Theory and Applications of Categories, Vol. 5, No. 7, 1999, pp. 163–175.

DOUBLE CATEGORIES, 2-CATEGORIES, THIN STRUCTURES AND CONNECTIONS

RONALD BROWN AND GHAFAR H. MOSA

Transmitted by Robert Paré

Abstract. The main result is that two possible structures which may be imposed on an edge symmetric double category, namely a connection pair and a thin structure, are equivalent. A full proof is also given of the theorem of Spencer, that the category of small 2-categories is equivalent to the category of edge symmetric double categories with thin structure.

1. Introduction

The main result is the equivalence of two structures which may be available on *edge symmetric* double categories, i.e. those in which the horizontal and vertical edge categories coincide. The structures are a *connection pair*, and a *thin structure*. The first notion was introduced in [10] and the second in [16]. Thin structures are important for applications, particularly in relation to the notion of 'commutative cube', while a connection pair is easy to generalise to higher dimensions and to related structures.

We also note that an edge symmetric double category with connection pair satisfies the general associativity and commutativity conditions of Dawson and Paré [11]. This allows for the computation of arbitrary compositions in such a double category and in particular justifies a number of our calculations.

It is stated in [16] that the categories of these double categories with thin structure and of 2-categories are equivalent, but for the proof he refers only to similar results in the literature, since his main aim is the homotopy applications. We give a full proof here (section 5) since it fits nicely with our earlier results, the techniques will be used in other situations elsewhere, and it should be useful to workers in higher dimensional algebra.

There is an equivalence between 'cubical ∞-groupoids' with connections and 'globular ∞-groupoids' – this follows from the main results of [6, 7], which give equivalences with crossed complexes. In the category case, the above equivalence for 2-categories has been extended to 3-categories in the thesis of Al-Agl [1]. It is conjectured that there is an equivalence of categories between globular ∞-categories and cubical ∞-categories with connections, but the proof presents difficulties. There is an analogous result in [2], but it uses a much broader structure for the description of the cubical ∞-categories.

An advantage of double categories with connection is the relatively easy description of

Received by the editors 1999 February 4 and, in revised form, 1999 April 30.
Published on 1999 May 17.
1991 Mathematics Subject Classification: 18D05.
Key words and phrases: Double category, 2-category, thin structure, connection.

a monoidal closed structure yielding lax natural transformations. Indeed, this is given for what are called ω-categories in [8], which includes the general version of all the structure we have given for double categories with connection, except for the connection Γ'. However, this lack is easily remedied, and this gives the rules stated in [1] for what are there called ω-categories. Of course, under the equivalence given above, the tensor product of double categories with connection corresponds to the Gray tensor product of 2-categories, except that, from the point of view of ω-categories, the tensor product of double categories with connections is more naturally a quadruple category with connections – this has to be quotiented to give a double category. It is hoped to pursue these matters elsewhere.

We will develop in [9] the equivalence given in [15] between edge symmetric double algebroids with thin structure and certain crossed modules of algebroids – this is an analogue of the equivalence between edge symmetric double groupoids with connection and crossed modules [10, 6].

For more discussion on matters of higher dimensional group theory see the web article [4]. We would like to thank Rafael Sivera for help with the layout of section 4 and the referee for helpful comments.

2. Double categories

A double category is a category object internal to the category of small categories. It may also be represented as consisting of four category structures

$$(D_2, D_1^V, \partial_1^0, \partial_1^1, \circ_1, \varepsilon_1) \quad (D_2, D_1^H, \partial_2^0, \partial_2^1, \circ_2, \varepsilon_2)$$

$$(D_1^V, D_0, \partial_2^0, \partial_2^1, ., \varepsilon) \quad (D_1^H, D_0, \partial_1^0, \partial_1^1, ., \varepsilon)$$

as partially shown in the diagram

$$\begin{tikzcd} D_2 \arrow[r, shift left] \arrow[r, shift right] \arrow[d, shift left] \arrow[d, shift right] & D_1^V \arrow[d, shift left] \arrow[d, shift right] \\ D_1^H \arrow[r, shift left] \arrow[r, shift right] & D_0 \end{tikzcd} .$$

Here D_1^H, D_1^V are called the *horizontal* and *vertical edge categories*. The functions written ∂ are the source and target maps of the categories, and the ε denote the functions giving the identity elements.

In the final section we shall use 2-categories, which may be defined as double categories in which the vertical edge category is discrete. We shall initially be interested in *edge symmetric* double categories, i.e. those in which the horizontal and vertical edge categories coincide – these were called *special double categories* in [10]. In this case we write D_1 for $D_1^H = D_1^V$.

