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Mathematical Model of a Waveguide Junction Containing a Dielectric Layer

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In the paper, we consider a three-port waveguide junction with a matching inhomogeneity in the form of a dielectric layer included in the side branch. The system is excited from different ports simultaneously. We construct a mathematical model for the structure under consideration, and additionally, we present the fields in a convenient form for further analysis and optimization. For the structure being studied, we have developed a so-called scattering matrix, which allows us to draw a parallel between the structure and the neural center. Furthermore, it enables the development of a strategy for optimizing the structure parameters.

Keywords: Waveguide junction, dielectric layer, supply mode.

AMS Subject Classification: 78A45

1. Introduction

Gratings consisting of open ends of waveguides belong to one of the fundamental classes of antenna gratings [1]. Such gratings are the basic elements of mesas in the construction of communication systems that require high transmission power. Gratings consisting of open ends of waveguides are also widely used in aircraft, since they do not require additional protrusions on the body and therefore do not create problems in the aerodynamics of the aircraft.

The increase in the speed, maneuvering and flight intensity of flying machines leads to an increase in the area of problems, the solution of which significantly depends on the efficient operation of the radio-electronic equipment of these machines.

During the building and construction of flying machines, the problem of placing a large number of antennas and systems necessary for their provision arises.

One way to solve this problem is to create multifunctional (complex) antenna-feeder systems, which will combine antennas, filters, switches, splitters, control units and other functional elements, which will enable the distribution of electromagnetic energy between different radio-electronic devices and their radiation (reception).

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The effectiveness of using the results achieved in the fields of radio engineering, radio physics and radio electronics in communication systems depends to a large extent on the capabilities of the base of elements (transmission and reception sources, channels and oscillatory systems). Therefore, special importance is given to new methods of constructing super-high-frequency (SHF) devices based on planar (PIS) and volume (VIS) integrated circuits.

VIS can have a complex purpose (transmission line reversal, transfer between cascades, T-junction, phase reversal, line break, etc.). Therefore, for the analysis of VIS, a certain value is assigned to the study of multiport waveguides [2, 3].

Multi-cascade waveguide systems containing inhomogeneities can be used as VIS elements in SHF devices, the transition between cascades of which is carried out by means of waveguide fragments containing inhomogeneities.

In order to increase the efficiency of multi-port waveguide junctions within the framework of radio-electronic and communication systems, it is necessary first of all to increase their bandwidth and reduce energy losses. In addition, when branching, splitting and converting the signal in such systems, it is necessary to ensure an acceptable agreement between the arms, that is, a fairly high level of transmission in the desired arms and a low level in the others. Finally, for the stable operation of the system, it is necessary to achieve stable characteristics in a fairly wide range of frequencies.

To achieve this goal, it is possible to include artificial inhomogeneities in different branches of the structure, which also perform the functions of filters, phase reversers, multiplexers and other functions. But, the analysis of the characteristics of such structures shows that it is often not possible to achieve the desired level of electromagnetic compatibility in the system only by including inhomogeneities [4–6].

On the basis of the studies conducted in the works [7, 8], it becomes clear that the electrodynamic properties of the multi-level waveguide junction/splitter can be changed by the variation of the system's power supply mode.

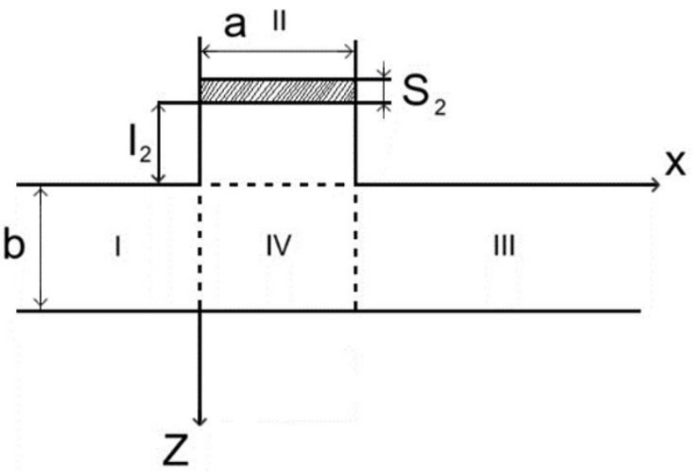

Figure 1. The considered system and the obtained geometric notations.

In the presented paper, a three-port waveguide junction is considered, with matching inhomogeneity in the form of a dielectric layer included in the side branch, and the system is excited from the different ports at the same time. The considered system and the obtained geometric notations are shown in figure 1.

