

Book reviews

Berry, J.; Houston, K.:

Mathematical Modelling

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(Modular Mathematics Series)

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Adrian Oldknow, Chichester (UK)

1. Introduction

The Modular Mathematics Series is designed to support undergraduate courses of around one semester. The books are intended to be accessible, motivating and, above all, affordable. They contain a preface by the series editors, Chris Collinson, Johnston Anderson and Peter Holmes, describing how they are intended to include historical contexts, real life situations and linkages with other areas of mathematics, as well as encouraging the use of software packages. The books are designed to provide both traditional worked examples and exercises which are augmented by open-ended exercises and tutorial problems suitable for group work or self-study. Thus the series aims to provide a valuable flexible learning resource for mathematics in higher education - which is very welcome. This book is one of the "first level" texts, and, as such, assumes only the core content of a current A-level syllabus.

The authors of this book have a wealth of experience, both in the field of mathematical modelling, and in producing "user-friendly" student material. In particular Ken Houston has been involved with the development of a pioneering A-level Further Mathematics course in Northern Ireland in which students are required to read mathematical articles and to demonstrate their understanding through comprehension questions. This technique is well illustrated in the book, and, as such, provides a valuable source of ideas for lecturers and tutors seeking to broaden the assessment styles of their courses.

The material is written in a very direct, lively and unfussy way which should make it accessible to students from a wide variety of backgrounds. The mathematical content is also pitched at a level which should make it undaunting to a wide range of students.

2. Content

The book is very clearly structured and organised. There are four distinctive chapters. The first, entitled "What is Mathematical Modelling?", is an introductory chapter which aims to give the student a feel for the process of

mathematical modelling through a variety of simple examples. The second, "Modelling Population Growth", goes into greater mathematical detail through the study of the classical Malthus and Verhulst models (though they are not named as such) with both discrete and continuous mathematics. The third chapter, "Mathematical Modelling in Action", contains extracts from six articles taken from books on mathematical modelling, together with a number of comprehension and extension questions for the student. The final chapter, "Developing Modelling Skills", contains a number of more general problems, as well as introducing two particular modelling approaches: simulation and dimensional analysis. There is also an appendix containing solutions to a fair number of the problems set in the text.

The authors suggest that the first two chapters should be studied in their entirety, and that then students might consider in detail, say, two of the six articles in chapter three and one of the three sections in chapter four. In this way the material is suitable for students from a variety of different subject foci (science, social science, humanities, mathematics etc.) taking a common mathematical modelling course within a modular degree structure. There are a few general points of weakness which lecturers adopting this book might seek to remedy.

The most serious weakness is the lack of examples of how to use technology to assist in modelling. Several of the questions suggest the students use a calculator (presumably graphic?), or spreadsheet or computer algebra system, such as *Derive*. However there are no suggestions of, for example, how to set up a recurrence model on a calculator or spreadsheet, nor illustrations of the output.

The second weakness is the lack of emphasis on some of the key issues implicit in the material. For instance there is an implicit message in the first two worked examples that extrapolating from a model fitted to data might lead to unrealistic forecasts. But that message appears buried in text several pages after the examples and it would be easy for a student either to skip-read the message, or at least not to read it until some time after believing that the models were valid. Similarly there is an implicit message about the importance of formulating qualitative models before getting involved in the quantitative approaches - but this, too, could easily be overlooked. Again, those aspects of modelling given explicit emphasis in chapter one are "formulating", "solving" and "interpreting" - and "validating" does not appear.

The third weakness is the quality of the production. The mathematical notation is inconsistent e.g. in the use of italics in setting equations, and the text has been poorly

proof-read with many annoying (mainly minor) errors. These will probably irritate lecturers more than students. But students might not take too kindly to having their attention drawn, in one of the model solutions to a question on an article in chapter three, to a misprint in the original of that article when there are so many in the body of this text. There is no bibliography, nor suggestions for further reading or resources. These points all suggest that the material was assembled in rather a hurry, and it is to be hoped that they will be remedied in a second edition.

