mathematics in the first place, and that creativity is not important in learning mathematics. On the other hand, if we consider a mathematician who develops new results in mathematics, we cannot overlook his/her use of the creative potential. Thus, the main questions are as follows: What is the meaning of creativity within school mathematics? What methods could be used to foster mathematical creativity within school situations? What scientific knowledge, i.e. research results, do we have on the meaning of mathematical creativity?

Kurzreferat: *Einführung: Mathematische Kreativität – eine Übersicht.* Kreativität wird im Mathematikunterricht häufig vernachlässigt. Lehrer sind in der Regel der Ansicht, daß an erster Stelle in der Mathematik Logik gebraucht würde, und daß Kreativität beim Mathematiklernen nicht so wichtig sei. Wenn wir andererseits Mathematiker betrachten, die neue Erkenntnisse in Mathematik entwickeln, so können wir ihr kreatives Potential nicht übersehen. Die wesentlichen Fragen sind also: Was bedeutet Kreativität in der Schulmathematik? Welche Methoden zur Förderung mathematischer Kreativität in der Schule können benutzt werden? Welches wissenschaftliche Wissen, d.h. welche Forschungsergebnisse, haben wir über die Bedeutung mathematischer Kreativität?

ZDM-Classification: C40, C80

1. What is creativity?

Creativity is not a characteristic only found in artists and scientists, but it is also a part of everyday life. For example, a do-it-yourself man is making use of his creative thinking when he solves practical problems with defective tools. Therefore, creativity should be an intrinsic part of the "mathematics for all" program.

Commonly, people think that creativity and mathematics have nothing to do with each other. But the mathematicians disagree strongly. For example, Kiesswetter (1983) states that, in his own experience, flexible thinking which is one component of creativity is one of the most important abilities – perhaps the most important – which a successful problem-solver ought to have. According to Bishop (1981), one needs two very different complementary modes of thinking in mathematics: Creative thinking, for which "intuition" is typical, and analytic thinking, for which "logic" is typical. Verbality, which is always one-dimensional, is connected to logic, and visuality which is usually two- or three-dimensional, to intuition. The same idea is put forward by Wachsmuth (1981), who speaks about a "logic mode" and a "relax mode" in thinking.

If we observe the performance of a mathematician (or a scientist in any other discipline) when he encounters a new task, we can surely note that he is experimenting at first. These first experimentations are random, but they gradually settle in one direction as an idea of the possible solution is awakening in the mind. Based on the experimentations, the mathematician may set a hypothesis which he tries to prove. Thus, we see that creative performance is an essential part of doing mathematics.

1.1 Description of creativity

From the very beginning of research on creativity, it has been typical to describe creativity through such persons' behavior who are generally considered to have been creative. In books on creativity (e.g. Branthwaite 1986), one may read about the Heureka experience of Arkhimedes and about Darwin's tedious years of collecting and arranging data before he got his idea of evolution.

In the relevant literature, there are many definitions of creativity, but according to Haylock (1987) there seems to be no commonly accepted definition. Every scientist has put forward his own version. In the following, we will use the definition of the Finnish neurophysiologist Matti Bergström. He defines creativity as "performance where the individual is producing something new and unpredictable" (Bergström 1984, 159). Furthermore, he introduces the concepts of "everyday creativity" and "Sunday creativity": The first concept covers the finding of new associations which can be predicted if we know the elements to be associated. Real creativity, on the other hand ("Sunday creativity"), requires special circumstances and can neither be achieved through intention nor by mechanical methods. For further discussion on the concept of "creativity" see, e.g., the paper of Haylock (in this ZDM-issue).

2. Problem solving as a fostering method

Problem solving has a long tradition in school mathematics. Usually, it has been taught (and is still taught in some schools even today) by the method of "learning from the master": The teacher shows a method, with some examples, which pupils then apply to similar problems. Every now and then, such a teaching style is criticized as formal and schematic, but so far, attempts to shake off formal teaching methods have never been successful.

Our starting point in this contribution will be the definition of a problem which is commonly used in the mathematical literature (e.g. Kantowski 1980): We will use the concept "*a problem*" for a task situation where the individual is compelled to connect the known information in a way that is new (for him) in order to do the task. If he immediately recognizes the actions needed to do the task, then it will be a routine task for him. Thus, the concept "problem" is bound to time and person.

2.1 Reasons for teaching problem solving

In many countries all over the world, problem solving is an objective contained explicitly in the mathematics curriculum. But if one asks why problem solving has such a central position, satisfactory answers are not easily found. Some years ago, I investigated this problematic situation and published my findings in Pehkonen 1987.

In the mathematical literature, there are few acceptable reasons for teaching problem solving. Most of the reasons given are opinions voiced by individuals. If we want to gather the reasons given in the mathematical literature to support problem solving, we might put them into four categories:

(1) Problem solving develops general cognitive skills.
(2) Problem solving fosters creativity.
(3) Problem solving is a part of the mathematical application process.
(4) Problem solving motivates pupils to learn mathematics.

