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FINDING AN ELLIPSE TANGENT TO FINITELY MANY GIVEN LINES

Alan Horwitz

Abstract. Given a finite set Σ of lines in the plane, we discuss necessary and sufficient conditions on when there is an ellipse E, with specified center (h,k) and angle of rotation α, which is tangent to each line in Σ. In all cases we assume that no three of the lines are parallel or have a common intersection point, else no such ellipse could exist. If such an ellipse exists, we say that (h,k) is α admissible, or just admissible if $\alpha = 0$. For two given lines $T_1 : y - k = m_1(x-h) + b_1$ and $T_2 : y - k = m_2(x-h) + b_2$, with $|m_1| \neq |m_2|$, (h,k) is admissible if and only if $\frac{b_2^2 - b_1^2}{m_2^2 - m_1^2}$ and $\frac{b_1^2 m_2^2 - b_2^2 m_1^2}{m_2^2 - m_1^2}$ are both positive. Further, the ellipse is unique. We prove similar results when $|m_1| = |m_2|$, in which case the ellipse may not be unique. In certain cases we allow α to vary. We then show that every $(h,k) \notin T_1 \cup T_2$ is α admissible for some α (if $m_1 \neq m_2$). We prove various results for three given lines. In particular, if none of the slopes of the lines are equal in absolute value, and none of the lines are horizontal, then there are cubic polynomials $q(k)$ and $r(h)$ such that the set of admissible centers is precisely the set of points on a hyperbola where q and r are positive. q and r are obtained using the intersection points of the given lines. Finally, for four given lines, we show that there is always some ellipse, rotated of course, tangent to the given lines.

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