The horizontal composition is written $\circ_2$ and the vertical composition as $\circ_1$, in accordance with matrix conventions. It is convenient to use matrix notation for composition of squares. Thus if u, v satisfy $\partial_2^1 u = \partial_2^0 v$, we write

$$\begin{bmatrix} u & v \end{bmatrix} \quad \text{for} \quad u \circ_2 v$$

and if $\partial_1^1 u = \partial_1^0 w$, we write

$$\begin{bmatrix} u \\ w \end{bmatrix} \quad \text{for} \quad u \circ_1 w$$

The interchange law for double categories thus gives a unique composition for the matrix

$$\begin{pmatrix} u & v \\ w & z \end{pmatrix}$$

namely

$$\begin{bmatrix} u & v \\ w & z \end{bmatrix} = (u \circ_2 v) \circ_1 (w \circ_2 z) = (u \circ_1 w) \circ_2 (v \circ_1 z),$$

whenever the compositions are all defined.

Two standard examples of edge symmetric double categories are the double categories $\boxdot C$, $\square\, C$ of squares and of commuting squares in a category C, with the obvious double category structures. It is convenient to write a square in $\boxdot C$ in the form

$$\begin{pmatrix} & a & \\ c & & b \\ & d & \end{pmatrix}$$

where a, b, c, d are arrows in C such that ab, cd are defined and have the same source and target.

In fact $\boxdot$ is right adjoint to the forgetful functor assigning to every (small) edge symmetric double category its edge category. The unit of this adjunction gives $\mathcal{D} \to \boxdot D_1$ which assigns to each element of D_2 its *shell*, namely its square of boundary edges.

3. Thin structure on a double category

The edge symmetric double categories we study will have another structure which we call a *thin* structure. This is defined to be a morphism of double categories

$$\Theta : \square D_1 \to \mathcal{D}$$

which is the identity on D_1 and D_0. The elements of D_2 lying in $\Theta(\square D_1)$ are called *thin elements*. The definition implies immediately that:

T1) *any commutative shell has a unique thin filler,*

T2) *any composition of thin squares is thin.*

There are some special thin squares for which we use a special notation, introduced in [16]. These have boundaries as follows and the notation is given underneath:

$$\begin{pmatrix} 1 & {1 \atop 1} & 1 \end{pmatrix} \quad \begin{pmatrix} a & {1 \atop 1} & a \end{pmatrix} \quad \begin{pmatrix} 1 & {b \atop b} & 1 \end{pmatrix} \quad \begin{pmatrix} a & {a \atop 1} & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & {1 \atop a} & a \end{pmatrix}$$

$$\square \qquad\quad \overline{\underline{\;\;}} \qquad\quad | \, | \qquad\quad \lrcorner \qquad\quad \ulcorner$$

The last two thin squares are called *connections* and are written also $\Gamma a, \Gamma' a$ respectively. We will develop this structure further. The second and third squares are of course $\varepsilon_2 a, \varepsilon_1 b$, while the first square is $\varepsilon_1\varepsilon_2 x$ for some $x \in D_0$.

The rules T1),T2) now have some surprising consequences – for example we have the following equations, which we give in both notations:

3.1. Proposition.

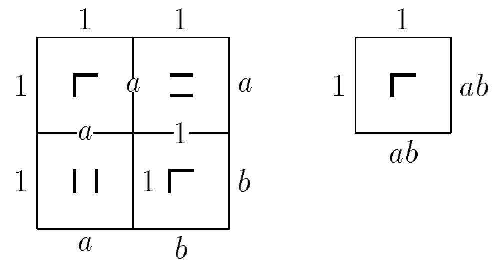
(i) $[\ulcorner \; \lrcorner] = |\,|$, $[\Gamma' a \; \Gamma a] = \varepsilon_1 a$;

(ii) $\begin{bmatrix} \ulcorner \\ \lrcorner \end{bmatrix} = \overline{\underline{\;\;}}$, $\begin{bmatrix} \Gamma' a \\ \Gamma a \end{bmatrix} = \varepsilon_2 a$;

(iii) $\begin{bmatrix} \ulcorner & \overline{\underline{\;\;}} \\ |\,| & \ulcorner \end{bmatrix} = \ulcorner$, $\begin{bmatrix} \Gamma' a & \varepsilon_2 a \\ \varepsilon_1 a & \Gamma' b \end{bmatrix} = \Gamma'(ab)$;

(iv) $\begin{bmatrix} \lrcorner & |\,| \\ \overline{\underline{\;\;}} & \lrcorner \end{bmatrix} = \lrcorner$, $\begin{bmatrix} \Gamma a & \varepsilon_1 b \\ \varepsilon_2 b & \Gamma b \end{bmatrix} = \Gamma(ab)$.

