Vladikavkaz Mathematical Journal
2019, Volume 21, Issue 4, P. 5–10

УДК 517.98
DOI 10.23671/VNC.2019.21.44595

2-LOCAL ISOMETRIES OF NON-COMMUTATIVE LORENTZ SPACES

A. A. Alimov[1], **V. I. Chilin**[2]

[1] Tashkent Institute of Design, Construction
and Maintenance of Automobile Roads,
20 Amir Temur Av., Tashkent 100060, Uzbekistan,
[2] National University of Uzbekistan,
Vuzgorodok, Tashkent 100174, Uzbekistan
E-mail: alimovakrom63@yandex.ru,
vladimirchil@gmail.com, chilin@ucd.uz

Dedicated to E. I. Gordon on the occasion of his 70th birthday

Abstract. Let $\mathcal{M}$ be a von Neumann algebra equipped with a faithful normal finite trace τ, and let $S(\mathcal{M},\tau)$ be an $*$-algebra of all τ-measurable operators affiliated with $\mathcal{M}$. For $x \in S(\mathcal{M},\tau)$ the generalized singular value function $\mu(x) : t \to \mu(t;x)$, $t > 0$, is defined by the equality $\mu(t;x) = \inf\{\|xp\|_{\mathcal{M}} : p^2 = p^* = p \in \mathcal{M},\ \tau(\mathbf{1}-p) \leqslant t\}$. Let ψ be an increasing concave continuous function on $[0,\infty)$ with $\psi(0) = 0$, $\psi(\infty) = \infty$, and let $\Lambda_\psi(\mathcal{M},\tau) = \{x \in S(\mathcal{M},\tau) :\ \|x\|_\psi = \int_0^\infty \mu(t;x)d\psi(t) < \infty\}$ be the non-commutative Lorentz space. A surjective (not necessarily linear) mapping $V : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ is called a surjective 2-local isometry, if for any $x, y \in \Lambda_\psi(\mathcal{M},\tau)$ there exists a surjective linear isometry $V_{x,y} : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ such that $V(x) = V_{x,y}(x)$ and $V(y) = V_{x,y}(y)$. It is proved that in the case when $\mathcal{M}$ is a factor, every surjective 2-local isometry $V : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ is a linear isometry.

Key words: measurable operator, Lorentz space, isometry.

Mathematical Subject Classification (2010): 46L52, 46B04.

For citation: *Alimov, A. A. and Chilin, V. I.* 2-Local Isometries of Non-Commutative Lorentz Spaces, *Vladikavkaz Math. J.*, 2019, vol. 21, no. 4, pp. 5–10. DOI: 10.23671/VNC.2019.21.44595.

1. Introduction

Let $\mathcal{H}$ be a complex separable infinite-dimensional Hilbert space, let $(\mathcal{C}_E, \|\cdot\|_{\mathcal{C}_E})$ be a Banach ideal of compact linear operators in $\mathcal{H}$ generated by symmetric sequence space $(E, \|\cdot\|_E) \subset c_0$, and let V be a surjective 2-local isometry on $\mathcal{C}_E$, that is, $V : \mathcal{C}_E \to \mathcal{C}_E$ is a surjective (not necessarily linear) mapping such that for any $x, y \in \mathcal{C}_E$ there exists a surjective linear isometry $V_{x,y}$ on $\mathcal{C}_E$ for which $V(x) = V_{x,y}(x)$ and $V(y) = V_{x,y}(y)$. In the papers [1, 2] it is shown that in the case when $\mathcal{C}_E$ is separable or has the Fatou property, $\mathcal{C}_E \neq \mathcal{C}_{l_2}$, every surjective 2-local isometry on $\mathcal{C}_E$ is a linear isometry. In the proof of this statement is essentially used explicit description of all surjective linear isometries on $\mathcal{C}_E$ [1, 3].