Assume that the fields entering the system from each port have a single tangential E_y component of the electric strength vector.

$$E_y^{(10)} = p1(1)\sin\left(\alpha_{\mu_1}^{(1)} z\right) e^{-ih_{\mu_1}^{(1)} x} \tag{1}$$

$$E_y^{(20)} = p1(2)\sin\left(\alpha_{\mu_2}^{(2)} x\right) e^{-ih_{\mu_2}^{(2)} (z+l_2+S_2)} \tag{2}$$

$$E_y^{(30)} = p1(3)\sin\left(\alpha_{\mu_1}^{(1)} z\right) e^{-ih_{\mu_1}^{(1)} (x-a)} \tag{3}$$

The full field in system ports can be written as

$$E_y^{(1)} = E_y^{(10)} + \sum_{m=1}^{\infty} \left(A_m^{(1)} + B_m^{(3)} e^{-ih_m^{(1)} a}\right) \sin\left(\alpha_m^{(1)} z\right) e^{ih_m^{(1)} x};\ x \le 0;\ \ 0 \le z \le b \tag{4}$$

$$E_y^{(21)} = E_y^{(20)} + \sum_{m=1}^{\infty} A_m^{(21)} \sin\left(\alpha_m^{(2)} x\right) e^{ih_m^{(21)} (z+l_2+S_2)};\ z \le -\left(l_2 + S_2\right);\ 0 \le x \le a \tag{5}$$

$$E_y^{(22)} = \sum_{m=1}^{\infty} \left(A_m^{(22)} e^{ih_m^{(22)}(z+l_2)} + B_m^{(21)} e^{-ih_m^{(22)}(z+l_2+S_2)}\right) \sin\left(\alpha_m^{(2)} z\right);$$
$$\left(l_2 + S_2\right) \le z \le l_2;\ \ 0 \le z \le b \tag{6}$$

$$E_y^{(23)} = \sum_{m=1}^{\infty} \left(A_m^{(23)} e^{ih_m^{(21)} z} + B_m^{(22)} e^{-ih_m^{(21)}(z+l_2)}\right) \sin\left(\alpha_m^{(2)} z\right);$$
$$l_2 \le z \le 0;\ 0 \le x \le a \tag{7}$$

$$E_y^{(3)} = E_y^{(30)} + \sum_{m=1}^{\infty} \left(A_m^{(3)} e^{ih_m^{(1)} a} + B_m^{(1)}\right) \sin\left(\alpha_m^{(1)} z\right) e^{-ih_m^{(1)} x};\ x \ge a;\ \ 0 \le z \le b. \tag{8}$$

Here we introduce the notations: $p1(j)$ $(j = 1, 2, 3)$ are the logical multipliers of exciting, $p1(j) = 0$, means that from port j power is not supplied to the system, $p1(j) =$ 1-supplied); $\alpha_m^{(1)} = \pi m/a$, $\alpha_m^{(2)} = \pi m/b$-transverse wavenumbers of ports I and III and II respectively; $h_m^{(1)} = \sqrt{k^2 - \alpha_m^{(1)}}$, $h_m^{(21)} = \sqrt{k^2 - \alpha_m^{(2)}}$, $h_m^{(22)} = \sqrt{k^2 \varepsilon_r - \alpha_m^{(2)}}$ are longitudinal wavenumbers across the ports I, III and in different areas of the port II. It should be noted that from the condition of radiation in the infinity there follows-Im $\left(h_m^{(1)}, h_m^{(21)}\right) \le 0$.

The tangential component of the electric strength vector in the resonator part of the structure $(0 \le x \le a;\ 0 \le z \le b)$ can be represented in the following form

$$E_y^{(4)} = \int_{-\infty}^{\infty} D(t)\,\mathrm{sh}\left[\sqrt{t^2 - k^2}(b - z)\right] e^{\mathrm{itx}} dt$$

$$+\sum_{m=1}^{\infty}\left(B_m^{(1)}e^{-ih_m^{(1)}x}+B_m^{(3)}e^{ih_m^{(1)}(x-a)}\right)\sin\left(\alpha_m^{(1)}z\right) \tag{9}$$

(1)–(9) represent the field record over the entire physical area for the problem under consideration. This representation is fictitious since it contains sequences of unknown coefficients, which in their essence represent the complex amplitudes of reflected and passed fields from real or imaginary surfaces separating different areas.

Let's find these sequences of coefficients.