Proof. Note that for example the left hand and right hand sides of the third equation are abbreviations for



and similarly for the others. The arguments for the proofs are that any composite of thin element is thin, and so is determined by its shell. ∎

Finally in this section we note that the thin structure can be recovered from the connection pair Γ, Γ' as follows:

3.2. Proposition. *A thin structure Θ on $\mathcal{D}$ can be recovered from the associated connection pair by*

$$\Theta\begin{pmatrix} a & {c \atop b} & d \end{pmatrix} = (\varepsilon_2 a \circ_1 \Gamma' b) \circ_2 (\Gamma c \circ_1 \varepsilon_2 d) = (\varepsilon_1 c \circ_2 \Gamma' d) \circ_1 (\Gamma a \circ_2 \varepsilon_1 b). \qquad (*)$$

Proof. The proof is immediate from the fact that a thin square is determined by its shell, as long as that commutes. ∎

4. Connections on a double category

We now reverse the previous procedure and start with the connection pair, which we call simply a *connection*. The rules we impose are those given mainly by proposition 3.1.

A *connection* on an edge symmetric double category $\mathcal{D}$ is given by a pair of maps

$$\Gamma, \Gamma' : D_1 \to D_2$$

whose edges are given by the following diagrams for $a \in D_1$:

$$\Gamma(a) \quad = \quad \begin{tikzcd} \cdot \arrow[r, dash, "a"] \arrow[d, dash, "a"'] & \cdot \arrow[d, dash, "1"] \\ \cdot \arrow[r, dash, "1"'] & \cdot \end{tikzcd} \quad = \quad \boxed{\lrcorner} \quad = \quad \lrcorner$$

$$\Gamma'(a) \quad = \quad \begin{tikzcd} \cdot \arrow[r, dash, "1"] \arrow[d, dash, "1"'] & \cdot \arrow[d, dash, "a"] \\ \cdot \arrow[r, dash, "a"'] & \cdot \end{tikzcd} \quad = \quad \boxed{\ulcorner} \quad = \quad \ulcorner.$$

Their boundary conditions are those indicated by the graphical representation, i.e.

$$\partial_2^0 \Gamma(a) = \partial_1^0 \Gamma(a) = a \quad \text{and} \quad \partial_2^1 \Gamma(a) = \partial_1^1 \Gamma(a) = \varepsilon \partial^1 a \qquad \text{(CON 1.)}$$

$$\partial_2^1 \Gamma'(a) = \partial_1^1 \Gamma'(a) = a \quad \text{and} \quad \partial_2^0 \Gamma'(a) = \partial_1^0 \Gamma'(a) = \varepsilon \partial^0 a. \qquad \text{(CON}'\text{ 1.)}$$

We also require

$$\Gamma \varepsilon(x) = 1_x \qquad \text{(CON 2.)}$$

$$\Gamma' \varepsilon(x) = 1_x. \qquad \text{(CON}'\text{ 2.)}$$

The connections of the composition of two elements are given by the "transport laws":

$$\Gamma(ab) = \begin{bmatrix} \Gamma a & \varepsilon_1 b \\ \varepsilon_2 b & \Gamma b \end{bmatrix} \qquad \text{(CON 3.)}$$

$$\Gamma'(ab) = \begin{bmatrix} \Gamma' a & \varepsilon_2 a \\ \varepsilon_1 a & \Gamma' b \end{bmatrix}. \qquad \text{(CON}'\text{ 3.)}$$

The last condition is that they are "inverse" to each other in both directions, i.e.

$$\Gamma'(a) \circ_2 \Gamma(a) = \varepsilon_1(a) \qquad \text{(CON 4.)}$$

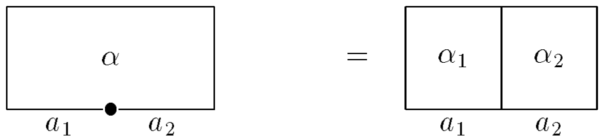
$$\Gamma'(a) \circ_1 \Gamma(a) = \varepsilon_2(a). \qquad \text{(CON}'\text{ 4.)}$$

The above laws should be compared with the results of proposition 3.1.

