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Linear Evolution of IGW Structures in the Ionospheri Plasma at Interaction with Shear Flows

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Theoretical explanation intensification of low frequency (LF) internal gravity waves (IGW) is presented. The method used is based on generalizing results on shear flow phenomena from the hydrodynamics community. In the 1990s, it was realized that fluctuation modes of spectrally stable nonuniform sheared flows are non-normal. That is, the linear operators of the flows modal analysis are non-normal and the corresponding eigenmodes are not orthogonal. The non-normality results in linear transient growth with bursts of the perturbations and the mode coupling, which causes the amplification of LF IG waves shear flow driven ionospheric plasma and generation of the higher frequency oscillations. Transient growth substantially exceeds the growth of the classical dissipative trapped-particle instability of the system.

Keywords: Gravity waves, self organisation into vortical structures.

AMS Subject Classification: 35Q35, 37K10, 37K40.

1. Introduction

Internal gravity waves (eddies) are important in their own right as major components of the total circulation. They are also major transporters of energy and momentum. For a medium to propagate a disturbance as a wave there must be a restoring "force", and in the higher atmospheric levels, this arises, primarily, from two sources: conservation of potential temperature in the presence of positive static stability and from the conservation of potential vorticity in the presence of a mean gradient of potential vorticity. The latter leads to what are known as Rossby waves. The former leads to internal gravity waves (and surface gravity waves as

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well). Internal gravity waves are simpler to understand and clearly manifest the various ways in which waves interact with the beckground shear flow. (Holton, 1983, Hines, 2011).

The energy transfer from the Earth's lithosphere to the atmosphere and ionosphere is a fundamental problem of geophysics and applied research. Internal gravity waves (IGWs) play an important role in such a process. For example, IGWs, propagating upward from the Earth's surface to the upper atmosphere and ionosphere, are able to carry a large amount of energy and momentum. These waves are crucial in atmosphere convection, generation of atmospheric turbulence and may affect global circulation. IGWs and related nonlinear structures are widely observed in the upper and lower layers of the atmosphere as well as in the lower ionosphere (Onishcenko et al, 2014, Garret, 1968)

Nonmodal approach correctly describes transient exchange of energy between basic shear flow and perturbations. The energy transfer channel is resonant by nature and leads to energy exchange between different wave modes (chagelishvili et al, 1996; Gogoberidze et al, 2004). The mutual transformation of the ULF electromagneic Rossby type waves is studied numerically and analytically in (Aburjania et al, 2006; Aburjania, 2006) in detail. The mutual transformation occurs at small shear rates if the dispersion curves of the wave branches have pieces near one another. Other possibility of energy transfer channel is nonresonant vortex and wave mode characteristic times are significantly different and nonsymmetric a vortex mode is able to generate a wave mode but not vice versa. This channel leads to energy exchange between vortex and wave modes, as well as between different wave modes. We concentrate on this channel of mode coupling because it is important at high shear rates.

2. Generation and intensification of IGW at linear stage of evolution

To study the linear stage of interaction of internal gravity waves with the local non-uniform zonal wind and geomagnetic field, the model of the medium and basic hydrodynamic equations for the lower ionosphere, is given in Aburjania et all 2013.

On the basis of non-modal approximation, shear flows can become unstable transiently till the condition of the strong relationship between the shear flows and wave perturbations is satisfied (Chagelishvili et al., 1996), e. i. the perturbation falls into amplification region in the wave number space. Leaving this region, e.i. when the perturbation passes to the damping region in the wave vector space, it returns an energy to the shear flow (Aburjania et al., 2010). The experimental and observation data shows the same (Gossard and Hooke, 1975).

Non-uniform zonal wind or shear flow can generate and/or intensify the internal gravity waves in the ionosphere and provoke transient growth of amplitude, i.e. transient transport the medium into an unstable state. In the next subsection we confirm this view by using a different, more self-consistent method for the shear flow.