General opinions about the importance of problem solving are left out of consideration in this classification. For example, in the United States, a large study of the opinions of different interest groups on how to emphasize mathematics teaching in school was carried out in 1979 (NCTM 1981). The respondents were teachers from primary to upper secondary school, school directors, representatives of parents, school councils and teacher educators (altogether $N \approx 10'000$). The purpose of the study was to determine which parts of the mathematics curriculum were ranked high by the different groups in the school mathematics of the Eighties. In all groups, problem solving received the highest rank.

In Finland, the author had the possibility to conduct a survey among mathematics teachers ($N = 44$) about the teaching of problem solving. The answers to the question "Why do you think that problem solving is important? Give at least three reasons" could be classified mainly into the first and fourth categories (almost 40% each). For the second category, there were about 10% of the responses. The description of the survey is to be found in Pehkonen 1993.

2.2 Different approaches to problem solving

A couple of years ago, ZDM journal contained a discussion of the subject of "Using open-ended problems in mathematics" also edited by the author (cf. Pehkonen 1995a), which was based on the presentations of the PME discussion group in Japan 1993. The theme "open-ended problems" has very near connections with creativity, and will therefore be explained briefly in the following.

The method of using open-ended problems in the classroom for promoting mathematical discussion, the so-called "open- approach" method, was developed in Japan in the 1970's (Shimada 1977). About the same time, the use of investigations, a kind of open-ended problems, became popular in mathematics teaching in England (Wiliam 1994), and the idea was spread more rapidly by the Cockcroft report (1982). In the Eighties, the idea to use some form of open-ended problems in the mathematics classroom spread all over the world, and research on its possibilities became very vivid in many countries (e.g. Nohda 1988, Pehkonen 1989, Silver & Mamona 1989, Williams 1989, Mason 1991, Nohda 1991, Stacey 1991, Zimmermann 1991, Clarke & Sullivan 1992, Leung 1993, Silver 1993, Pehkonen 1995b, Silver & Cai 1996). In some countries, a different name is used for open-ended problems; for example, in the Netherlands, they call their method "realistic mathematics" (Treffers 1991).

The idea of using open-ended problems in school mathematics in any form has been introduced into the curriculum in several countries. For example, in the mathematics curriculum for the comprehensive school in Hamburg (Germany), about one fifth of the teaching time is left content-free, in order to encourage the use of mathematical activities (Anon. 1990). In California, there are suggestions that open-ended problems should be used in assessment besides the ordinary multiple-choice tests (Anon. 1991). In Australia, some open problems (e.g. investigative projects) are used in the final assessment since the late Eighties (Stacey 1995).

One aim of the PME discussion group was to find answers to the question "What are 'open-ended problems'?" since the group of open-ended problems does not seem to be well defined. In the course of the discussion, several types of problems were put forward: *Investigations, problem posing, real-life situations, projects, problem fields (or problem sequences), problems without question*, and *problem variations ("what-if"-method)*. Examples of these groups of problems can be found in the papers published on this subject (Nohda 1995, Silver 1995, Stacey 1995).

3. Results of neurological research

Fostering creativity is usually mentioned as one objective of mathematics teaching in the curriculum. The importance of creativity in the problem solving process becomes clear if we consider the theory of functional asymmetry in the human brain. According to this, the left hemisphere is usually connected with logical thinking, whereas the right hemisphere acts mainly with the help of visual thinking.

Some years ago, I wrote some papers on the meaning of the theory of hemispheric asymmetry for mathematics teaching and learning. The description here is based on these publications (see Pehkonen 1987, 1991b).

Here our focus will be one point of emphasis in the recent neurological research, i.e. the theory of human cerebral asymmetry. Recent work on hemispheric specialization suggests that verbal processing is largely the province of the left hemisphere, whereas the right hemisphere predominates in subserving nonverbal (spatial) functions. These conclusions are supported by several different types of evidence.

3.1 The theory of functional asymmetry in the brain

In neurology, studies are made, e.g., on the meaning of different parts of the brain for human performance. Several investigations of different types show that the two cerebral hemispheres process stimuli in different ways. In more than 90% of the normal adult population, the left cerebral hemisphere processes stimuli sequentially – one after another –, whereas the right hemisphere is specialized in parallel processing. One can deduce that the left hemisphere is better suited, e.g., for reading, speaking, analytic deduction and arithmetic, whereas the right hemisphere is better, e.g., in spatial tasks, recognition of faces and music. However, the dichotomy verbal/nonverbal is inadequate for completely describing hemispheric specialization. It is better to say that there is a continuum of functions between the hemispheres, the differences being quantitative rather than qualitative (Springer & Deutsch 1985; Wheatley & al. 1978).