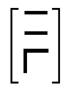
One immediate application of this notion uses a result of Dawson and Paré [11]. They are concerned with all possible computations of compositions in a double category. They show that there is a unique composition if a basic decomposition result holds:

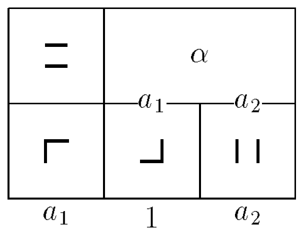
4.1. Assumption. [Dawson-Paré] *Suppose a square α has a decomposition of an edge a as $a = a_1 a_2$. Then α has a compatible decomposition $\alpha = \alpha_1 \circ_i \alpha_2$, i.e. such that α_j has edge a_j, $j = 1, 2$.*

As an example we give:



4.2. Proposition. *The assumption 4.1 holds in a double category with connection.*

Proof. The required decomposition can be obtained in the presence of a connection for example as shown below, in which we take $\alpha_1 =$  :



Similar decompositions may be obtained for the other possibilities. ■

The result of Dawson and Paré justifies the calculations we will carry out below.

We now show the more difficult result that a connection defines a thin structure. For double groupoids the proof may be traced back to work of Brown and Higgins [5]. Here we adopt a different approach.

4.3. Theorem. *If there is given a connection pair (Γ, Γ') on $\mathcal{D}$, then the functions $\Theta_1, \Theta_2 : \square D_1 \to D_2$ given by*

$$\Theta_1 \begin{pmatrix} & c & \\ a & & d \\ & b & \end{pmatrix} = (\varepsilon_1 c \circ_2 \Gamma' d) \circ_1 (\Gamma a \circ_2 \varepsilon_1 d), \tag{1}$$

$$\Theta_2 \begin{pmatrix} & c & \\ a & & d \\ & b & \end{pmatrix} = (\varepsilon_2 a \circ_1 \Gamma' b) \circ_2 (\Gamma c \circ_1 \varepsilon_2 b) \tag{2}$$

satisfy

(i) $\Theta_1 = \Theta_2$;

(ii) $\Theta_2(u \circ_1 w) = \Theta_2(u) \circ_1 \Theta_2(w)$;

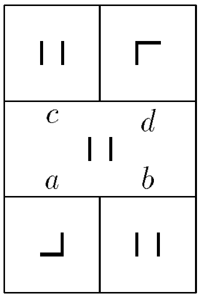
(iii) $\Theta_1(u \circ_2 v) = \Theta_1(u) \circ_2 \Theta_1(v)$;

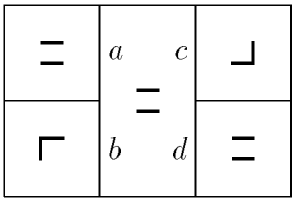
(iv) $\Theta_1\begin{pmatrix} & b & \\ 1 & & 1 \\ & b & \end{pmatrix} = \varepsilon_1(b), \Theta_1\begin{pmatrix} & 1 & \\ a & & a \\ & 1 & \end{pmatrix} = \varepsilon_2(a),$

$\Theta_1\begin{pmatrix} & a & \\ a & & 1 \\ & 1 & \end{pmatrix} = \Gamma(a), \Theta_1\begin{pmatrix} & 1 & \\ 1 & & b \\ & b & \end{pmatrix} = \Gamma'(b)$

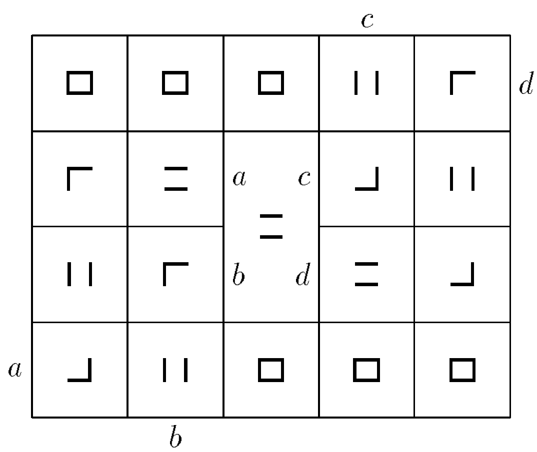
for all $u, v, w \in \square D_1$ such that $u \circ_1 w$, $u \circ_2 v$ are defined, and $a, b \in D_1$.