According the above discussions, further analysis of the features of magnetized IGW wave at the linear stage in the ionosphere should be conducted in accordance with a non-modal approach. Considering the initial model equations into moving coordinate system, where the coordinates become time dependent, the coefficients of the initial system of linear equations will obtain a temporal dependence. Such mathematical transformations replace this spatial non-uniform property into temporal one. Thus, the initial-boundary problem is reduced to the initial problem of Cauchy type. Hence, the coefficients of are independent of spatial variables, the Fourier transformation of these equations with respect to spatial variables x_1, z_1 is already possible without any local approximation, the temporal evolution of these spatial Fourier harmonics (SFH) are considered independently (Chargazia et al, 2018):

$$\frac{\partial V_x}{\partial \tau} = -SV_z + k_x P - [b_0 + \nu k^2(\tau)]V_x, \tag{1}$$

$$\frac{\partial V_z}{\partial \tau} = k_z(\tau)P - \rho - [b_y + \nu k^2(\tau)]V_z, \tag{2}$$

$$\frac{\partial \rho}{\partial \tau} = V_z, \tag{3}$$

$$k_x V_x + k_z(\tau)V_z = 0. \tag{4}$$

Here, $\mathbf{V}_0(z) = v_0(z)e_x = A \cdot z \cdot e_x$ is a background of a plane zonal shear flow (wind) velocity, which is non-uniform along the vertical, $\mathbf{V} = \mathbf{V}_0(z) + \mathbf{V}(x, z, t)\,, \rho = \rho_0(z) + \rho(x, z, t)$,$P = P_0(z) + P(x, z, t)$,$v_0^{'}(z) = dv_0(z)/dz \omega_g = (g/H)^{1/2} > 0$ is frequency of Brunt-Vaisala for stably stratified incompressible isothermal atmosphere; $K^2 = k_x^2 + k_z^2 + 1/(4H^2)$, $K_1^2 = K_2^2 - ik_z/H$, $K_2^2 = k_x^2 + k_z^2 - 1/(4H^2)$,$(x, z) \Rightarrow (x_1, z_1)/H$, $S \Rightarrow A/\omega_g$,$k_{x,z} \Rightarrow k_{x_1,z_1}H$;$k_z = k_z(0) - k_x S\tau$,$k^2(\tau) = (k_x^2 + k_z^2(\tau))$, $\nu \Rightarrow \nu/\omega_g H^2$, $b_0 \Rightarrow (\sigma_P B_0^2)/(\rho_0 \omega_g)$, $b_y \Rightarrow (\sigma_P B_y^2)/(\rho_0 \omega_g)$.

Closed system of equations (1) - (4) describes the linear interaction of IGW with a shear flow and the evolution of the generated disturbances in the dissipative ionosphere medium. We note once again that after these transformations the wave vector $k(k_x, k_z(\tau))$ of the perturbation became dependent on time: $k_z(\tau) = k_z(0) - k_x S \cdot \tau$; $k^2(\tau) = (k_x^2 + k_z^2(\tau))$. Variation of the wave vector according to time (i.e. splitting of the disturbances' scales in the linear stage) leads to significant interaction in the medium even of such perturbations, the characteristic scale of which are very different from each other at the initial time (Aburjania et al., 2006).

On the basis of (1) - (4) an equation of energy transfer of the considered wave structures can be obtained, which gives possibility to identify the pattern of energy density variation with time:

$$\frac{dE(\tau)}{d\tau} = -\frac{S}{2}\left(V_x^*(\tau) \cdot V_z(\tau) + V_x(\tau) \cdot V_z^*(\tau)\right) - b_1(\tau)\left|V_x\right|^2 - b_2(\tau)\left|V_z\right|^2, \tag{5}$$

Here the asterisk denotes the complex conjugate values of the indignations, $b_1(\tau) = b_0 + \nu k^2(\tau)$, $\quad b_2(\tau) = b_y + \nu k^2(\tau)$and the density of the total dimensionless energy of the wave perturbations $E(\tau)$ in the wave number space is given by:

$$E[k(\tau)] = \frac{1}{2}\left(\left|V_x\right|^2 + \left|V_z\right|^2 + \left|\rho\right|^2\right). \tag{6}$$

It's obvious that the transient evolution of wave energy structures in the ionosphere is due to the shear flow($S \neq 0$, $A \neq 0$), dissipative processes - induction decay ($b_0 \neq 0$, $b_y \neq 0$) and viscosity ($\nu \neq 0$). In the absence of shear flow ($S = 0$, $A = 0$), and dissipative processes ($\nu = 0$, $\sigma_P = 0$), the energy of the considered wave disturbances in the ionosphere conserves $dE(\tau)/d\tau = 0$. The total energy density of the perturbations (6) consists of two parts: $E[k] = E_k + E_t$, where the first term is the kinetic energy of perturbation$E_k = \left(\left||V_x|^2 + |V_z|^2\right|\right)/2$, and the second - thermobaric energy $E_t = |\rho|^2/2$, stipulated due to the elasticity of perturbations.