2.1 Chapter 1

Having defined a mathematical model as “a mathematical representation of the relationship between two or more variables relevant to a given situation or problem” the authors discuss linear models, and standard ways of transforming data to get a linear relationship. The first example, “Modelling the greenhouse effect”, uses data on the average rise in the earth’s temperature T between years $n = 1860$ and $n = 1980$. The authors suggest that plotting $\log T$ against n will ‘straighten the curve’, and they clearly use predictions based on knowledge of the parameters for a suitable linear model to forecast 2078 as the year when the temperature will have risen by 7 degrees. However there is no information for the student on a suitable way to find such a quantitative model. There is no mention about regression, least squares or other criteria for curve-fitting etc. [I would expect my students to tackle this using one of the built-in regression models on a graphic calculator.]

The second example “World Record for the Mile” uses data on the world record times for the one mile athletic race from 1913 to 1993 which have a high linear correlation. Again the students are expected to fit a linear model to extrapolate 2028 as the year when the record will get below 3 minutes and 30 seconds. Both these examples are described as “data driven” and are given as examples of “empirical modelling”. The first tutorial problem asks the students to “List the mathematical structures that could form mathematical models.” - which is very vague. I guess they wanted types of functions such as quadratic (projectile), sinusoidal (lighting-up times) etc. The other tutorial problems in the section on empirical modelling are much better formulated and include Kepler’s third law, safe stopping distances for vehicles, rate at which height of liquid descends in a leaking bottle, Bode’s law, the counter on a cassette recorder, the period of a pendulum and Hooke’s law.

The remainder of the chapter is concerned with what the authors call “Modelling from Theory”. There are worked examples illustrating various techniques. The first uses probability and inequalities to specify the conditions under which a pedestrian crossing would be needed for an idealised street. The second example uses differential calculus to determine the dimensions of a cake of given volume to have a minimum surface area for icing - either square or circular. The third returns to the cassette tape counter problem and uses some results about arc length to derive a differential equation which is solved by integration. This is an example where poor proof-reading could be quite confusing since some data are given for elapsed time t in minutes, the 3 digit tape

counter reading c and the ratio t/c . However this last column of data is clearly computed using t measured in seconds. The tutorial problems include modelling tying a knot in a rope, estimating the length of a toilet roll, whether to buy or rent a TV, the economics of buying new and second-hand cars and how to organise the prizes in a raffle.

As well as the detailed examples and problems there are clearly written sections about the modelling process in general.

2.2 Chapter 2

Starting with census data for the population of England and Wales between 1801 and 1951 the authors introduce the notion of an average proportionate growth rate (for which they do not specify units). The first model, which is quickly questioned, assumes that this rate is constant. However there is no mention of the link between this assumption and the definition of the exponential function. Nor is there any discussion about factors which might change the growth rate - such as wars, changes in birth-control methods etc. There is a nice sequence of graphs illustrating 14 different changes in populations of different species, such as moose, wolves, lynx and lemming. Through an example of the introduction of a rare species of bird to a small populated island the authors provide a clear illustration of a methodology for formulating a word model, and hence a mathematical model in conventional symbolism. This model, expressed as $P_{n+1} - P_n = (b-d)P_n$, is given as an example of a “recurrence relation” for which “numerical solutions can be found using a calculator or spreadsheet.” - but unfortunately the student is not shown how. The student is set as an exercise to show that this has solution: $P_n = (1 + b-d)^n P_0$ and is told that “the general power solution of the simple model with constant birth and death rates is called *the exponential model*.” without any attempt to discuss the exponential function. The next exercise asks students to sketch graphs of the general solution in the cases $b < d$, $b = d$, $b > d$, again without reference to the exponential function, or of growth and decay.

The model is then modified to make the net growth factor $b-d$ a function of the population size. This leads to: “the recurrence relation model for population growth is $P_{n+1} - P_n = k P_n (P_{\max} - P_n)$. This non-linear equation is called *the logistic model*.” This is the first time the term “non-linear” appears - and no mention is made of it as a quadratic model, nor about the possibility of chaos, nor about the impossibility of finding an explicit solution for P_n in the discrete case. The student is left to validate the model against 3 different sets of population data. The explicit form of the solution to the continuous model is not quoted, but set as an exercise “for those readers who have met calculus”.

The chapter concludes with an excellent detailed summary of the stages of modelling process, and a set of exercises. There is no mention given of other mathematical models of population modelling, such as Leslie matrices, nor of other references or sources.