Proof. Let us first prove that $\Theta_1 = \Theta_2$. We can write

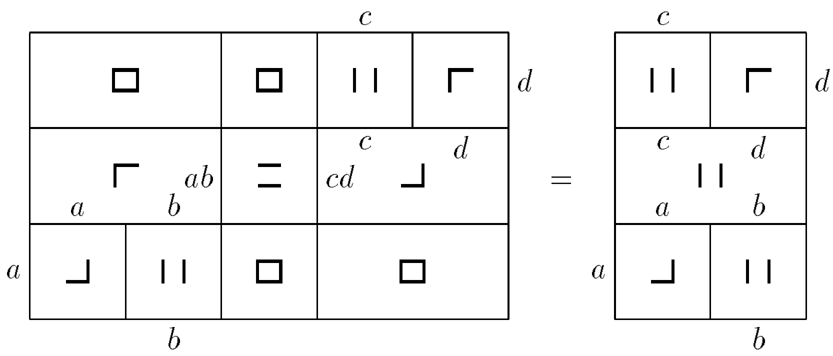
$\Theta_1\begin{pmatrix} & c & \\ a & & d \\ & b & \end{pmatrix} =$ 

$\Theta_2\begin{pmatrix} & c & \\ a & & d \\ & b & \end{pmatrix} =$ 

To prove $\Theta_1 = \Theta_2$ we construct a common subdivision. One that is appropriate for this case is

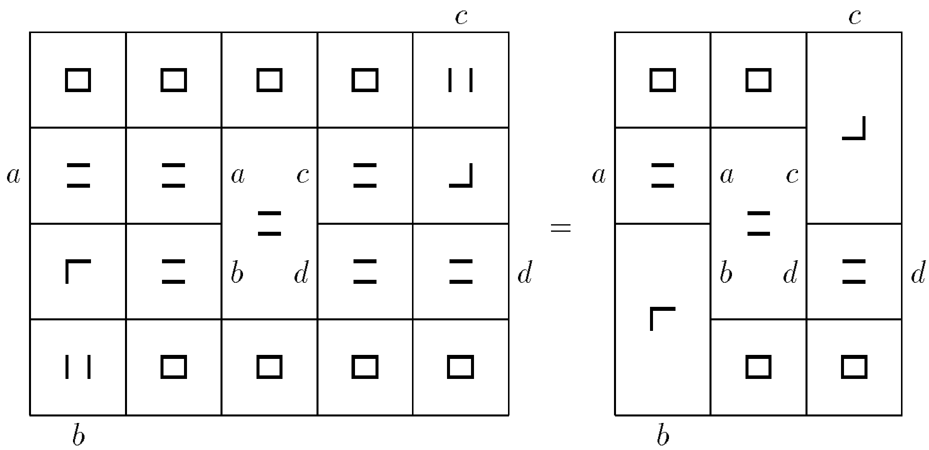


From this diagram, we may compose the second and third row using the transport law and then rearrange things, getting Θ_1 as indicated



$$= \Theta_1 \begin{pmatrix} a & \begin{smallmatrix} c \\ b \end{smallmatrix} & d \end{pmatrix}.$$

Similarly, operating in the bottom left and the top right corner, we get

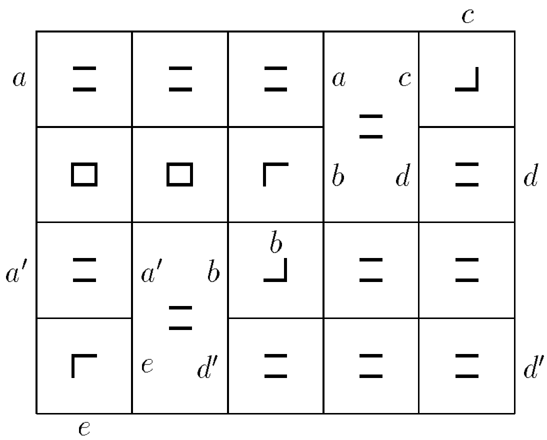


and this last diagram is clearly Θ_2.

We next prove (ii), that Θ_2 commutes with the vertical composition. So we want to prove