3. Linear spectrum of the perturbations

The dispersion equation of our system may be obtained in the shearless limit S=0 using the full Fourier expansion of the variables, including time (Horton et al, 2009). Although the roots of the dispersion equation obtained in the shearless limit do not adequately describe the mode behavior in the shear case, we use this limit to understand the basic spectrum of the considered system. Hence, using Fourier expansion of the field vector we derive for the shearless limit the second order dispersion relation:

$$(\omega - k_x v_0)^2 - \frac{k_x^2}{K^2}\omega_g^2 + i\frac{(\omega - k_x v_0)}{K^2}\left[k_x^2\left(\frac{\sigma_P B_{0y}^2}{\rho_0} + \nu K_1^2\right) - \left(k_z^2 + \frac{1}{4H^2}\right)\left(\frac{\sigma_P B_0^2}{\rho_0} + \nu K_1^2\right)\right] = 0. \qquad (7)$$

Here, we introduce the notation: $\omega_g = (g/H)^{1/2} > 0$ is frequency of Brunt-Vaisala for stably stratified incompressible isothermal atmosphere; $K^2 = k_x^2 + k_z^2 + 1/(4H^2)$, $K_1^2 = K_2^2 - ik_z/H$, $K_2^2 = k_x^2 + k_z^2 - 1/(4H^2)$.

This second order dispersion equation describes two different modes of perturbations: two low frequency modes of IG waves. IG fluctuations are dispersive with ω dependent on K_x. This fact is very important for mode coupling since K_x is time dependent in nonuniform flow, which, in turn, makes ω also time dependent. The dispersion equation is solved numerically for the parameters taking S=0 and the real parts of the dispersive curves, respectively, are plotted in Fig. 1.The plots show that the magnitude of the frequencies of IGW are close at some critical point. Consequently, the IGW's are linearly coupled solely to each other at sizeable shear flow rates.

Figure 1 shows that maximum values of frequency for the least stable IGW modes are achieved at $K_x/K_z \sim 1$. Thus, and the trapped-particle instability has no significant influence on the dynamical phenomena.

4. Transient growth and mode coupling

Spectral Fourier harmonics dynamics are studied by numerically solving the three complex time evolution equations (1)-(4). Separation of the fields into the real and

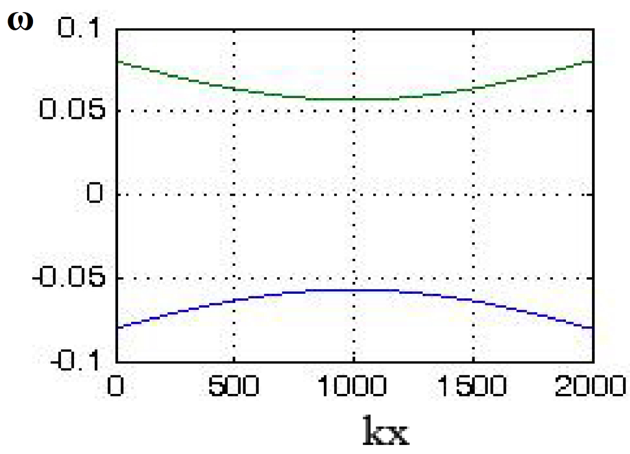

Figure 1. Dispersion Curves

imaginary parts is made in the following way (Aburjania et al, 2006):

$$V_x = V_{x1} + iV_{x2}, \quad V_z = V_{z1} + iV_{z2}, \quad \rho = \rho_1 + i\rho_2, \tag{8}$$

For each Fourier harmonics we will have:

$$\frac{\partial V_{x1}}{\partial \tau} = -SV_{z1} + k_x P - [b_0 + \nu k^2(\tau)]V_{x1}, \tag{9}$$

$$\frac{\partial V_{z1}}{\partial \tau} = k_z(\tau)P - \rho_1 - [b_y + \nu k^2(\tau)]V_{z1}, \tag{10}$$

$$\frac{\partial \rho_1}{\partial \tau} = V_{z1}, \tag{11}$$

$$k_x V_{x1} + k_z(\tau)V_{z1} = 0. \tag{12}$$

Equations (9)–(12), together with the appropriate initial values, pose the initial value problem describing the dynamics of a perturbation SFH. The character of the dynamics depends on inittial SFH mode impose into the equations: pure IGW SFH (Spatial Fourier Harmonics of IGW) or a mixture of these wave SFHs. Let us concentrate on the linear dynamics when we initially insert in Eqs. (9)–(12) a SFH nearly corresponding to a IGW perturbation with wavenumbers satisfying the condition $K_x(0)/K_z >> 1$. The numerical simulations are performed using the MATLAB numerical ordinary differential equation solver. Note that the action of the flow shear on the dynamics of IGW SFH at wavenumbers $K_x(0)/K_z >> 1$is negligible.