Banach ideals $(\mathcal{C}_E, \|\cdot\|_{\mathcal{C}_E})$ of compact linear operators are examples of non-commutative symmetric spaces $\mathcal{E}(\mathcal{M},\tau)$ of measurable operators affiliated with a von Neumann algebra $\mathcal{M}$

equipped with a faithful normal semifinite trace τ (see, for example, [4, Ch. 2, § 2.5]). It is natural to expect that for these non-commutative symmetric spaces with the Fatou property, every surjective 2-local isometry $V : \mathcal{E}(\mathcal{M},\tau) \to \mathcal{E}(\mathcal{M},\tau)$ is a linear map. Unfortunately, the method of proof of a similar statement for Banach ideals $(\mathcal{C}_E, \|\cdot\|_{\mathcal{C}_E})$ can not be applied here, since there is no description of surjective linear isometries $V : \mathcal{E}(\mathcal{M},\tau) \to \mathcal{E}(\mathcal{M},\tau)$. At the same time, in the case of non-commutative Lorentz and Marcinkiewicz spaces, such a description of surjective linear isometries was obtained in the paper [5]. Using this description, we obtain the following description of surjective 2-local isometries of non-commutative Lorentz spaces.

Theorem 1. *Let $\mathcal{M}$ be an arbitrary factor with a faithful normal finite trace τ, and let $(\Lambda_\psi(\mathcal{M},\tau), \|\cdot\|_\psi)$ be a non-commutative Lorentz space. Then every surjective 2-local isometry $V : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\varphi(\mathcal{M},\tau)$ is a linear isometry.*

2. Preliminaries

Let $\mathcal{H}$ be an infinite-dimensional complex Hilbert space, let $\mathcal{B}(\mathcal{H})$ be the C^*-algebra of all bounded linear operators in $\mathcal{H}$, and let $\mathbf{1}$ be the unit in $\mathcal{B}(\mathcal{H})$. Let $\mathcal{M} \subseteq \mathcal{B}(\mathcal{H})$ be a von Neumann algebra on Hilbert space $\mathcal{H}$ equipped with a faithful normal semifinite trace τ (see, for example, [6]). A linear operator $x : \mathfrak{D}(x) \to \mathcal{H}$, where the domain $\mathfrak{D}(x)$ of x is a linear subspace of $\mathcal{H}$, is said to be *affiliated* with $\mathcal{M}$ if $yx \subseteq xy$ for all $y \in \mathcal{M}'$, where $\mathcal{M}'$ is the commutant of $\mathcal{M}$. A linear operator $x : \mathfrak{D}(x) \to \mathcal{H}$ is termed *measurable* with respect to $\mathcal{M}$ if x is closed, densely defined, affiliated with $\mathcal{M}$ and there exists a sequence $\{p_n\}_{n=1}^\infty$ in the lattice $\mathcal{P}(\mathcal{M})$ of all projections of $\mathcal{M}$, such that $p_n \uparrow \mathbf{1}$, $p_n(\mathcal{H}) \subseteq \mathfrak{D}(x)$ and $\mathbf{1} - p_n$ is a finite projection (with respect to $\mathcal{M}$) for all n. The collection $S(\mathcal{M})$ of all measurable operators with respect to $\mathcal{M}$ is a unital $*$-algebra with respect to strong sums and products.

Let x be a self-adjoint operator affiliated with $\mathcal{M}$ and let $\{e^x\}$ be a spectral measure of x. It is well known that if x is a closed operator affiliated with $\mathcal{M}$ with the polar decomposition $x = u|x|$, then $u \in \mathcal{M}$ and $e \in \mathcal{M}$ for all projections $e \in \{e^{|x|}\}$. Moreover, $x \in S(\mathcal{M})$ if and only if x is closed, densely defined, affiliated with $\mathcal{M}$ and $e^{|x|}(\lambda,\infty)$ is a finite projection for some $\lambda > 0$.

An operator $x \in S(\mathcal{M})$ is called τ-measurable if there exists a sequence $\{p_n\}_{n=1}^\infty$ in $\mathcal{P}(\mathcal{M})$ such that $p_n \uparrow \mathbf{1}$, $p_n(\mathcal{H}) \subseteq \mathfrak{D}(x)$ and $\tau(\mathbf{1} - p_n) < \infty$ for all n. The collection $S(\mathcal{M},\tau)$ of all τ-measurable operators is a unital $*$-subalgebra of $S(\mathcal{M})$. It is well known that a linear operator x belongs to $S(\mathcal{M},\tau)$ if and only if $x \in S(\mathcal{M})$ and there exists $\lambda = \lambda(x) > 0$ such that $\tau(e^{|x|}(\lambda,\infty)) < \infty$.