2.3 Chapter 3

This contains reprints of six articles from books or

journals, described as “Case Studies”, which demonstrate a good range of modelling approaches and mathematical techniques.

The first, “The Percy Grainger feat”, is described as “inspired by Chapter 8 of” Hart & Croft, 1988. The Australian born composer, Percy Grainger, claimed to be able to throw a cricket ball over the roof of his house and to run through the house to catch the ball on its way down! The piece uses quite simple mathematics of idealised projectiles and introduces the notion of the “Bounding Parabola”. As well as the text of the article, the authors provide a helpful commentary, which again is marred by possibly confusing errors such as the following, in which r is given as the earth’s radius i.e. $r = 6.4 \times 10^6$ m. “Also, the ratio of the magnitudes of the force of gravity on the ball at the top of its flight to the magnitude at the bottom is approximately $[r/(r+30)]^2$ which again is very small.” Clearly the authors mean us to deduce that the change in the force of gravity is negligible - but the ratio is, of course, very close to 1. The section concludes with some questions for the student to develop.

The second, “Safe Driving Speeds on Newly Surfaced Roads”, is from the same source and applies the notion of the bounding parabola to finding a safe distance between cars travelling on a recently resurfaced road so that loose stones do not hit the paintwork or windscreen. The material is again accessible, realistic and interesting. There is no commentary, but there are a set of questions taken from a further mathematics A-level paper to test the student’s comprehension. A similar set of questions is used in connection with the third extract, “Drug Therapy”. This is from Burghes, Huntley & MacDonald, 1982, and uses an exponential decay model of drug concentration in the treatment of asthma. It is also an accessible and interesting application. Again a typing error, this time a surplus right parenthesis in a crucial equation on line 74, could throw a student left to read this piece alone. The fourth extract “Rats” is again taken from the same source and provides an interesting investigation into the way that birth and death rates vary with population. The same source is also used for the fifth extract, “Handicapping Weightlifters”, which contains some very interesting alternative models which have been proposed to enable the weights lifted by weightlifters, classed in different body-weights, to be compared.

The final case study, “Insulating a house”, is taken from a chapter in Huntley & James 1990 written by John Berry. To this reader it seems rather more long-winded, less mathematically challenging and generally rather less interesting than the other five.

2.4 Chapter 4

This contains three sections. The first consists of 10 case studies of more generalised modelling problems, including examples such as the spread of an epidemic and parachute jumping with air-resistance. The second, entitled “Simulation Modelling”, is based on a case study “A queuing problem at an airport” taken from Open University 1990. As may be expected this provides a very clear introduction to the use of distributions and

sampling with the use of random numbers to model stochastic processes. There are further tutorial problems and suggestions for a further 5 situations to which the techniques might be applied. The final section, entitled “Dimensional Analysis Modelling”, gives details of how to use a dimensional analysis. This is an important technique which is often overlooked, or under-emphasised. However it is more usually introduced as one technique for helping to validate a model, rather than a modelling technique in its own right. Again there are some useful and interesting examples and problems.

3. Summary

This is a very useful, and affordable, introductory student text around which a first-year modelling course could easily be designed. The structure, content and style of the book make it a valuable and distinctive addition to the student modelling literature. The authors have a wealth of experience in mounting modelling courses for their own students, and those at the Open University - and this book provides an effective way of disseminating that practice. The book fully lives up to the general aims of the series, but could have easily included some more details on both the application of computer technology, and historical developments of some of the models. A lecturer planning to adopt this text will need (a) to plan how to introduce computer usage to support the book, (b) to make sure that a technique for finding the equation of a linear model is introduced at a very early stage, (c) to warn students to be on their guard for misprints and mistakes in the text and (d) to produce their own supporting bibliography. Even with these reservations I shall be recommending this book to my own students. I very much hope that the book will prove popular enough to go to a second edition in which my reservations can be addressed.

References

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Author

Oldknow, Adrian, Prof., The Mathematics Centre, Chichester Institute of Higher Education, Upper Bognor Road, Bognor Regis, Sussex PO21 1HR, Great Britain
 E-mail: 101341.622@compuserve.com