Bulletin of TICMI
Vol. 22, No. 2, 2019, 143–158

Discrete Systems of Information Warfare and Optimal Control Problems

Nugzar Kereselidze*

Sokhumi State University, Tbilisi, Georgia
Department of Mathematical and Computer Sciences, Tbilisi, Georgia

(*Received October 23, 2018; Revised December 21, 2019; Accepted December 25, 2019*)

In the article on the basis of the new unified approach in mathematical and computer modeling of information warfare a specific integrated nonlinear discrete model of information warfare is constructed. At creation of the integrated discrete model the ideas of modeling of information opposition the offer by academician A.A. Samarsky, together with professor A.P. Mikhaylov and Professor T.I. Chilachava are used.

On the basis of combining these approaches in modeling, when information flows and recipients of information were separately considered, the integrated discrete model was constructed. This model was investigated for controllability and a specific problem of optimal control of the combined discrete process was formulated.

Keywords: Information warfare, Integration discrete models, Mathematical and computer models, Problem of optimal control.

AMS Subject Classification: 93A30, 93B03, 93C55, 97M70, 68U20.

1. Introduction

Recently decision-making centers of different levels are paying more attention to the issues, which contain elements of information warfare. Naturally, the higher the level of decision-making center, the greater the impact of its decision on the ongoing processes, in addition, more capital investment is required for decision realization.

Therefore it is logical, that the decision existing as a project in the decision-making center requires expert evaluation, in terms of effective impact on the processes, which also implies optimal financing for decision realization. It is obvious that expert evaluation should be preceded by a scientific research of the issue, which will result in accumulation of both theoretical knowledge about information warfare and practical experience of using this knowledge.

With the topical standpoint of the issue, we will bring the following example. As we know, in 2015, the heads of governments and states of European Union countries considered it necessary to oppose the targeted misinformation campaign against the European Union and a special service was established for this purpose. The campaign "European Union Against misinformation" is executed by a special targeted operative group East StratCom (Eastern Strategic Communication), which denies the misinformation coming from Russia. This misinformation is aimed

*Email: nkereselidze@sou.edu.ge

ISSN: 1512-0082 print

to be harmful to the European Union's established policy towards eastern Neighborhood countries. (Azerbaijan, Belarus, Moldova, Georgia, Armenia, Ukraine). As it seems from the official site of the operative group (https://euvsdisinfo.eu/), from the day of its foundation they have been able to reveal about 4,000 cases of misinformation, which has been spread in 18 languages. Every revealed case of misinformation and its rejection is given on the operative group's website - https://euvsdisinfo.eu/disinformation-cases/ Operative Group East StratCom's work demonstrated the real scale of current cyber attacks and the hybrid war, that gives the possibility to become a basis for increasing the operative group's financing to the 1.1 Million Euros.

The information on the justification of increasing the funding of the operative group to this amount has not been spread, but we hope, that defining the sum was preceded by researches, which prove that the apportion of this sum is an optimal solution of the problem.

In the previous work, we represent one of the directions of scientific research on information warfare and some issues of making decision in this field. Particularly, we will consider a particular type of information war, its discrete mathematical and computer models; we will build an integrated discrete mathematical model of information warfare by combining the existing modelling paradigms of information warfare and we will research it by means of computer experiments. We will consider termination task of the information warfare and will consider its optimal solution. In this work, we will call information warfare dissemination of misinformation and discrediting information by one party to another. In some cases, in the information warfare between two sides, a third party - peacekeeping side may be involved, which disseminates calmative information to the opposing parties in order to stop them from these information confrontations.

2. Discrete mathematical model of the adepts in information warfare

In this part of the work we will try to build a discrete analogue of the continuous mathematical model of adept attraction of Samarskiy-Mikhailov. Let's discuss the task: in time segment $[0;T]$ some information is disseminated, in discrete points of this segment of time $t_i = i * \tau$, where $\tau = T/m$, is the step distance between the discrete points, and $i = 0, 1, 2, ..., m$, where m natural number, relevant number of population $n_0, n_1, n_2, ..., n_m$ knows this information. Information about any event is transmitted by any structure or personality. Initially, before starting the information dissemination campaign, no one knows about this information, so we have the initial condition

$$n_0 = 0. \tag{1}$$

At the time point $i+1$ $(i \geq 0)$ number of information holders n_{i+1} depends on the number of information holders n_i at the time point i, on the campaign's intensity coefficient α_1 of this information dissemination, on the the number of uninformed population : $\alpha_1 (N - n_i)$ and on the intensity coefficient α_2 of information dissemination in the uninformed population by the informed population via personal contacts : $\alpha_2 (N - n_i) n_i$, on the relation with the amount and the intensity coefficient of this relation. If we denote the total number of the population with N, then we can build a discrete mathematical model of informed (adepts) number, which is an analogue of the continuous mathematical model of Samarskiy-Mikhailov:

$$\frac{dN(t)}{dt} = [\alpha_1(t) + \alpha_2(t) N(t)] (N_0 - N(t))$$

$$n_{i+1} - n_i = \tau(\alpha_1 + \alpha_2 n_i)(N - n_i), i = 0, 1, 2, ... \quad (2)$$

To receive the difference equation (2) the following way of the model's constructing is used - Information dissemination is uniform in the population environment, which is permissible for that type of models which are the first approximation of the studied object. In model (2) the next factor is considered - Part of the informed population disseminates information among the uninformed part of the population through personal contacts during mutual meetings. By transforming difference equation (2) we get:

$$n_{i+1} = n_i + \tau\alpha_1 N - \tau\alpha_1 n_i + \tau\alpha_2 N n_i - \tau\alpha_2 n_i^2,$$

$$n_{i+1} = -\tau\alpha_2 n_i^2 + (1 - \tau\alpha_1 + \tau\alpha_2 N) n_i + \tau\alpha_1 N. \quad (3)$$

