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GENERALIZED NONLINEAR PSEUDOCOCOERCIVE VARIATIONAL INEQUALITIES AND GENERAL PROXIMAL POINT METHODS

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Abstract. Let $T: K \to H$ be a hemicontinuous, η-pseudococoercive, $\eta - L$-relaxed monotone and d-Hölder continuous mapping from a nonempty closed invex subset K of an infinite-dimensional Hilbert space H into H. Let $f: K \to R$ be proper, invex and lower semicontinuous on K and $h: K \to R$ be continuously Fréchet-differentiable on K with h' (gradient of h) $\eta - \alpha$-strongly monotone and $\eta - \beta$-Lipschitz continuous on K. Suppose that there exist an $x^* \in K$ and numbers $a > 0$, $r \geq 0$, ρ^k $(a < \rho^k < \alpha/L)$ such that for all $t \in [0, 1]$, and for all $x \in K^{\#}$, the set $S^{\#}$ defined by

$$S^{\#} = \{(h, \eta): h'(x^* + t(x - x^*))(x - x^*) \geq \langle h'(x^* + t\eta(x, x^*)), \eta(x, x^*)\rangle\}$$

is nonempty, where $K^{\#} = \{x \in K: \|x - x^*\| \leq r\}$, and $\eta: K \times K \to H$ is λ-Lipschitz continuous with the following assumptions:

(i) $\eta(x, y) + \eta(y, x) = 0$, $\eta(x, y) = \eta(x, z) + \eta(z, y)$, and $\|\eta(x, x^*)\| \leq r$.
(ii) For each fixed $y \in K$, map $x \to \eta(y, x)$ is sequentially continuous from the weak topology to the weak topology.

If in addition, h' is continuous from H equipped with weak topology to H equipped with weak topology, then the sequence $\{x^k\}$ generated by the general proximal point algorithm converges weakly to a solution x^* of the variational inequality problem (VIP), that is,

$$\langle T(x^*), \eta(x, x^*)\rangle + f(x) - f(x^*) \geq 0 \quad \text{for all } x \in K.$$

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