The generalized singular value function $\mu(x) : t \to \mu(t;x)$, $t > 0$, of the operator $x \in S(\mathcal{M},\tau)$ is defined by setting [7]

$$\mu(t;x) = \inf\Big\{\|xp\| : p \in \mathcal{P}(\mathcal{M}),\ \tau(\mathbf{1}-p) \leqslant t\Big\} = \inf\Big\{s > 0 : \tau(e^{|x|}(s,\infty)) \leqslant t\Big\}.$$

A non-zero linear subspace $\mathcal{E}(\mathcal{M},\tau) \subset S(\mathcal{M},\tau)$ with the Banach norm $\|\cdot\|_{\mathcal{E}(\mathcal{M},\tau)}$ is called *a symmetric space* if the conditions

$$x \in \mathcal{E}(\mathcal{M},\tau), \quad y \in S(\mathcal{M},\tau), \quad \mu_t(y) \leqslant \mu_t(x) \ \text{ for all } \ t > 0,$$

imply that $y \in \mathcal{E}(\mathcal{M},\tau)$ and $\|y\|_{\mathcal{E}(\mathcal{M},\tau)} \leqslant \|x\|_{\mathcal{E}(\mathcal{M},\tau)}$.

It is known that in the case $\tau(\mathbf{1}) < \infty$ it is true

$$S(\mathcal{M}) = S(\mathcal{M},\tau) \quad \text{and} \quad \mathcal{M} \subseteq \mathcal{E}(\mathcal{M},\tau) \subseteq L_1(\mathcal{M},\tau)$$

for each symmetric space $\mathcal{E}(\mathcal{M},\tau)$, where

$$L_1(\mathcal{M},\tau)=\left\{x\in S\,(\mathcal{M},\tau):\ \|x\|_1=\int\limits_0^\infty \mu_t(x)\,dt<\infty\right\}.$$

In addition,

$$\mathcal{M}\cdot\mathcal{E}(\mathcal{M},\tau)\cdot\mathcal{M}\subseteq\mathcal{E}(\mathcal{M},\tau),$$

and

$$\|axb\|_{\mathcal{E}(\mathcal{M},\tau)}\leqslant\|a\|_{\mathcal{M}}\cdot\|b\|_{\mathcal{M}}\cdot\|x\|_{\mathcal{E}(\mathcal{M},\tau)}$$

for all $a,b\in\mathcal{M}$, $x\in\mathcal{E}(\mathcal{M},\tau)$.

Let ψ be an increasing concave continuous function on $[0,\infty)$ with $\psi(0)=0$, $\psi(\infty)=\lim\limits_{t\to\infty}\psi(t)=\infty$, and let

$$\Lambda_\psi(\mathcal{M},\tau)=\left\{x\in S\,(\mathcal{M},\tau):\ \|x\|_\psi=\int\limits_0^\infty \mu(t;x)\,d\psi(t)<\infty\right\}$$

be the non-commutative *Lorentz space*. It is known that $(\Lambda_\psi(\mathcal{M},\tau),\|\cdot\|_\psi)$ is a symmetric space [8], and the norm $\|\cdot\|_\psi$ has the Fatou property, that is, the conditions $0\leqslant x_k\in\Lambda_\psi(\mathcal{M},\tau)$ for all k, and $\sup_{k\geqslant1}\|x_k\|_\psi<\infty$, imply that there exists $0\leqslant x\in\Lambda_\psi(\mathcal{M},\tau)$ such that $x_k\uparrow x$ and $\|x\|_\psi=\sup_{k\geqslant1}\|x_k\|_\psi$.

Denote by $M_\psi(\mathcal{M},\tau)$ the set of all $x\in S\,(\mathcal{M},\tau)$ for which

$$\|x\|_{M_\psi}=\sup_{t>0}\frac{1}{\psi(t)}\int\limits_0^t \mu(s;x)\,ds$$

is finite. The set $M_\psi(\mathcal{M},\tau)$ with the norm $\|\cdot\|_{M_\psi}$ is a symmetric space which is called a *Marcinkiewicz space*.