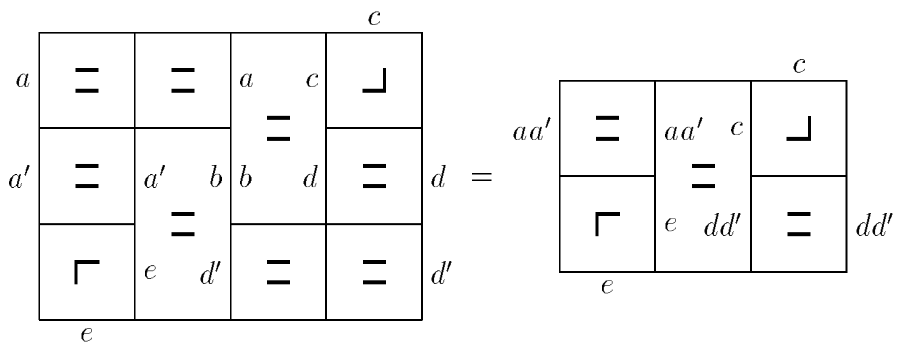
$$\Theta_2\left(a \; \begin{smallmatrix} c \\ b \end{smallmatrix} \; d \right) \circ_1 \Theta_2\left(a' \; \begin{smallmatrix} b \\ e \end{smallmatrix} \; d' \right) = \Theta_2\left(aa' \; \begin{smallmatrix} c \\ e \end{smallmatrix} \; dd' \right).$$

As before we compute a common subdivision in two ways. The common subdivision we choose is



If we compose the first two rows, they produce $\Theta_2\left(a \; \substack{c \\ b} \; d\right)$. Similarly, the two last rows give $\Theta_2\left(a' \; \substack{b \\ e} \; d'\right)$.

On the other hand, making some easy adjusts on the three middle rows, we get



which clearly is $\Theta_2\left(aa' \; \substack{c \\ e} \; dd'\right)$.

The proof of (iii), that Θ_1 commute with the horizontal composition is similar to the above and is left to the reader, as is the much simpler proof of (iv). ■

4.4. Corollary. *A thin structure and connection pair on an edge symmetric double category determine each other.*

For other applications of thin structures we refer to [16, 17, 3].

5. The equivalence of 2-categories and edge symmetric double categories with connection

It was observed by C. Ehresmann [12] that a 2-category gives rise in two ways to a double category. Following the lead given in [10] for double groupoids and crossed modules, it was observed in [16] that double categories with thin structure yield connections, and that the Ehresmann construction gives an equivalence between 2-categories and double categories with thin structure. This work has not been much exploited, perhaps because Spencer left the detailed proof to the reader, and so we give a proof here.

Recall that a 2-category may be defined as a double category $\mathcal{D} = (D, H, V, X)$ in which the vertical edge category V is discrete, i.e. consists only of identities. It follows that any double category $\mathcal{D}$ as above contains a 2-category $\gamma\mathcal{D} = (\gamma D, H, X, X)$ where γD consists of the squares of $\mathcal{D}$ whose vertical edges are identities.

In order to reconstruct an edge symmetric double category with connection from the 2-category it contains, we introduce the important *folding map*

$$\Phi : D \to \gamma D$$

given by $\Phi(u) = [\ \ulcorner \ u \ \lrcorner\ ]$. The description of the behaviour of Φ with respect to compositions requires the 'whiskering' operations of the category H on γD given as usual by

$${}^{a}u = |\,|_a \circ_2 u,\ \ u^b = u \circ_2 |\,|_b.$$

5.1. Proposition. *If $u, v, w \in D$ and $u \circ_1 v, u \circ_2 w$ are defined then*

$$\Phi(u \circ_1 v) = (\Phi u)^{\partial_2^1 v} \circ_1 {}^{\partial_2^0 u}(\Phi v), \tag{3}$$

$$\Phi(u \circ_2 w) = {}^{\partial_1^0 u}(\Phi w) \circ_1 (\Phi u)^{\partial_1^1 w}, \tag{4}$$

Proof. This consists of composing in two ways each of the diagrams:

$$\begin{pmatrix} \ulcorner & = & u & \lrcorner & |\,| \\ |\,| & \ulcorner & v & = & \lrcorner \end{pmatrix} \qquad \begin{pmatrix} \square & |\,| & \ulcorner & w & \lrcorner \\ \ulcorner & u & \lrcorner & |\,| & \square \end{pmatrix}$$

■

We also require:

5.2. Proposition. *If an element $u \in D$ is thin then $\Phi u = |\,|$.*

Proof. If u is thin, then so also is Φu, since it is a composition of thin elements. Since its vertical edges are constant, it follows that $\Phi u = |\,|$. ■

The converse of this proposition will be proved later.