As a result of the implementation of the boundary conditions for the tangential components of the electric and magnetic strength vectors on the surfaces of the dielectric layer located in port II, we obtain

$$A_m^{(21)}=\frac{h_m^{(21)}+h_m^{(22)}}{2h_m^{(21)}}A_m^{(22)}e^{-ih_m^{(22)}S_2}+\frac{h_m^{(21)}-h_m^{(22)}}{2h_m^{(21)}}B_m^{(22)} \tag{10}$$

$$B_m^{(21)}=p1(2)\frac{2h_m^{(21)}}{h_m^{(21)}+h_m^{(22)}}\delta_{m\mu_2}+\frac{h_m^{(22)}-h_m^{(21)}}{h_m^{(21)}+h_m^{(22)}}A_m^{(22)}e^{-ih_m^{(22)}S_2} \tag{11}$$

$$A_m^{(22)}=\frac{h_m^{(21)}+h_m^{(22)}}{2h_m^{(22)}}A_m^{(23)}e^{-ih_m^{(21)}l_2}+\frac{h_m^{(22)}-h_m^{(21)}}{2h_m^{(22)}}B_m^{(22)} \tag{12}$$

$$B_m^{22}=p1(2)\frac{\delta_{m\mu_2}}{\gamma_m}+\frac{\beta_m}{\gamma_m}A_m^{(23)}e^{-ih_m^{(21)}l_2} \tag{13}$$

where

$$\gamma_m=\cos\left(h_m^{(22)}S_2\right)+\frac{i}{2}\left(\frac{h_m^{(21)}}{h_m^{(22)}}+\frac{h_m^{(22)}}{h_m^{(21)}}\right)\sin\left(h_m^{(22)}S_2\right)$$

$$\beta_m=\left(\frac{h_m^{(21)}}{h_m^{(22)}}-\frac{h_m^{(22)}}{h_m^{(21)}}\right)\sin\left(h_m^{(22)}S_2\right)$$

$$\delta_{mn}=\begin{cases}1; & m=n\\ 0; & m\neq n\,.\end{cases}$$

Using the continuity condition of E_y on the imaginary surface separating port II and the resonator part of the structure, we have

$$D(t)=\frac{1}{2\pi sh\left(\sqrt{t^2-k^2b}\right)}\sum_{m=1}^{\infty}\frac{\alpha_m^{(2)}\left(1-(-1)^m e^{-ita}\right)}{\alpha_m^{(2)}-t^2}\overline{A}_m^{(23)} \tag{14}$$

Here we introduce the notation

$$\overline{A}_m^{(23)}=p1(2)\frac{e^{-ih_m^{(21)}l_2}}{\gamma_m}\delta_{m\mu_2}+\left(1+\frac{\beta_m}{\gamma_m}e^{-2ih_m^{(21)}l_2}\right)A_m^{(23)} \tag{15}$$

If we consider (14) in (9), move to the complex plane, use the theory of residues, Jordan's lemma, Cauchy's theorem and correctly take into account the areas of analyticity of the integral function, the tangential component of the electric strength vector in the resonator part can be rewritten in the following form

$$E_y^{(4)} = \sum_{m=1}^{\infty} \overline{A}_m^{(23)} \left\{ \frac{\operatorname{sh}\left[\sqrt{t^2-k^2}(b-z)\right]}{\operatorname{sh}\left(\sqrt{t^2-k^2}b\right)} \sin\left(\alpha_m^{(2)} z\right) \right.$$
$$\left. -i\sum_{n=1}^{\infty} \frac{(-1)^n \alpha_n^{(1)} \alpha_m^{(2)} \left[e^{-ih_n^{(1)}x} - (-1)^m e^{-2ih_n^{(1)}(x-a)}\right]}{h_n^{(1)} b\left(\alpha_m^{(2)2} - h_n^{(1)2}\right)} \times \sin\left(\alpha_n^{(1)} z\right) \right\}$$
$$+ \sum_{m=1}^{\infty} \left(B_m^{(1)} e^{-ih_m^{(1)}x} + B_m^{(3)} e^{ih_m^{(1)}(x-a)}\right) \sin\left(\alpha_m^{(1)} z\right) \tag{16}$$

As a result of the realization of the continuity condition of E_y on the imaginary surface separating imaginary surfaces separating ports I, III and the resonator part, we write

$$\overline{A}_s^{(1)} = B_s^{(1)} - i\sum_{m=1}^{\infty} \overline{A}_m^{(23)} \frac{(-1)^s \alpha_s^{(1)} \alpha_m^{(2)} \left[1 - (-1)^m e^{-2ih_s^{(1)}a}\right]}{h_s^{(1)} b\left(\alpha_m^{(2)2} - h_s^{(1)2}\right)} \tag{17}$$

$$\overline{A}_s^{(3)} = B_s^{(3)} + i\sum_{m=1}^{\infty} (-1)^m \overline{A}_m^{(23)} \frac{(-1)^s \alpha_s^{(1)} \alpha_m^{(2)} \left[1 - (-1)^m e^{-2ih_s^{(1)}a}\right]}{h_s^{(1)} b\left(\alpha_m^{(2)2} - h_s^{(1)2}\right)} \tag{18}$$

here

$$\overline{A}_s^{(1)} = p1(1)\delta_{s\mu_2} + A_s^{(1)}$$
$$\overline{A}_s^{(3)} = p1(3)\delta_{s\mu_2} + A_s^{(3)}$$