Denote by $M^0_\psi(\mathcal{M},\tau)$ the closure of $\mathcal{M}$ in $M_\psi(\mathcal{M},\tau)$. It is known [9] that the conjugate space of $(\Lambda_\psi(\mathcal{M},\tau),\|\cdot\|_\psi)$ is identified with $(M_\psi(\mathcal{M},\tau),\|\cdot\|_{M_\psi})$, and the conjugate space of $(M^0_\psi(\mathcal{M},\tau),\|\cdot\|_{M_\psi})$, under the condition $\lim\limits_{t\to0}\frac{t}{\psi(t)}=0$, is identified with$(\Lambda_\psi(\mathcal{M},\tau),\|\cdot\|_\psi)$. The duality in these pairs of spaces is realized via the bilinear form $(x,y)=\tau(xy)$. It should be pointed out that the spaces $(\Lambda_\psi(\mathcal{M},\tau),\|\cdot\|_\psi)$, $(M_\psi(\mathcal{M},\tau),\|\cdot\|_{M_\psi})$ and $(M^0_\psi(\mathcal{M},\tau),\|\cdot\|_{M_\psi})$ are symmetric spaces [4, Ch. 2, § 2.6], [8].

3. Isometries of Non-Commutative Lorentz Spaces

Let $\mathcal{M}\subseteq\mathcal{B}(\mathcal{H})$ be a von Neumann algebra on Hilbert space $\mathcal{H}$. A linear bijective mapping $\Phi\colon\mathcal{M}\to\mathcal{M}$ is called *a Jordan isomorphism* if $\Phi(x^2)=(\Phi(x))^2$ and $\Phi(x^*)=(\Phi(x))^*$ for all $x\in\mathcal{M}$.

If $\Phi\colon\mathcal{M}\to\mathcal{M}$ is a Jordan isomorphism, then there exists a central projection $z\in\mathcal{M}$ such that $\Phi_z(x)=\Phi(x)\cdot z$, $x\in\mathcal{M}$, is an $*$-homomorphism, and $\Phi_{z^\perp}(x)=\Phi(x)\cdot(\mathbf{1}-z)$, $x\in M$, is an $*$-antihomomorphism (see, for example, [10, Ch. 3, § 3.2.1]). Consequently, if $\mathcal{M}$ is a factor then a Jordan isomorphism $\Phi:\ \mathcal{M}\to\mathcal{M}$ is an $*$-homomorphism or $*$-antihomomorphism.

If τ is a faithful normal finite trace on von Neumann algebra $\mathcal{M}$ then a Jordan isomorphism $\Phi\colon\mathcal{M}\to\mathcal{M}$ is continuous with respect to measure topology t_τ generated by trace τ (see, for example, [11, Ch. 5, § 3, Proposition 1]). Therefore, Φ extends to a t_τ-continuous Jordan isomorphism $\widetilde{\Phi}\colon S(\mathcal{M},\tau) \to S(\mathcal{M},\tau)$. In addition, if $\tau(\Phi(x)) = \tau(x)$ for all $x \in \mathcal{M}$ then $\mu(t;\widetilde{\Phi}(x)) = \mu(t;x)$ for all $x \in S(\mathcal{M},\tau)$, in particular, $\widetilde{\Phi}(\mathcal{E}(\mathcal{M},\tau) = \mathcal{E}(\mathcal{M},\tau))$ and $\|\widetilde{\Phi}(x)\|_{\mathcal{E}(\mathcal{M},\tau)} = \|x\|_{\mathcal{E}(\mathcal{M},\tau)}$ for all $x \in \mathcal{E}(\mathcal{M},\tau)$, that is, $\widetilde{\Phi}\colon \mathcal{E}(\mathcal{M},\tau) \to \mathcal{E}(\mathcal{M},\tau)$ is a surjective linear isometry for any symmetric space $(\mathcal{E}(\mathcal{M},\tau), \|\cdot\|_{\mathcal{E}(\mathcal{M},\tau)})$.

Thus, it is true the following

Proposition 1. *Let $\mathcal{M}$ be an arbitrary von Neumann algebra with a faithful normal finite trace τ, and let $\Phi\colon \mathcal{M} \to \mathcal{M}$ be a Jordan isomorphism such that $\tau(\Phi(x)) = \tau(x)$ for all $x \in \mathcal{M}$. Then for every symmetric space $(\mathcal{E}(\mathcal{M},\tau), \|\cdot\|_{\mathcal{E}(\mathcal{M},\tau)})$ the mapping $V\colon \mathcal{E}(\mathcal{M},\tau) \to \mathcal{E}(\mathcal{M},\tau)$ given by the equality $V(x) = u \cdot \widetilde{\Phi}(x) \cdot v$, $x \in \mathcal{E}(\mathcal{M},\tau)$, u, v are unitary operators in $\mathcal{M}$, is a surjective linear isometry.*

We need the following description of surjective linear isometries of the spaces $(\Lambda_\psi(\mathcal{M},\tau), \|\cdot\|_\psi)$ and $(M^0_\psi(\mathcal{M},\tau), \|\cdot\|_{M_\psi})$ [5, Theorems 5.1, 6.1].