Now suppose C is a 2-category with set of i-cells denoted C_i for $i = 0, 1, 2$. We construct an edge symmetric double category with thin structure λC whose objects and edge category are C_0 and C_1 respectively, and whose set of squares is given by quintuples

$$\left(u : c \,{a \atop d}\, b\right) \tag{5}$$

such that $a, b, c, d \in C_1$, $u \in C_2$ and $u : ab \Rightarrow cd$ in C. The boundary maps on squares are defined in the obvious way, and the thin squares are those of the form

$$\left(1 : c \,{a \atop d}\, b\right).$$

The compositions are defined by

$$\left(u : c \genfrac{}{}{0pt}{}{a}{d} b\right) \circ_1 \left(v : f \genfrac{}{}{0pt}{}{d}{g} e\right) = \left(u^e \circ_1 {}^c v : cf \genfrac{}{}{0pt}{}{a}{g} be\right), \tag{6}$$

$$\left(u : c \genfrac{}{}{0pt}{}{a}{d} b\right) \circ_2 \left(w : b \genfrac{}{}{0pt}{}{k}{h} l\right) = \left({}^a w \circ_1 u^h : c \genfrac{}{}{0pt}{}{ak}{dh} l\right). \tag{7}$$

It is straightforward to check that these rules give a double category with thin structure. The hardest part is the interchange law, and that follows from the interchange law for the 2-category C. We leave further details to the reader.

It is clear that if C is a 2-category, then there is a natural isomorphism of 2-categories $\gamma\lambda C \cong C$, in which the map on 2-cells is $\left(u : 1 \genfrac{}{}{0pt}{}{a}{b} 1\right) \mapsto u$. The difficult part of the proof is the isomorphism $\phi : \mathcal{D} \to \lambda\gamma\mathcal{D}$.

This is to be the identity on objects and edges and on squares is given by

$$u \mapsto \left(\Phi u : \partial_2^0 u \genfrac{}{}{0pt}{}{\partial_1^0 u}{\partial_1^1 u} \partial_2^1 u\right). \tag{8}$$

The composition rules for Φ imply immediately that ϕ is a morphism for the two compositions.

We now construct an inverse ψ to ϕ. Let $\left(v : c \genfrac{}{}{0pt}{}{a}{d} b\right)$ be an element of $\lambda\gamma D$, so that $v : ab \Rightarrow cd$. Let t, t' be the thin elements $\left(1 \genfrac{}{}{0pt}{}{a}{ab} b\right)$, $\left(c \genfrac{}{}{0pt}{}{cd}{d} 1\right)$ respectively. We define

$$\psi\left(v : c \genfrac{}{}{0pt}{}{a}{d} b\right) = \begin{pmatrix} t \\ v \\ t' \end{pmatrix}. \tag{9}$$

The proofs that $\psi\phi(u) = u, \phi\psi(v) = v$ are given in essence by the following diagrams:

$$\begin{pmatrix} \square & \mid\mid & \ulcorner \\ \ulcorner & u & \lrcorner \\ \lrcorner & \mid\mid & \square \end{pmatrix} \qquad \begin{pmatrix} \square & t & \lrcorner \\ \square & v & \square \\ \ulcorner & t' & \square \end{pmatrix}$$

where t, t' are as defined above, and since we can also write

$$t = (\square \ \mid\mid \ \ulcorner) = (\mid\mid \ \ulcorner),$$
$$t' = (\lrcorner \ \mid\mid \ \square) = (\lrcorner \ \mid\mid).$$

We also note that if $v = \mid\mid$ then the right hand side of (9) is thin. This proves that if Φu is thin, then so also is u. It follows that ϕ determines an isomorphism of double categories with thin structure $\mathcal{D} \to \lambda\gamma\mathcal{D}$.

The above maps ϕ, ψ are clearly natural. So we have proved the theorem of Spencer [16]:

5.3. Theorem. *The functor γ gives an equivalence of categories from edge symmetric double categories with thin structure to 2-categories.*

5.4. Remark. A generalisation of the folding map Φ is given in all dimensions in [1], following leads given in [15]. The problems not solved in dimensions greater than 3 are the formulation and proof of properties corresponding to those of proposition 5.1, and the definitions of and relations with thin structures.

5.5. Remark. Andrée Ehresmann has pointed out that the paper [14, Proposition 9, p.103] proves that any double category is a subdouble category of the double category associated in the above equivalence to a well defined 2-category. See also the comments in [13], II-1, comment 105.1, and IV-2, p.798-800.

References

[1] Al-Agl, F., *Aspects of multiple categories*, University of Wales PhD Thesis, 1989.

[2] Al-Agl, F. and Steiner, R., 'Nerves of multiple categories', *Proc. London Math. Soc.* 66 (1992) 92-128.

[3] Brown, R., 'Higher dimensional group theory', in *Low dimensional topology*, London Math Soc. Lecture Note Series 48 (ed. R. Brown and T.L. Thickstun, Cambridge University Press, 1982), pp. 215-238.