With the initial conditions (1) finding solution to the difference equation (3) analytically is impossible. That is why we solve it using numerical methods by means of given computers. Numerical experiment will be conducted in the environment of software package Matlab, where for different values of intensity coefficients we get adept values of disseminated information in the population in discrete moments. We have taken different values for the intensity coefficients α_1, α_2. The program code is given in Listing 1:

Listing 1. Code of solving the discrete model of Samarskiy-Mikhailov. N=100;

```
T=20; tau=0.01;
n=fix(T/tau); m=n+1;
A11=0.2; A12=0.1; x(1)=0; A13=1-tau*A11+tau*A12*N; A14=tau*A11*N;
A21=0.4; A22=0.2; y(1)=0; A23=1-tau*A21+tau*A22*N; A24=tau*A21*N;
A31=0.6; A32=0.4; z(1)=0; A33=1-tau*A31+tau*A32*N; A34=tau*A31*N;
i=1:m;
for j=2:m
x(j)=-tau*A12*x(j-1)*x(j-1)+A13*x(j-1)+A14;
y(j)=-tau*A22*y(j-1)*y(j-1)+A23*y(j-1)+A24;
z(j)=-tau*A32*z(j-1)*z(j-1)+A33*z(j-1)+A34;
end;
plot(i,x,'+',i,y,'o',i,z,'*','line Width',2)
title('Diskret Model of Samarskiy-Mikhailov')
xlabel('Diskret Time')
ylabel('Amount of adepts')
legend('Variant 1','Variant 2','Variant 3')
grid on
```

One of the cases of the given listing 1 code realization is presented in Figure 1. Where in the logistical graph with crosses (blue color - variant 1), an increase in the number of adepts is given α_1, α_2, for the values of intensity coefficients - (0,2;0,1); On the graph with rings (green color - variant 2), an increase in the number of adepts is given α_1, α_2, for the values of intensity coefficients - (0,2;0,4); On the graph with times (asterisk) (red color - variant 3), an increase in the number of adepts is given α_1, α_2, for the values of intensity coefficients - (0,6;0,4);

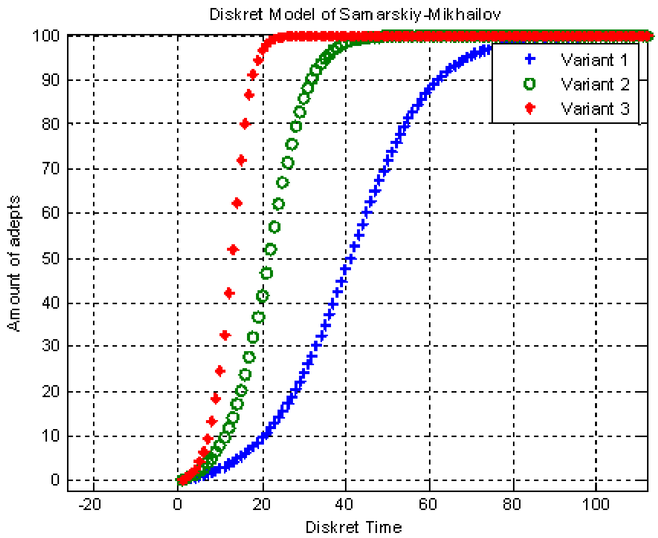

Figure 1. Logistic graphs for solving the discrete model of Samarskiy-Mikhailov

From the discrete model of Samarskiy-Mikhailov (1), (2) the solution graphs of the Cauchy task -it is clear, that the more the value of the coefficients α_1, α_2, the faster the number of adepts approach the total number of population. It is therefore possible to consider the intensity of information dissemination coefficients α_1, α_2 as the controlled process parameters. At the same time, we should note that in the discreet model of adept growth of Samarskiy-Mikhailov information streams are not explicitly provided and they are represented by the coefficients α_1, α_2. In the next part of the work we will present the approach of the modelling of information warfare in which the focus is on the description of the information flow intensity.

3. Discrete model of information flow dissemination

T. Chilachava is the author of studying informational flow dissemination of information warfare with mathematical and computer models paradigm (see [2-4]). There are many specific mathematical and computer models based on this paradigm in proposed works [5-7]. In these models two opposing sides are considered which disseminate discrediting information towards each other and the third party involved in the informational warfare spreads peacemaking statements to the opposing sides. Studying these types of models shows that - according to the peacekeeper's level of activity and how authoritative is the peacekeeper to the opposing sides, the information warfare may be terminated - or, the opposing sides end disseminating of disinformation and discrediting information towards each other. For certain values of the model parameters the possibility of ending the information warfare is determined. It is believed that ending of the information warfare between the parties significantly reduces the transformation of this confrontation to the "hot" warfare phase.

Let's build the information stream dissemination model in the conditions of opposing side ignorance by considering the level of information technologies. Let's say the process of activity of the two opposing parties and the third peacemaker is discussed in $[0;T]$ time period. In this period of time, we take discrete points $t_i = i * \tau$, where $\tau = T/m$ is the distance between the discrete points $i = 0, 1, 2, ..., m = T/\tau$ where m is a natural number. Assume that in discrete moments of time $i = 0, 1, 2, ..., m$, the first party respectively disseminates N_{10}^i amount of discrediting information towards the other party. Similarly, in discrete moments of time $i = 0, 1, 2, ..., m$, the second party respectively disseminates N_{20}^i amount of discrediting information towards the first party. Third, the peace-keeping party respectively disseminates N_3^i amount of information in the same discrete moments of time. Let's agree that neither party can disseminate more information, in any of the discrete moments of time, than their technologies are able to give. Let us assume that the parties are able to disseminate the maximum I_1, I_2, I_3 amount of information according to the possibilities of their Internet technologies in discrete moments. Because the opposing sides do not take into account the information disseminated by the opponent, the amount of information disseminated by them in the time point t_{i+1} depends on the amount of proportionally disseminated information in the time point t_i, the maximum importance given by information technology level at its proximity and the number of disseminated information by the peace-keeping side. The number of information disseminated at the time point t_{i+1} by the third party depends proportionally on the disseminated information by the opposing parties at time point t_i, the maximum importance determined by the information technology level at its proximity. Thus, we can build a mathematical model of the spreading information stream taking into account the level of information technologies under the conditions of the opponent ignorance:

$$\begin{cases} N_{10}^{i+1} = N_{10}^i + \tau\alpha_1 N_{10}^i \left(1 - \frac{N_{10}^i}{I_1}\right) - \beta_1 \tau N_3^i, \\ N_{20}^{i+1} = N_{20}^i + \tau\alpha_2 N_{20}^i \left(1 - \frac{N_{20}^i}{I_2}\right) - \beta_2 \tau N_3^i, \\ N_3^{i+1} = N_3^i + \tau \left(\gamma_1 N_{10}^i + \gamma_2 N_{20}^i\right) \left(1 - \frac{N_3^i}{I_3}\right). \end{cases} \tag{4}$$

with the initial conditions:

$$N_{10}^0 = N_{10}\,,\, N_{20}^0 = N_{20}\,,\, N_3^0 = N_{30}\,. \tag{5}$$

The solution of the Cauchy discrete task (4), (5) is found using numerical methods in the environment of software packages MatLab. The solution code is presented in Listing 2.

Listing 2. The Code of Solving the Information Flow dissemination Model.

```
T=100; tau=0.2;
n=fix(T/tau); m=n+1;
A11=0.3; A12=0.2; N10(1)=30; N20(1)=25; N3(1)=15;
A21=0.04; A22=0.02; G1=0.03; G2=0.02;
A31=600; A32=700; A33=800;
i=1:m;
for j=2:m
```

```
N10(j)=N10(j-1)+tau*A11*(1-N10(j-1)/A31)-tau*A21*N3(j-1);
N20(j)=N20(j-1)+tau*A12*(1-N20(j-1)/A32)-tau*A22*N3(j-1);
N3(j)=N3(j-1)+tau*(G1*N10(j-1)+G2*N20(j-1))*(1-N3(j-1)/A33);
end
plot(i,N10,'+',i,N20,'o',i,N3,'*','lineWidth',2)
title('Diskret Model of IW flow')
xlabel('Diskret Time')
ylabel('Amount of informations')
legend('I side','II side','III side')
grid on
```

A computer experiment is conducted in MatLab's environment with the code provided in Listing 2 and it is established that the system, which is described with (4), (5) discrete mathematical model, is controlled. This means that the system (5) from the initial state with the change of some parameters of model (4) can be transferred to the state, when the opposing sides do not disseminate disinformation or discrediting information anymore - or the information warfare has ended. Information warfare ends by increasing the activity of the third - peacekeeping side. In particular, by changing its control "levers" γ_1, γ_2, I_3. On Figure 2 solution graph of the discrete model of information warfare's information flow dissemination is presented, for the following values of parameters: $\alpha_1 = 0,3;\alpha_2 = 0,2;\beta_1 = 0,04;\beta_2 = 0,02;I_1 = 600;I_2 = 700;I_3 = 800;\gamma_1 = 0,03;\gamma_2 = 0,02;$

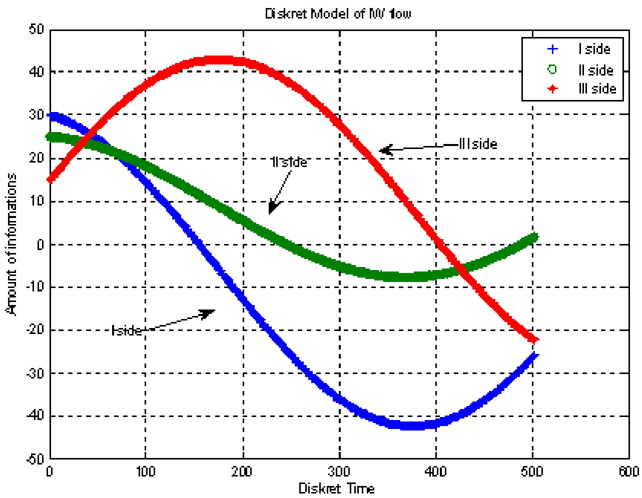

Figure 2. Solution graph of the information stream dissemination model

In Figure 3-5 we see "zero crossing" moments of the solution which exists on Figure 2. In particular, the first party performs information dissemination in t^{*}_{155} discrete moment of "time", the second side in the discrete moment t^{**}_{245} – and the third side in the discrete moment t^{***}_{405}.

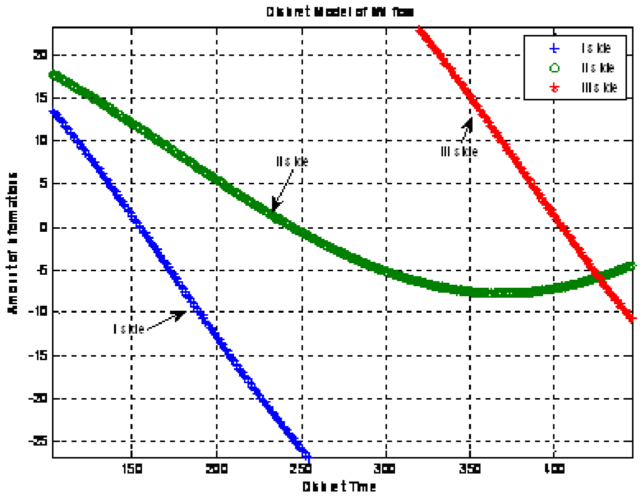

Figure 3. "Zero crossing" of the solution of information flow dissemination model