Theorem 2. *Let $\mathcal{M}$ be an arbitrary von Neumann algebra with a faithful normal finite trace τ, and let $V : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ (respectively, $V : M^0_\psi(\mathcal{M},\tau) \to M^0_\psi(\mathcal{M},\tau)$) be a surjective linear isometry. Then there exist uniquely an unitary operator $u \in \mathcal{M}$ and a Jordan isomorphism $\Phi : \mathcal{M} \to \mathcal{M}$ such that $V(x) = u \cdot \Phi(x)$ and $\tau(\Phi(x)) = \tau(x)$ for all $x \in \mathcal{M}$.*

4. Local Isometries of Non-Commutative Lorentz Spaces

Let $(X, \|\cdot\|_X)$ be an arbitrary Banach space over the field $\mathbb{K}$ of complex or real numbers. A surjective (not necessarily linear) mapping $T\colon X \to X$ is called a surjective 2-local isometry [2], if for any $x, y \in X$ there exists a surjective linear isometry $V_{x,y}\colon X \to X$ such that $T(x) = V_{x,y}(x)$ and $T(y) = V_{x,y}(y)$. It is clear that every surjective linear isometry on X is a surjective 2-local isometry on X. In addition,

$$T(\lambda x) = V_{x,\lambda x}(\lambda x) = \lambda V_{x,\lambda x}(x) = \lambda T(x)$$

for any $x \in X$ and $\lambda \in \mathbb{K}$.

Consequently, in order to establish linearity of a 2-local isometry T, it is sufficient to show that $T(x + y) = T(x) + T(y)$ for all $x, y \in X$.

Since

$$\|T(x) - T(y)\|_X = \|V_{x,y}(x) - V_{x,y}(y)\|_X = \|x - y\|_X \quad \text{for all } \ x, y \in X,$$

it follows that T is continuous map on $(X, \|\cdot\|_X)$. In addition, in the case a real Banach space X ($\mathbb{K} = \mathbb{R}$), every surjective 2-local isometry on X is a linear map (see Mazur–Ulam Theorem [12, Ch. 1, § 1.3, Theorem 1.3.5.]). In the case a complex Banach space X ($\mathbb{K} = \mathbb{C}$), this fact is not true.

Using the description of all surjective linear isometries on a separable Banach symmetric ideal $\mathcal{C}_E$ [3] (respectively, on a Banach symmetric ideal $\mathcal{C}_E$ with Fatou property [1]), $\mathcal{C}_E \neq \mathcal{C}_{l_2}$, in the papers [1, 2] it is proved that every surjective 2-local isometry $T : \mathcal{C}_E \to \mathcal{C}_E$ is a linear isometry.

The following Theorem is a version of the above results for the spaces $\Lambda_\psi(\mathcal{M},\tau)$ and $M^0_\psi(\mathcal{M},\tau)$.

Theorem 3. *Let $\mathcal{M}$ be an arbitrary factor with a faithful normal finite trace τ, and let $T : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ (respectively, $T : M^0_\psi(\mathcal{M},\tau) \to M^0_\psi(\mathcal{M},\tau)$) be a surjective 2-local isometry. Then T is a linear isometry.*

$\lhd$ Fix $x, y \in \mathcal{M}$ and let $V_{x,y} : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ be a surjective isometry such that $T(x) = V_{x,y}(x)$ and $T(y) = V_{x,y}(y)$. By Theorem 2, there exist uniquely an unitary operator $u \in \mathcal{M}$ and a Jordan isomorphism $\Phi : \mathcal{M} \to \mathcal{M}$ such that $V_{x,y}(a) = u \cdot \Phi(a)$ and $\tau(\Phi(a)) = \tau(a)$ for all $a \in \mathcal{M}$. Since $\mathcal{M}$ is a factor it follows then $\Phi : \mathcal{M} \to \mathcal{M}$ is an $*$-isomorphism or Φ is an $*$-anti-isomorphism.

We assume that Φ is an $*$-isomorphism (in the case when Φ is an $*$-anti-isomorphism, the proof is similar).