[4] Brown, R., 'Higher dimensional group theory', `http://www.bangor.ac.uk/~mas010/hdaweb2.html`

[5] Brown, R. and Higgins, P.J., 'On the connection between the second relative homotopy groups of some related spaces', *Proc. London Math. Soc.* (3) 36 (1978) 193-212.

[6] Brown, R. and Higgins, P.J., 'On the algebra of cubes', *J. Pure Appl. Algebra* 21 (1981) 233-260.

[7] Brown, R. and Higgins, P.J., 'The equivalence of ∞-groupoids and crossed complexes', *Cah. Top. Géom. Diff.* 22 (1981) 371-386.

[8] Brown, R. and Higgins, P.J., 'Tensor products and homotopies for ω-groupoids and crossed complexes', *J. Pure Appl. Algebra* 47 (1987) 1-33.

[9] Brown, R. and Mosa, G.H., 'Double algebroids and crossed modules', UWB Maths Preprint 99.15.

[10] Brown, R. and Spencer, C.B., 'Double groupoids and crossed modules', *Cahiers Top. Géo. Diff.* 17 (1976) 343-362.

[11] Dawson, R. and Paré, R., 'General associativity and general composition for double categories', *Cahiers Top. Géom. Diff. Cat.* 34 (1993) 57-79.

[12] Ehresmann, C., 'Catégories structurées III: Quintettes et applications covariantes', *Cahiers Top. Géom. Diff.* 5 (1963) 1-22.

[13] Ehresmann, C., *Oeuvres Complètes et Commentées*, Amiens, 1980.

[14] Ehresmann, A. and Ehresmann, C., 'Multiple functors IV: Monoidal closed structures on Cat_n', *Cahiers Top. Géom. Diff.* 20 (1979) 59-104.

[15] Mosa, G.H., *Higher dimensional algebroids and crossed modules*, Univ. of Wales, Bangor, PhD. Thesis (1986).

[16] Spencer, C.B., 'An abstract setting for homotopy pushouts and pullbacks', *Cahiers Top. Géo. Diff.* 18 (1977) 409-430.

[17] Spencer, C.B. and Wong, Y.L., 'Pullback and pushout squares in a double category with connection', *Cahiers Top. Géo. Diff.* 24 (1983) 161-192.

School of Mathematics
University of Wales, Bangor
Gwynedd LL57 1UT
United Kingdom
and
Department of Mathematics
Zarka Private University
Zarka, Jordan
Email: `r.brown@bangor.ac.uk` and `kallami@hotmail.com`

This article may be accessed via WWW at `http://www.tac.mta.ca/tac/` or by anonymous ftp at `ftp://ftp.tac.mta.ca/pub/tac/html/volumes/1999/n7/n7.{dvi,ps}`

THEORY AND APPLICATIONS OF CATEGORIES (ISSN 1201-561X) will disseminate articles that significantly advance the study of categorical algebra or methods, or that make significant new contributions to mathematical science using categorical methods. The scope of the journal includes: all areas of pure category theory, including higher dimensional categories; applications of category theory to algebra, geometry and topology and other areas of mathematics; applications of category theory to computer science, physics and other mathematical sciences; contributions to scientific knowledge that make use of categorical methods.

Articles appearing in the journal have been carefully and critically refereed under the responsibility of members of the Editorial Board. Only papers judged to be both significant and excellent are accepted for publication.

The method of distribution of the journal is via the Internet tools `WWW/ftp`. The journal is archived electronically and in printed paper format.

SUBSCRIPTION INFORMATION. Individual subscribers receive (by e-mail) abstracts of articles as they are published. Full text of published articles is available in .dvi and Postscript format. Details will be e-mailed to new subscribers and are available by `WWW/ftp`. To subscribe, send e-mail to `tac@mta.ca` including a full name and postal address. For institutional subscription, send enquiries to the Managing Editor, Robert Rosebrugh, `rrosebrugh@mta.ca`.

INFORMATION FOR AUTHORS. The typesetting language of the journal is TEX, and LATEX is the preferred flavour. TEX source of articles for publication should be submitted by e-mail directly to an appropriate Editor. They are listed below. Please obtain detailed information on submission format and style files from the journal's WWW server at URL `http://www.tac.mta.ca/tac/` or by anonymous ftp from `ftp.tac.mta.ca` in the directory `pub/tac/info`. You may also write to `tac@mta.ca` to receive details by e-mail.