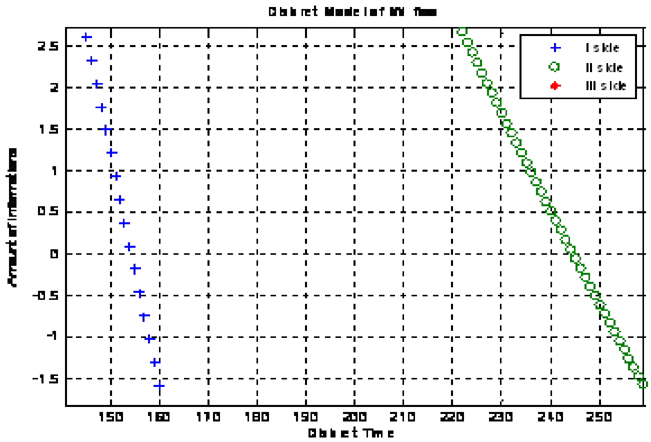

Figure 4. Zero crossing of information flow model dissemination for first and second sides

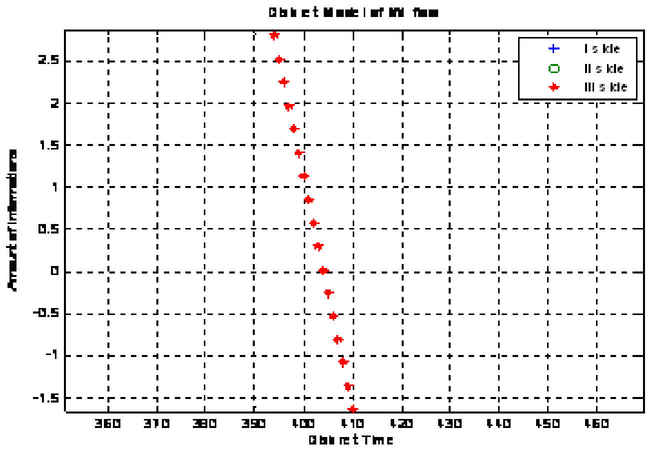

Figure 5. Zero crossing of information flow model dissemination for the third side

As it is obvious in the information flow dissemination model of information warfare with taking into account the level of information technologies in the conditions of ignoring the opponent adept engagement in disseminating similar information in uninformed population is not considered. Moreover, the population does not participate in this model, for which the information is intended. On the other hand, the information flow, which is disseminated in the population and with its influence information adepts are created, is not explicitly represented in the Samarskiy-Mikhailov model. In the next part of the work we will try to build a discrete model of information war, in which flow dissemination, also adept existence and their respective activity after the dissemination of this information is taken into account.

4. An integrated discrete model of information warfare

When constructing an integrated discrete model of information warfare, we will rely on the approaches, which were proposed while constructing a continuous nonlinear model of information warfare [8]. Let's consider the information warfare process at discrete time moments of any $[0;T]$ section: $t_i = i \times \tau$, where τ is an increment and the distance between discrete moments, $i = 0, 1, 2, ..., n$ and $\tau = T/n$. The two opposing sides participating in information warfare disseminate so-called "official" discrediting information towards each other in discrete moments of time, the first side N^i_{10}amount of information and second side - N^i_{20}. Let's say the first party is represented by a population, the maximum number of which is ${}_1N$, and the other side is represented by the maximum population - ${}_2N$.

Let's denote by ${}^{i}_{11}N$ the amount of the first side population, which in the time point i, assimilated "official" discrediting information disseminated by the first side towards the second one, or became the first side adept (so-called "Patriot") in the first side and also started to circulate similar information in the amount of N^i_{11} on his own. Let's denote by ${}^{i}_{12}N$ the amount of the first side population, which in the time point i, assimilated "official" discrediting information disseminated by the second side towards the first one, or became the second side adept (so-called "Fifth column" or "Dissidents") in the first side and also started to circulate similar information in the amount of N^i_{12} on his own.

Similarly, Let's denote by ${}^{i}_{21}N$ the amount of the second side population, which in the time point i, assimilated "official" discrediting information disseminated by the first side towards the second one, or became the first side adept (so-called "Fifth column" or "Dissidents") in the second side and also started to circulate similar information in the amount of N^i_{21} on his own. Let's mark with ${}^{i}_{22}N$ the amount of the second side population, which in the time point i, assimilated "official" discrediting information disseminated by the second side towards the first one, or became the second side adept (so-called "Patriot") in the second side and also started to circulate similar information in the amount of N^i_{22} on his own.

Third peacekeeping side in time point i spreads N^i_3 amount of "calmative" information towards opposing sides. The first, second and third - "official" sides have the upper limit of the maximum number of information dissemination, which is stipulated by their IT technologies and financial resources. Let's denote these upper limits respectively by I_1, I_2, I_3 . The upper limit of maximum number of dissemination is also possessed by adepts and "dissidents", who are in the first and second sides, let's denote them respectively by $I_{11}, I_{12}, I_{21}, I_{22}$. Graphical illustration of participant parties and information streams in information warfare is shown in Figure 6.

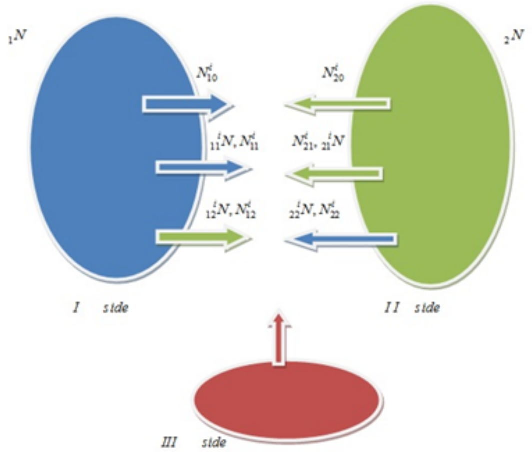

Figure 6. Graphical illustration of participant parties and information streams in information war