The simulations reveal a novel linear effect - the excitation of higher frequency fluctuations - that accompanies the linear evolution of IGW mode perturbations in the sheaar flow. The evolution of the initial IGW SFH according to the dynamic

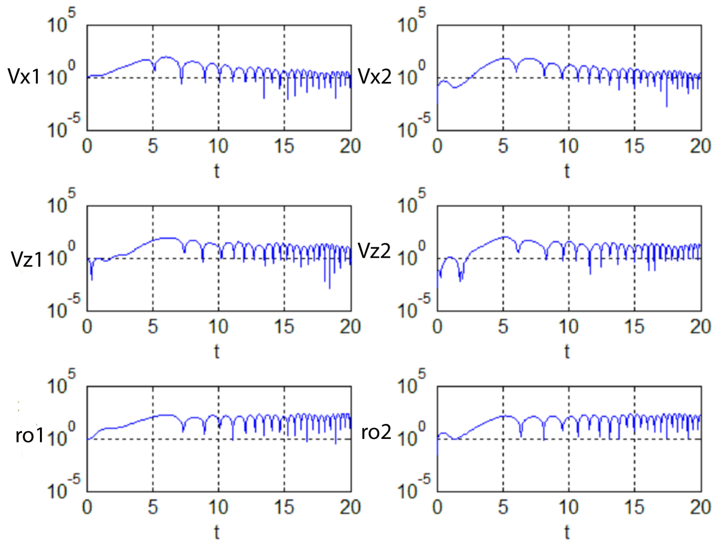

Figure 2. The evolution of a single SFH

equations (1)-(4) for the ionospheric parameters is presented in Fig. 2. Recall that K_x changes in time according to Eq. 7: the shear flow sweeps K_x to low values and then back to high values but with negative K_x/K_z. While $K_x/K_z >> 1$, over time τ, $0 < \tau < \tau^* = k_z(0)/(Sk_x)$, the energy density of IGW increases monotonically and reaches its maximum value (exceeding its initial value by an order) at the time $\tau = \tau^*$- the IGW SFH undergoes substantial transient growth without any oscillations and the magnetic fluctuations are small. Significant magnetic field fluctuations appear when $K_x/K_z = 1$. While $K_x/K_z = 1 < 0$, the IGW SFH generates the related SFH higer frequency wave modes like inertial modes through the second channel of the mode coupling. This generation of higer frequency wave modes is especially prominent, where significantly higher frequency oscillations of all the fields are clearly seen at times when $K_x/K_z = 1 < 0$. A substantial transient burst of the perturbations energy, density of fluctuations is evident and an appearance of high frequency fluctuations. Further, at $\tau^* < \tau < \infty$ the energy density begins to decrease (when$k_z(\tau) < 0$), and monotonically returns to its initial approximately constant value. In other words, in the early stages of evolution, temporarily, when $k_z(\tau) > 0$and IGW perturbations are in the intensification region in wave-number space, the disturbances draw energy from the shear flow and increase own amplitude and energy by an order during the period of time $0 < \tau < \tau^* = k_z(0)/(Sk_x)$ (Horton et al, 2009).

Then (if the nonlinear processes and the self-organization of the wave structures are not turned on), when $k_z(\tau) < 0$, IGW perturbation enters the damping region in wave number space and the perturbation returns energy back to the shear flow over time $\tau^* < \tau < \infty$ and so on. Such transient redistribution of energy in the medium with the shear flow is due to the fact that the wave vector of the perturbation becomes a function of time $k = k(\tau)$, i.e. disturbances' scale splitting takes place. The structures of comparable scales effectively interact and redistribute free energy between them. Taking into account the induction and viscous damping the perturbation's energy reduction in the time interval $\tau^* < \tau < \infty$ is more intensive than that shown on fig. 1, the decay curve in the region $\tau^* < \tau < \infty$

becomes more asymmetric (right-hand side curve becomes steeper), and part of the energy of the shear flow passes to the medium in the form of heat.

Thus, even in a stable stratified ionosphere ($\omega_g^2 > 0$), temporarily, during the time interval $0 < t^* \approx 100/(\omega_g) \sim 5 \cdot 10^3 s \sim 1.5$ hour IGW-intensively draws energy from the shear flow and increases own energy and amplitude by an order. Accordingly, the wave activity will intensify in the given region of the ionosphere due to the shear flow (inhomogeneous wind) energy.