3.2 Implications for teaching mathematics

Many weak points observed in pupils' problem solving skills and higher level thinking might be implications of excessive left hemisphere activity. The constant emphasis on rules and algorithms which are usually sequential may prevent the development of creativity, problem solving skills and spatial ability. Rich and varied learning programs which offer pupils possibilities for investigations, nonverbal expression, laboratory work and multisense learning, can give pupils possibilities to reach new levels in mathematics (Branthwaite 1986).

Creative thinking might be defined as a combination of logical thinking and divergent thinking which is based on intuition but has a conscious aim. When one is applying creative thinking in a practical problem solving situation, divergent thinking produces many ideas. Some of these seem to be useful for finding solutions. Of these, a summary will be made by a process of logical thinking. In a creative process, both hemispheres will be needed alternatingly.

The balance between logic and creativity is very important. If one places too much emphasis on logical deduction, creativity will be reduced. What one wins in logic will be lost in creativity and vice versa. In order to develop, creativity demands freedom from superfluous selection pressure and control.

The meaning of knowledge for the problem solving process is well known and generally approved. But too little or too much knowledge may decrease the information processing ability and effectiveness of the human brain, and therefore both might form an obstacle to creativity. An invidual who has had a one-sided education with too much emphasis on knowledge might be unable to use his creativity, as the respective parts of his brain have not been trained enough while the preventive part has been overstimulated. Therefore, a school education which emphasizes knowledge and logic will neglect creativity education (Bergström 1985).

In successful problem solving both hemispheres will be needed: First, the right hemisphere has a leading role as this is where holistic data processing takes place. The left hemisphere is better in logical tasks, therefore it dominates the work in the second stage of problem solving. When the solution has been reached, the solver will again consider the situation in a holistic manner (the right hemisphere) in order to check the reasonableness of the constructed solution.

Our modern society especially stimulates and rewards actions of the left hemisphere. In school, the emphasis is placed on pupils' verbal skills (both oral and written) and on their ability to follow different rules. The activation of the right hemisphere seems to be a necessary prerequisite for successful problem solving. On the other hand, solving problems fosters pupils' creativity and thus activates their right hemisphere. The level of the problems used should correspond to the pupils' skill, since they should experience success in order to be motivated to continue with problem solving. Actions which stimulate the right hemisphere are, e.g., tasks which demand inventing (Wheatley & al. 1978).

3.3 Final remarks

When we move on from solving routine problems to creative problem solving, there is a danger that the teaching of problem solving methods (heuristic strategies) will give a new set of rules, and then we are back to teaching routines. There are some signs of this kind of proposition already in the literature of the Eighties (cf. Pehkonen 1991a). We ought to take care in order to avoid making problem solving strategies a new teaching subject. Pupils ought to be allowed to find and to form their own problem solving methods. Only the knowledge worked out by the pupils themselves is valuable.

With the development of computers, people will, in future, have still more time for creative performance, e.g. problem solving. Some people speak enthusiastically about the possibilities of computers in problem solving (artificial intelligence), too. A computer can, at its best, develop the thinking of its user, for example, with games of the 'Master Mind'-type, but a computer can never be able to achieve creative performance of "Sunday creativity" (Bergström 1984). A computer always works within its programming, its thinking is similar to "left-hemisphere thinking", not to "right-hemisphere thinking".

4. The structure of the theme

The theme "Fostering of Mathematical Creativity" originates from the Topic Group (TG7) of the International Congress on Mathematical Education (ICME-8) in Sevilla, July 1996. As I organized the program for the TG7, I realized that for some years, not much had been written about mathematical creativity. Therefore, I suggested the theme to the editor of the ZDM journal who kindly accepted my offer.

In the first paper “Recognising mathematical creativity in school children”, Derek Haylock (University of East Anglia, UK) describes general guidelines for the theory of mathematical creativity. The other papers deal with problems and methods designed for fostering mathematical creativity.

The main method offered for fostering creativity is, naturally, problem solving. But in problem solving, one may again find many possible approaches of its realization which I sketched in Chapter 2. Ed Silver and Susan Leung are both emphasizing the use of problem posing besides problem solving. Their papers are, as follows: Edward A. Silver (University of Pittsburgh, USA) “Fostering mathematical creativity through instruction rich in mathematical problem solving and problem posing”, and Susan S. Leung (National Chiayai Teachers’ College, Taiwan) “On the role of creative thinking in problem posing”. Yoshihiko Hashimoto (University of Yokohama, Japan), in his paper “The methods of fostering creativity through mathematical problem solving”, describes the Japanese problem solving and posing method, the so-called “open-end approach”. At the end, there are two examples of discovering and of fostering pupils’ creativity, the first is given by Hartmut Köhler (Institute for Education, Stuttgart, Germany), “Acting artist-like in the classroom”, and the second by Teh Pick Ching (University of Brunei, Brunei) “An experiment to discover mathematical talent in a primary school in Kampong Air”.

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5. References

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Author

Pehkonen, Erkki, Dr., Dept. of Teacher Education, University of Helsinki, PB 38 (Ratakatu 6A), 00014 Helsinki, Finland. EPehkonen@bulsa.helsinki.fi)