While constructing the information warfare integrated discrete model, during the description of so-called "official" information flow changes of the sides we will take into account the approach, which was given in the second point of the work. While describing changed adepts or "dissidents" number we will take into account the Samarskiy-Mikhailov approaches discrete model constructing, which is given in the second point of the work and also consider the fact that the number of information flows disseminated by the parties will be clearly visible. While describing the changes of disseminated information flow by adepts and so-called "dissidents", approaches existing in the second and third section will be combined and used. Thus, the integrated discrete model is shown as:

$$\begin{cases} N^{i+1}_{10} = N^i_{10} + \tau\alpha_1 N^i_{10}\left(1 - \frac{N^i_{10}}{I_1}\right) - \beta_1\tau N^i_3, \\ N^{i+1}_{20} = N^i_{20} + \tau\alpha_2 N^i_{20}\left(1 - \frac{N^i_{20}}{I_2}\right) - \beta_2\tau N^i_3, \\ N^{i+1}_{3} = N^i_{3} + \tau\left(\gamma_1 N^i_{10} + \gamma_2 N^i_{20}\right)\left(1 - \frac{N^i_{3}}{I_3}\right), \\ {}^{i+1}_{11}N = {}^{i}_{11}N + \tau\left(\alpha_3 N^i_{10} + \alpha_4 N^i_{11}\,{}^{i}_{11}N\right)\left(1 - \frac{{}^{i}_{11}N}{{}_1N}\right), \\ {}^{i+1}_{12}N = {}^{i}_{12}N + \tau\left(\alpha_5 N^i_{20} + \alpha_6 N^i_{12}\,{}^{i}_{12}N\right)\left(1 - \frac{{}^{i}_{12}N}{{}_1N}\right), \\ {}^{i+1}_{21}N = {}^{i}_{21}N + \tau\left(\alpha_7 N^i_{20} + \alpha_8 N^i_{21}\,{}^{i}_{21}N\right)\left(1 - \frac{{}^{i}_{21}N}{{}_2N}\right), \\ {}^{i+1}_{22}N = {}^{i}_{22}N + \tau\left(\alpha_9 N^i_{10} + \alpha_{10} N^i_{22}\,{}^{i}_{22}N\right)\left(1 - \frac{{}^{i}_{22}N}{{}_2N}\right), \\ N^{i+1}_{11} = N^i_{11} + \tau\alpha_{11} N^i_{11}\left(1 - \frac{{}^{i}_{11}N}{{}_1N}\right){}^{i}_{11}N + \tau\alpha_{12} N^i_{10}\left(1 - \frac{N^i_{11}}{I_{11}}\right), \\ N^{i+1}_{12} = N^i_{12} + \tau\alpha_{13} N^i_{12}\left(1 - \frac{{}^{i}_{12}N}{{}_1N}\right){}^{i}_{12}N + \tau\alpha_{14} N^i_{20}\left(1 - \frac{N^i_{12}}{I_{12}}\right), \\ N^{i+1}_{21} = N^i_{21} + \tau\alpha_{15} N^i_{21}\left(1 - \frac{{}^{i}_{21}N}{{}_2N}\right){}^{i}_{21}N + \tau\alpha_{16} N^i_{20}\left(1 - \frac{N^i_{21}}{I_{21}}\right), \\ N^{i+1}_{22} = N^i_{22} + \tau\alpha_{17} N^i_{22}\left(1 - \frac{{}^{i}_{22}N}{{}_2N}\right){}^{i}_{22}N + \tau\alpha_{18} N^i_{10}\left(1 - \frac{N^i_{22}}{I_{22}}\right). \end{cases} \tag{6}$$

With initial conditions

$$\begin{cases} N_{10}^{0} = N_{10}, & N_{20}^{0} = N_{20}, & N_{3}^{0} = N_{30}, \\ N_{11}^{0} = 0, & N_{12}^{0} = 0, & N_{21}^{0} = 0, & N_{22}^{0} = 0, \\ {}_{11}^{0}N = 0, & {}_{12}^{0}N = 0, & {}_{21}^{0}N = 0, & {}_{22}^{0}N = 0. \end{cases} \tag{7}$$

Therefore, (6), (7) represents the integrated discrete model of information warfare, in which information flows and also adepts and "dissidents" are presented in a clear form. Integrated discrete model of information warfare represents a discrete task of Cauchy. It is obvious that the amount of so-called "useful" information N_1^i in a discrete moment of time for the first side is the sum of "official" information disseminated by the first side, by adepts of the first side and disseminated information by the second side's "dissidents":

$$N_1^i = N_{10}^i + N_{11}^i + N_{22}^i. \tag{8}$$

the amount of so-called "useful" information N_2^i in a discrete moment of time for the second side is the sum of "official" information disseminated by the second side, by adepts of the second side and disseminated information by the first side's "dissidents":

$$N_2^i = N_{20}^i + N_{21}^i + N_{12}^i. \tag{9}$$

Let's assume that the opposing sides are carrying out the information warfare, if in any discrete time moment the so-called "useful" information is not disseminated - (8) and (9) become non-positive, which is possible to write as

$$N_1^{i^*} \leq 0\,,\, N_2^{i^{**}} \leq 0\,. \tag{10}$$

The algorithm code of information warfare integrated discrete model (8), (9) and solution finding is given in Listing 3.