We have

$$\tau(T(x)\cdot(T(y))^*) = \tau(V_{x,y}(x)\cdot(V_{x,y}(y))^*)$$
$$= \tau(u\cdot\Phi(x)\cdot(u\cdot\Phi(y))^*) = \tau(u\cdot\Phi(xy^*)\cdot u^*) = \tau(\Phi(xy^*)) = \tau(xy^*).$$

Consequently, $\tau(T(x)\cdot(T(y))^*) = \tau(xy^*)$ for all $x, y \in \mathcal{M}$.

If $x, y, z \in \mathcal{M}$, then

$$\tau(T(x+y)\cdot(T(z))^*) = \tau((x+y)z^*), \quad \tau(T(x)\cdot T(z)^*) = \tau(xz^*),$$
$$\tau(T(y)\cdot T(z)^*) = \tau(y\cdot z^*).$$

Therefore

$$\tau((T(x+y) - T(x) - T(y))\cdot(T(z))^*) = 0$$

for all $z \in \mathcal{M}$. Taking $z = x+y$, $z = x$ and $z = y$, we obtain

$$\tau((T(x+y) - T(x) - T(y)\cdot((T(x+y) - T(x) - T(y))^*) = 0,$$

that is, $T(x+y) = T(x) + T(y)$ for all $x, y \in \mathcal{M}$.

Since the Lorentz space $\Lambda_\psi(0,\infty)$ of measurable functions on a semi-axis $[0,\infty)$ is separable space [13, Ch. 2I, § 5], it follows that the non-commutative Lorentz $(\Lambda_\psi(\mathcal{M},\tau), \|\cdot\|_\psi)$ has an order continuous norm [14, Proposition 3.6], that is, $\|x_n\|_\psi \downarrow 0$ whenever $x_n \in \Lambda_\psi(\mathcal{M},\tau)$ and $x_n \downarrow 0$. Consequently, the factor $\mathcal{M}$ is dense in the space $\Lambda_\psi(\mathcal{M},\tau)$. Since T is a continuous mapping on $\Lambda_\psi(\mathcal{M},\tau)$ it follows that $T(x+y) = T(x)+T(y)$ for all $x, y \in \Lambda_\psi(\mathcal{M},\tau)$, that is, T is a surjective linear isometry.

For the space $M^0_\psi(\mathcal{M},\tau)$, the proof of the linearity of the surjective 2-local isometry $T : M^0_\psi(\mathcal{M},\tau) \to M^0_\psi(\mathcal{M},\tau)$ repeats the previous proof. $\rhd$

References

1. *Aminov, B. R. and Chilin, V. I.* Isometries of Perfect Norm Ideals of Compact Operators, *Studia Mathematica*, 2018, vol. 241, no. 1, pp. 87–99. DOI: 10.4064/sm170306-19-4.
2. *Molnar, L.* 2-Local Isometries of some Operator Algebras, *Proceedings of the Edinburgh Mathematical Society*, 2002, vol. 45, pp. 349–352. DOI: 10.1017/S0013091500000043.
3. *Sourour, A.* Isometries of Norm Ideals of Compact Operators, *Journal of Functional Analysis*, 1981, vol. 43, no. 1, pp. 69–77. DOI: 10.1016/0022-1236(81)90038-0.
4. *Lord, S., Sukochev, F. and Zanin, D. Singular Traces. Theory and Applications*, Berlin/Boston, Walter de Gruyter GmbH, 2013.
5. *Chilin, V. I., Medzhitov, A. M. and Sukochev, F. A.* Isometries of Non-Commutative Lorentz Spaces, *Mathematische Zeitschrift*, 1989, vol. 200, no. 4, pp. 527–545. DOI: 10.1007/BF01160954.
6. *Takesaki, M. Theory of Operator Algebras I*, New York, Springer-Verlag, 1979.
7. *Fack, T. and Kosaki, H.* Generalized s-Numbers of τ-Measurable Operators, *Pacific Journal of Mathematics*, 1986, vol. 123, no. 2, pp. 269–300. DOI: 10.2140/pjm.1986.123.269.
8. *Chilin, V. I. and Sukochev, F. A.* Weak Convergence in Non-Commutative Symmetric Spaces, *Journal of Operator Theory*, 1994, vol. 31, no. 1, pp. 35–55.
9. *Ciach, L. J.* On the Conjugates of Some Operator Spaces, I, *Demonstratio Mathematica*, 1985, vol. 18, no. 2, pp. 537–554. DOI: 10.1515/dema-1985-0213.