Let's realize the smoothness conditions of the tangential component of the electric strength vector on the imaginary surfaces separating ports I, III and the resonator part of the structure. As a result we get

$$B_m^{(1)} = p1(1)\delta_{m\mu_1} + i\sum_{n=1}^{\infty} \sigma_{\mathrm{nm}} \overline{A}_m^{(23)} \tag{19}$$

$$B_m^{(3)} = p1(3)\delta_{m\mu_1} - i\sum_{n=1}^{\infty} (-1)^n \sigma_{\mathrm{nm}} \overline{A}_m^{(23)} \tag{20}$$

where

$$\sigma_{\mathrm{nm}} = \frac{\alpha_m^{(1)} \alpha_n^{(2)}}{h_s^{(1)} b} \left[-\frac{1}{2\left(\alpha_m^{(1)2} - h_n^{(21)2}\right)} + \frac{(-1)^m}{\left(\alpha_n^{(2)2} - h_m^{(1)2}\right)} \right]$$

Let's insert (19), (20) into (16)

$$E_y^{(4)} = \left[p1(1)e^{-ih_{\mu_1}^{(1)}x} - p1(3)e^{-ih_{\mu_1}^{(1)}(x-a)}\right]\sin\left(\alpha_{\mu_1}^{(1)}z\right)$$
$$+\sum_{m=1}^{\infty}\overline{A}_m^{(23)}\left\{\frac{\text{sh}\left[\sqrt{t^2-k^2}(b-z)\right]}{\text{sh}\left(\sqrt{t^2-k^2}b\right)}\sin\left(\alpha_m^{(2)}z\right)\right.$$
$$\left.-i\sum_{n=1}^{\infty}\frac{\alpha_n^{(1)}\alpha_m^{(2)}\left[e^{-ih_n^{(1)}x}-(-1)^m e^{ih_n^{(1)}(x-a)}\right]}{2h_n^{(1)}b\left(\alpha_n^{(1)2}-h_m^{(21)2}\right)}\sin\left(\alpha_n^{(1)}z\right)\right\} \tag{21}$$

Finally, let's use the smoothness condition of the electric stress vector on the imaginary surface separating port II and the resonator part of the structure under consideration, from which we can write:

$$A_s^{(23)} + \sum_{m=1}^{\infty} Q_{\text{sm}} A_m^{(23)} = \theta_s, \quad s = 1, 2, \dots . \tag{22}$$

Here we introduce the notations

$$\theta_s = \frac{p1(2)\left[\left(\left(1+icth\left(\sqrt{t^2-k^2}b\right)\right)\right)\delta_{s\mu_2} - \xi_{\delta_{s\mu_1}}\right] - \eta_{s\mu_2}}{1-icth\left(\sqrt{t^2-k^2}b\right) - \frac{\beta_s}{\gamma_s}e^{-2ih_s^{(21)}l_2}\left(1+icth\left(\sqrt{t^2-k^2}b\right)\right)}$$

$$Q_{sm} = \frac{\frac{\beta_m}{\gamma_m}e^{-ih_m^{(21)}l_2}}{1-icth\left(\sqrt{t^2-k^2}b\right) - \frac{\beta_s}{\gamma_s}e^{-2ih_s^{(21)}l_2}\left(1+icth\left(\sqrt{t^2-k^2}b\right)\right)}\xi_{sm}$$

$$\eta_{s\mu_1} = \frac{\alpha_{\mu_1}^{(1)}\alpha_s^{(2)}\left[p1(1)-(-1)^s p1(3)\right]\left(1-(-1)^m e^{-ih_{\mu_1}^{(1)}a}\right)}{ih_s^{(21)}a\left(\alpha_s^{(1)2}-h_{\mu_1}^{(21)2}\right)}$$

$$\xi_{sm} = \sum_{n=1}^{\infty}\frac{\alpha_n^{(1)2}\alpha_m^{(2)}\alpha_s^{(2)}\left(1+(-1)^{m+s}\right)\left(1-(-1)^m e^{-ih_n^{(1)}a}\right)}{2h_s^{(21)}b\left(\alpha_s^{(2)2}-h_n^{(1)2}\right)h_n^{(1)}b\left(\alpha_n^{(1)2}-h_m^{(21)2}\right)}$$

(22) represents an infinite system of linear algebraic equations with respect to the sequence of coefficients $\left\{A_m^{(23)}\right\}_{m=1}^{\infty}$.

(17)–(20) and (10)–(13) relate this sequence to all other sequences of coefficients. Therefore, we can say that for the structure under consideration a full field has been represented in the entire physical area.

The analysis of the matrix elements and free terms of (22) reveals that the system is quasi-regular, so it can be solved on a computer by the reduction method.

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