Listing 3. Solution code of disseminated information by the parties in information warfare integrated discrete mode.

```
T=100; tau=0.2;
n=fix(T/tau); m=n+1;
A1=0.3; A2=0.2; A3=0.02; A4=0.022; A5=0.015; A6=0.018; A7=0.019; A8=0.0185;
A9=0.016; A10=0.017; A11=0.012; A12=0.013; A13=0.011; A14=0.012;
A15=0.0115; A16=0.0121; A17=0.0131; A18=0.0171;
N10(1)=30; N20(1)=25; N3(1)=15;
B1=0.04; B2=0.02; G1=0.03; G2=0.02;
I1=600; I2=700; I3=800; I11=100; I12=50; I21=120; I22=60;
N11(1)=0; N12(1)=0; N21(1)=0; N22(1)=0;
n11(1)=0; n12(1)=0; n21(1)=0; n22(1)=0; n1=900; n2=1000;
i=1:m;
for j=2:m
N10(j)=N10(j-1)+tau*A1*(1-N10(j-1)/I1)-tau*B1*N3(j-1);
N20(j)=N20(j-1)+tau*A2*(1-N20(j-1)/I2)-tau*B2*N3(j-1);
```

```
N3(j)=N3(j-1)+tau*(G1*N10(j-1)+G2*N20(j-1))*(1-N3(j-1)/I3);
n11(j)=n11(j-1)+tau*(A3*N10(j-1)+A4*N11(j-1)*n11(j-1))*(1-n11(j-1)/n1);
n12(j)=n12(j-1)+tau*(A5*N20(j-1)+A6*N12(j-1)*n12(j-1))*(1-n12(j-1)/n1);
n21(j)=n21(j-1)+tau*(A7*N20(j-1)+A8*N21(j-1)*n21(j-1))*(1-n21(j-1)/n2);
n22(j)=n22(j-1)+tau*(A9*N10(j-1)+A10*N22(j-1)*n22(j-1))*(1-n22(j-1)/n2);
N11(j)=N11(j-1)+tau*A11*(1-n11(j-1)/n1)*n11(j-1)+tau*A12*N10(j-1)*(1-
N11(j-1)/I11);
N12(j)=N12(j-1)+tau*A13*(1-n12(j-1)/n1)*n12(j-1)+tau*A14*N20(j-1)*(1-
N12(j-1)/I12);
N21(j)=N21(j-1)+tau*A15*(1-n21(j-1)/n2)*n21(j-1)+tau*A16*N20(j-1)*(1-
N21(j-1)/I21);
N22(j)=N22(j-1)+tau*A17*(1-n22(j-1)/n2)*n22(j-1)+tau*A18*N10(j-1)*(1-
N22(j-1)/I22);
end
N1=N10+N12+N22; N2=N20+N21+N12;
plot(i,N1,'+',i,N2,'o',i,N3,'*','lineWidth',2)
title('Integr Diskret Model of IW')
xlabel('Diskret Time')
ylabel('Amount of informations')
legend('I side','II side','III side')
grid on
```

So-called "useful" information dissemination by the sides graph for the parameter values of the model given in 1-11 lines of listing 3 is shown in Figure 7.

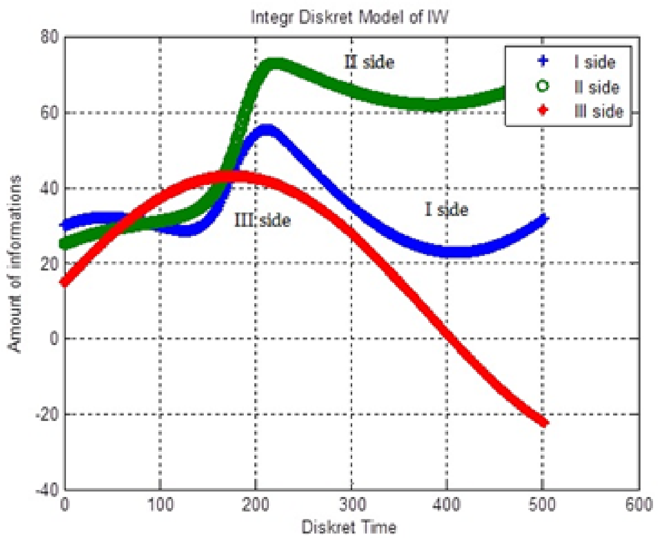

Figure 7. Useful information dissemination for the sides graph in the integrated discrete model of information warfare

5. Management task in the integrated discrete model of information warfare

In the case of an integrated discrete model, the task of completing the information warfare will be reduced to the specific (6), (7), (10) discrete boundary problem, which we will sometimes mention as Chilker type boundary problem. The specifics of the Chilker boundary problem lies in the fact that discrete moments of time i^*, i^{**}, for which condition (10) is fulfilled are different, at the same time they are free - they are not fixed. Let us note that i^*, i^{**} are the first discrete moments, for which condition (10) is fulfilled and for $\forall k \geq i^* \& \forall l \geq i^{**}$ we have $N_1^k \leq 0$, $N_2^l \leq 0$.

For Chilker (6), (7), (10) boundary problem we have the following management problem. Is it possible, for information warfare, which is described by (6), (7), (10) integrated discrete model, to select model parameters, which we can change to transfer the information warfare (7) state into information warfare ending - state (10). This issue was not positively resolved for specific model parameters available in Listing 3, clear illustration is given in Figure 6, where it is clear that "useful" information of the first and second opposing sides do not equal - do not cross zero. As the control parameter we can consider the peacekeeping activity coefficients of the third side - γ_1, γ_2 and the level of IT technology of the third side - I_3. By changing them we can change the scenario of the participant sides of information warfare. A computer experiment has been conducted, which confirms that the chosen parameters really are control parameters and with their help it is possible to end the information warfare. In particular, if we change the peacekeeping activity coefficients - γ_1, γ_2 and the IT technology level values I_3 of the third side, which are given in Listing 3 respectively with G1=0.3; G2=0.2; I3=1000 values – in fact, we increase it, then the opposing sides cross zero, this is shown clearly on the corresponding solution graphs – Figure 8 and Figure 9.