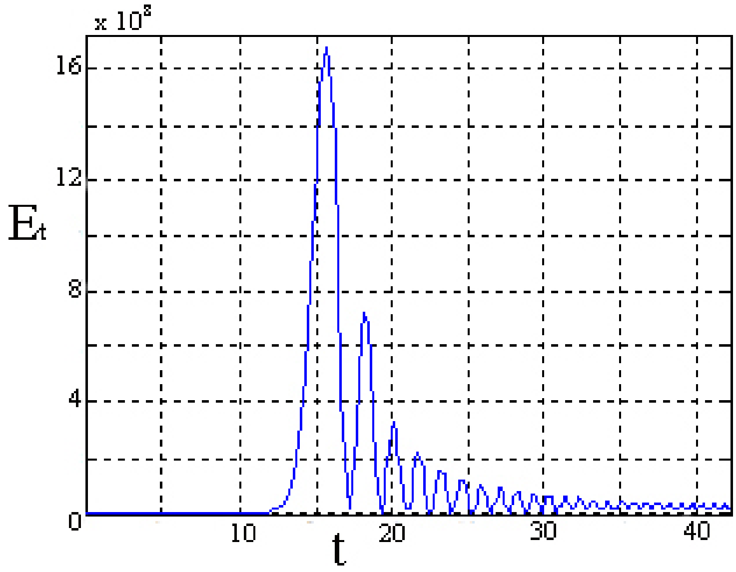

Figure 3. The evolution of total energy of SFH perturbations

Figure 3 shows the related dynamics of the total energy. It indicates a substantial transient burst of the electron thermal energy of fluctuations and an appearance of Alfvénic like fluctuations.

5. Conclusions

Linear amplification of IGW perturbation is not exponential as in the case of the AGW in the inverse-unstably stratified ($\omega_g < 0$, when IGW cannot be generated) atmosphere (Aburjania, 2010)), but in algebraic-power law manner. Intensification of IGW is possible temporarily, for certain values of environmental parameters, shear and waves, which form an unusual way of heating of the shear flow in the ionosphere: the waves draw their energy from the shear flow through a linear drift of SFH in the wave number space (fragmentation of disturbances due to scale) and pump energy into the region of small-scale perturbations, i.e. in the damping region. Finally, the dissipative processes convert this energy into heat. The process is permanent and can lead to strong heating of the medium. Intensity of heating depends on the level of the initial disturbance and the parameters of the shear flow.

A remarkable feature of the shear flow is the dependence of the frequency and wave number of perturbations on time $k_z = k_z(0) - k_x S\tau$, $k(\tau) = (k_x^2 + k_z^2(\tau))^{1/2}$. In particular, frequency and wave number transient growth leads to a reduction of scales of the wave disturbances due to time in the linear regime and, accordingly, to energy transfer into a short scale region - the dissipation region. On the other hand, a significant change in the frequency range of the generated disturbances stipulates in the environment the formation of a broad range of spectral lines of the perturbations, which is linked to the linear interactions and not to the strong turbulent effects. Moreover, amplification of the SFH perturbation and broadening of wave modes' spectra occur in a limited period of time (transient interval), yet satisfied the relevant conditions of amplification and a strong enough interaction between the modes.

It should be emphasized that the detection of the mechanism of the intensification and broadening of the spectrum of perturbations became possible within the non-modal mathematical analysis (these processes are overlooked by more traditional modal approach). Thus, non-modal approach, taking into account the nonorthogonality of the eigenfunctions of the linear wave dynamics, proved to be more appropriate mathematical language to study the linear stage of the wave processes in shear flows.

IGW is characterized by an exponential growth of the amplitude of the perturbed velocity at the vertical propagation in an environment with exponentially decaying vertical equilibrium density and pressure (Hines, Gossard, Hook, 1975). According to observational data, IGW disturbances manifest themselves in a wide range of heights - from the troposphere to the upper ionosphere heights $z \leq 600$km (Gossard and Hook, 1975). At ionospheric altitudes (above 90 km) the conductive medium strongly impacts on the IGW, causing its remarkable damping due to local Pedersen currents.

IGW structures are eigen degrees of freedom of the ionospheric resonator. Therefore, influence of external sources on the ionosphere above or below (magnetic storms, earthquakes, artificial explosions, etc.) will excite these modes (or intensified) in the first. For a certain type of pulsed energy source the nonlinear solitary vortical structures will be generated (Aburdjania, 2006), which is confirmed by experimental observations (Sundkvist et al., 2005). Thus, these wave structures can also be the ionospheric response to natural and artificial activity.

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