10. *Bratteli, O. and Robinson, D. W. Operator Algebras and Quantum Statistical Mechaniks*, N.Y.–Heidelber–Berlin, Springer-Verlag, 1979.
11. *Sarymsakov, T. A., Ayupov, Sh. A., Khadzhiev D. and Chilin V. I. Uporyadochennye Algebry* [Ordered Algebras], Tashkent, FAN, 1983 [in Russian].
12. *Fleming, R. J., Jamison, J. E. Isometries on Banach Spaces: Function Spaces*, Florida, Boca Raton, Chapman-Hall/CRC, 2003.
13. *Krein, M. G., Petunin, Ju. I. and Semenov, E. M. Interpolation of Linear Operators*, Translations of Mathematical Monographs, vol. 54, American Mathematical Society, 1982.
14. *Dodds, P., Dodds. Th. K.-Y and Pagter, B.* Noncommutative Kothe Duality, *Transactions of the American Mathematical Society*, 1993, vol. 339, no. 2, pp. 717–750. DOI: 10.1090/S0002-9947-1993-1113694-3.

Received 20 June, 2019

Akrom A. Alimov
Tashkent Institute of Design, Construction
and Maintenance of Automobile Roads,
20 Amir Temur Av., Tashkent 100060, Uzbekistan,
Associate Professor
E-mail: alimovakrom63@yandex.ru

Vladimir I. Chilin
National University of Uzbekistan,
Vuzgorodok, Tashkent 100174, Uzbekistan
Professor
E-mail: vladimirchil@gmail.com, chilin@ucd.uz

Владикавказский математический журнал
2019, Том 21, Выпуск 4, С. 5–10

2-ЛОКАЛЬНЫЕ ИЗОМЕТРИИ НЕКОММУТАТИВНЫХ ПРОСТРАНСТВ ЛОРЕНЦА

Алимов А. А.[1], Чилин В. И.[2]

[1] Ташкентский институт по проектированию, строительству
и эксплуатации автомобильных дорог,
Узбекистан, 100060, Ташкент, пр. Амира Темура, 20
[2] Национальный университет Узбекистана,
Узбекистан, 100174, Ташкент, Вузгородок

E-mail: alimovakrom63@yandex.ru,
vladimirchil@gmail.com, chilin@ucd.uz

Аннотация. Пусть $\mathcal{M}$ алгебра фон Неймана с точным нормальным конечным следом τ, и пусть $S(\mathcal{M},\tau)$ инволютивная алгебра всех τ-измеримых операторов, присоединенных к алгебре $\mathcal{M}$. Для оператора $x \in S(\mathcal{M},\tau)$ невозрастающая перестановка $\mu(x) : t \to \mu(t;x)$, $t > 0$, определяется с помощью равенства $\mu(t;x) = \inf\{\|xp\|_{\mathcal{M}} : p^2 = p^* = p \in \mathcal{M}, \tau(\mathbf{1}-p) \leqslant t\}$. Пусть ψ возрастающая вогнутая непрерывная функция на $[0,\infty)$, для которой $\psi(0) = 0$, $\psi(\infty) = \infty$. Пусть $\Lambda_\psi(\mathcal{M},\tau) = \big\{x \in S(\mathcal{M},\tau) : \|x\|_\psi = \int_0^\infty \mu(t;x)d\psi(t) < \infty\big\}$ некоммутативное пространство Лоренца. Сюръективное (не обязательно линейное) отображение $V : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ называется сюръективной 2-локальной изометрией, если для любых $x,y \in \Lambda_\psi(\mathcal{M},\tau)$ существует такая сюръективная линейная изометрия $V_{x,y} : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$, что $V(x) = V_{x,y}(x)$ и $V(y) = V_{x,y}(y)$. Доказано, что в случае, когда $\mathcal{M}$ есть фактор, каждая сюръективная 2-локальная изометрия $V : \Lambda_\psi(\mathcal{M},\tau) \to \Lambda_\psi(\mathcal{M},\tau)$ есть линейная изометрия.

Ключевые слова: измеримый оператор, пространство Лоренца, изометрия.

Mathematical Subject Classification (2010): 46L52, 46B04.

Образец цитирования: *Aminov B. R., Chilin V. I.* Isometries of Real Subspaces of Self-Adjoint Operators in Banach Symmetric Ideals // Владикавк. мат. журн.—2019.—Т. 21, № 4.—С. 5–10 (in English). DOI: 10.23671/VNC.2019.21.44595.