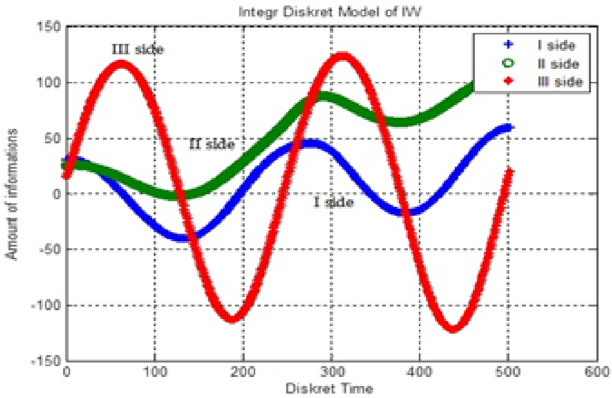

Figure 8. Crossing zero by the opposing sides is achieved by changing the managing parameters

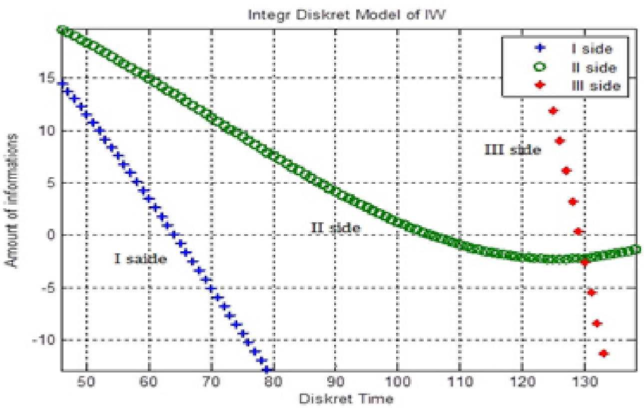

Figure 9. Crossing zero by the opposing sides is achieved by changing the control parameters (detailed)

In particular, $t^* = 65$, in this case $N_1^{64} = 0,5413$, which means that for the first side useful discrediting information is disseminated in the previous discrete moment - $i = 64$, and at the following discrete moment $i^* = i = 65$, this kind of information is not disseminated, because the corresponding $N_1^{65} = -0,2984$ is negative. In the following knots of the certain proximity discrete moment $t^* = 65$, in particular, until the second side "useful" information dissemination does not stop, the number of the "useful" information of the first party is negative. As for the second side $i^{**} = 107$, $N_2^{107} = -0,0585$, because for $i = 106$ N_2^{106} is still positive $N_2^{106} = 0,1489$. As for the third side, it participates in the information warfare at the discrete time moment $i = 131$ - $N_3^{131} = -2,7461$.

6. Optimal control problem of the integrated discrete model

As we have seen in the previous paragraph, the third party has control parameters of the discrete system, with which it is possible to change the state of the discrete system, even more – transfer the discrete system from the given state to the desired condition. This means that the integral discrete system in question is controlled and in this situation naturally a question arises - is it possible to transfer a discrete system from the given state to the desired condition, let's say with minimal cost or in minimum time. Generally, to answer these questions, it is necessary to set the optimal control problem of integrated discrete model and also solve it. As the control parameters we chose γ_1, γ_2, I_3 in our integrated discrete model. In our case they are constant, but in general they can get different discrete values. Let's consider them as control parameters and note:

$$u_1^i = \gamma_1^i, u_2^i = \gamma_2^i, u_3^i = I_3^i. \tag{11}$$

or, if we use vector marking then (11) will be shown as:

$$U^i = (u_1^i, u_2^i, u_3^i) = (\gamma_1^i, \gamma_2^i, I_3^i) \tag{12}$$

where we call $i = 0, 1, 2, ..., m-1$; $U^0, U^1, ..., U^{m-1}$, discrete control.

Because information warfare ending is essential for us, which is expressed in "useful" information of the first opposing side (8) meeting with the conditions of (10) in time and also the "useful" information of the second opposing side (9) meeting with the conditions of (10) in time. Discrete system condition can be described in these two sizes and depicted in two-dimensional phase plane.

$$N^i = (N_1^i, N_2^i) \tag{13}$$

Let's describe the transfer of a discrete system from one state to another by means of control with general ratios.

$$N^{i+1} = F^i(N^i, U^i). \tag{14}$$

where $i = 0, 1, 2, ...m-1$,

$$N^0 = (N_{10}^0, N_{20}^0), \tag{15}$$

F^i is a vector function

$$F^i = (f_1^i, f_2^i) \tag{16}$$

Where, for every i, f_1^i and f_2^i we get new forms. In our case, ratios existing in system (6) and initial values given in (7) unclearly take part in ratios (14). Ratios (14) and (16) determine the discrete control process. We consider as an admissible control such $U^i = (u_1^i, u_2^i, u_3^i) = (\gamma_1^i, \gamma_2^i, I_3^i)$ $i = 0, 1, 2, ..., m-1$ control, coordinates of which are not negative.

$$U^i = \left(u_1^i; u_2^i; u_3^i\right) \in U_+ \equiv (u_1 > 0; u_2 > 0; u_3 > 0), \tag{17}$$

And there exists i^*, i^{**}, $d = \max(i^*, i^{**})$ - the greatest among i^*, i^{**} for which the following is fulfilled:

$$N_1^{i^*+l} \leq 0, \;\; l = 1, 2, ..., d - i^* + 1, \;\; \text{and} \;\; N_1^{i^*+l} \leq 0, \;\; l = 1, 2, ..., d - i^* + 1, \tag{18}$$

$$N_2^{i^{**}-k} > 0, \;\; k = 0, 1, 2, ..., i^{**} \;\; \text{and} \;\; N_2^{i^{**}+l} \leq 0, \;\; l = 1, 2, ..., d - i^{**} + 1. \tag{19}$$

Let's denote the set of permissible controls by $D(U)$. Let's introduce in functions $f_0^i(N, U)$, $i = 0, 1, ..., m-1$. With the help of these functions we will construct a discrete process control criterion:

$$J(U) = f_0^0\left(N^0, U^0\right) + f_0^1\left(N^1, U^1\right) + ... + f_0^d\left(N^d, U^d\right) = \sum_{i=0}^{d} f_0^i\left(N^i, U^i\right). \tag{20}$$

Where in (20) $U = \left\{U^0, U^1, ..., U^d\right\})$ is a permissible discrete control.

Let's determine the discrete optimal control of information war's integrated discrete system as follows: of the initial state discrete control system (5)-(14), (16) optimal discrete control $U_* = \left\{U_*^0, U_*^1, ..., U_*^d\right\}$ is the name of the admissible control (17)-(19), for which functional (20) takes maximum (minimum) value:

$$J(U_*) = \sup_{U \in D(U)} J(U)\,, \left(J(U_*) = \inf_{U \in D(U)} J(U)\right). \tag{21}$$

Optimal control problem of information war's integrated discrete process (21), differs from the basic optimal control problem of discrete processes (14)-(19), which is given in [9]. In particular, on the one hand, discrete process in the basic optimal control problem is considered in all m knots, when discrete process of optimal control problem (21) (14)-(19) might end much earlier – at the $d + 1 \leq m$ knot, which, as a rule, is not fixed; On the other hand, in problem (21), (14)-(19) system state coordinates can be separately considered and their crossing on desired values get fixed at different discrete moments of time. Working out new solution methods for optimal control (21), (14)-(19) problem is conditioned by its features. The continuous analogue of optimal control (21), (14)-(19) problem for case $t^* = t^{**}$ was considered in [10] and for $t^* \neq t^{**}$ in [11;12]. Based on its specifics, optimal control (21), (14)-(19) problem sometimes is called optimal control Chilker type problem.

7. Conclusion

The present work shows the discrete models of information warfare. In particular, the Samarskiy-Mikhailov discrete model (1), (3) is constructed, which represents the continuous model analogue of the same title. Computer realization of model (1), (3) is given. Besides, the discrete model of information flow (4), (5) with taking into account the limitations of information technology levels is constructed and also its computer realization is given. It is mentioned, that these two types of discrete models are based on different approaches of modeling: in the first case only information receiver (adept) variability is considered and information flows are not clearly shown. In the second case information flow variability is considered, but following activity of information receivers, who were influenced by these flows are not presented. In order to overcome the vulnerability existing in these approaches an integrated discrete model of information warfare (6), (7) is constructed, which is studied via computer experiments. In integrated discrete model (6), (7) control parameters are picked out and corresponding discrete process manageability is established. Optimal control Chilker type problem of information warfare's integrated discrete process is given and its specificity towards general optimal problem of the discrete systems is mentioned.

The present work is created with the help of Shota Rustaveli National Science Funding of Georgia grant YS17_78. I would like to express my gratitude to the founder of Information warfare's informational flow mathematical modeling, Professor Temur Chilachava, with whom scientific discussions contributed to construct continuous and discrete integrated models of information warfare. I would like to thank the administration of Sokhumi State University, under the direction of the Rector, who created the conditions for the fruitful scientific work at the university.

References

[1] A.A. Samarskiy, A.P. Mikhailov, *Mathematical Modeling: Ideas. Methods.* Examples. 2nd ed. Correction. - M. FIZMATLIT, 2005, 320 pp.

[2] T. Chilachava, N. Kereselidze,*Mathematical model of information warfare* (Georgian), Georgian Electronic Scientific Journal: Computer Science and Telecommunications, **2010**, 1(24), 78-105

[3] T. Chilachava, N. Kereselidze, *Non-preventive continuous linear mathematical model of information warfare*, Sokhumi State University Proceedings, Mathematics and Computer Sciences, **7**, 7 (2009), 91-112

[4] T. Chilachava, N. Kereselidze, *Continuous linear mathematical model of preventive information warfare*, Sokhumi State University Proceedings, Mathematics and Computer Sciences **7**, 7 (2009), 113-141

[5] N. Kereselidze, *Mathematical model of information warfare taking into account the capabilities of the information technologies of the opposing sides* (Russian), Transactions II The International Technical Conference dedicated to the 90th anniversary of the Georgian Technical University "Basic Paradigms in Science and Technology", 21st Century, Tbilisi, Georgia, September 19-21, 2012, Publishing House "Technical University", Tbilisi, (2012), 188-190

[6] N. Kereselidze, *Mathematical model of information confrontation taking into account the possibilities of Information Technologies of the parties* (Russian), Proceedings of the XX International Conference Problems of Security Management of Complex Systems, Moscow, December 2012, art. 175-178

[7] N. Kereselidze, *Mathematical and Computer Models of Information Warfare*, Monograph (Georgian), ISBN 978-9941-0-9617-4 (PDF), 2017
http://kereseli.besaba.com/Math_and_comp_models_IW_monograph_2017.pdf

[8] N. Kereselidze, *Combined continuous nonlinear mathematical and computer models of the information warfare*, International journal of circuits, systems and signal processing, **12** (2018), 220-228

[9] V.G. Boltyanskiy, *Optimal Control of Discrete Systems* (Russian), publishing house "Science", Moscow, 1973

[10] T. Chilachava, N. Kereselidze, *Optimizing problem of mathematical model of preventive Information warfare*. Information and Computer Technologies – Theory and Practice: Proceedings of the International Scientific Conference ICTMC-2010 Devoted to the 80th Anniversary of I.V. Prangishvili, Nova Science Publishers, **55** (2011), 525-529

[11] N. Kereselidze, *An optimal control problem in mathematical and computer models of the information warfare*, Selected Contributions International Conference on Differential & Difference Equations and Applications ICDDEA, Amadora, Portugal, May 2015, Springer International Publishing, Springer Proceedings in Mathematics & Statistics book series (PROMS, **164**) (2016), 303-311

[12] N. Kereselidze, *Mathematical and computer models of the ChilKer-type in the information warfare* (Russian), Journal of Information Wars, Publisher Center for Security Studies of the Russian Academy of Sciences, **2**, 38 (2016), 18-25