Electronic Journal: Southwest Journal of Pure and Applied Mathematics
Internet: http://rattler.cameron.edu/swjpam.html
ISSN 1083-0464
Issue 2, December 2000, pp. 28–43.
Submitted: July 13, 2000. Published: December 15, 2000.

TENDER SELECTION PROCEDURE IN THE POWER GENERATING INDUSTRY

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Abstract. The paper presents a method evaluating tenders to bring on-line new power generating capacities. However, this methodology can be applied to all kind of multiobjective evaluation problems.

1991 A.M.S. Subject Classification Codes: 90.

Key Words and Phrases: Probability, density function, distribution.

In 1997, the Hungarian Power Companies (HPC) issued two tenders for installing new capacities in the period of 2002–2006: one for big capacities (200 MW<), and one for smaller ones. HPC is responsible for the electricity supply of the country. It buys power from plants

and sells it to distributors. Bid evaluations in Hungary are governed by the Electric Power Act passed in 1994. This act basically requires the HPC to ensure that any future power acquisitions abide by the principle of least cost. Of course, there is not a single cost indicator according to which bids could unambiguously be ranked. Costs can be divided into fixed and variable costs (expected energy fee and capacity charge), and also these two types of costs can be further broken down. The planned sites of the investments influence the costs, too, since new plants have to be connected into the national power transmission grid. In the evaluation process, other criteria have to be taken into consideration, too. One of these criteria is the environmental impacts of the production. In case of power plants, the amount of air pollutants have to be taken into account. In the past decades, Hungary joined most of the international conventions and agreements regularing the emission levels of different air pollutants. We mention, for example the UN Framework Convention of Climate Change regularing primarily the CO_2 emissions of anthropogenic origin. Since one third of the overall anthropogenic CO_2 emission comes from power plants, the efficiency and input fuel structure have to be taken into consideration, too. The

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level of other pollutants (SO_2, NO_X, CO, particulate) are also among the evaluation criteria.

The methodology described below is a decision supporting tool for policymakers. A lot of factors influence the electrical sector. The two main decisive factors are the development of demand on electrical energy, and the price escalation of fuels. Therefore, methodology has to take into account different paths of these variables. In the evaluation process we propose to consider three variants for both factors. The demand will be determined basically by the economic development in the next 10–20 years. After a deep recession in the late 80's and early 90's, the Hungarian economy has begun to recover, and 4–5% GDP growth is predicted in the next years. Certainly, an overall increase of efficiency of electricity consumption is expected as well, which will compensate in part the demand increase. Of course, the forecase of fuel prices is also very uncertain even for short-term.

1. INTRODUCTION

The evaluation procedure for tenders for the establishment of new electric power generating capacities is a very complex task. The presence of several interest groups (share holders), many evaluation criteria, and the dynamism of the need for electric power makes this problem challenging. This particular problem has been raised in Hungary, where the privatization of all fields of the economy is one of the most important trends in adjusting the Hungarian economy to the Western European community.

The selection of the most appropriate tenders and partners cannot be considered as a simple multiobjective programming problem because of the presence of multiple decision makers. Since each interest group's decision is based on several evaluation criteria, and the criteria are conflicting to each other, the problem cannot be approached by classical game theoretical methodology either. Therefore a hybrid methodology had to be developed in order to supply the final decision makers with a satisfactory solution.

This paper, after a brief methodology review, will describe the basics of the methodology and their details that was suggested by the authors of this paper to the leadership of the Hungarian Power Companies as well as to the Ministry of Economic Affairs of Hungary.

The presence of multiple evaluation criteria suggests the use of certain concepts and methodology of multicriteria decision making. The fundamentals of this topic are given for example in Szidarovszky et al. [1]. Depending on the preference informations of the decision makers different methods are available. To name a few, sequential optimization, the ε-constraint method, weighting, distance-, and direction-based methods can be used. Multicriteria problems are sometimes solved by using multiattribute utility theory (Keeney and Raiffa [3]), however not all preference orders can be characterized by single-valued utility functions. For example, the lexicographic order is not equivalent to any order generated by a real-valued function (Fishburn [4]). Decision problems with more than one decision makers are usually modeled by using the methodology of noncooperative- or cooperative games, including group decision making and conflict resolution. The game theoretical aspects are summarized for example in Selten [5], or in Myerson [6]. If the theoretical groups are considered as the decision makers, then since each of them has several objective functions, game theoretical methods cannot be used directly.

Recently Szidarovszky and Eskandari [7] have introduced a general framework to examine games with multiple payoffs. Future demands and economic conditions cannot be predicted exactly, therefore the uncertainty of these factors has to be incorporated into the methodology. In the decision science literature stochastization or fuzzification is used to handle such uncertainty. Szidarovszky et al. [1] contains the basics of both approaches.

The diversity of the methodology in each of the above-mentioned topics offers a large variety of particular methods to be adapted and combined to the tender evaluation process. In the next sections the major steps and the suggested methodology will be outlined.

2. STEPS OF THE PROCESS

For each interest group the evaluation process should be done separately, and it was suggested to follow the following pattern.

Step 1. A list of possible economic scenarios is constructed which are not controlled by the electric power suppliers and have significant effects on the outcome of each tender (e.g., fuel price, economic indices, regulation variants, etc.). Let $k = 1, 2, \dots, N$ denote these scenarios. For different interest groups the selected scenarios may be different.

One possibility is given below.

In the analysis three assumptions will be considered on fuel prices: low, medium, and high increase. Also, three different rates of demand expansions (low, medium, high) will be considered. The nine possible combinations of these assumptions will give the basic assumptions:

1: low fuel price increase and low demand expansions ($F_L \cap D_L$)
2: low fuel price increase and medium demand expansions ($F_L \cap D_M$)
3: low fuel price increase and high demand expansions ($F_L \cap D_H$)
4: medium fuel price increase and low demand expansions ($F_M \cap D_L$)
5: medium fuel price increase and medium demand expansions ($F_M \cap D_M$)
6: medium fuel price increase and high demand expansions ($F_M \cap D_H$)
7: high fuel price increase and low demand expansions ($F_H \cap D_L$)
8: high fuel price increase and medium demand expansions ($F_H \cap D_M$)
9: high fuel price increase and high demand expansions ($F_H \cap D_H$)

These scenarios will be then assumed in evaluating the tenders for estabilishing new power generating capacities. If we do not consider unchanged, or decreasing fuel prices and/or demands, then it will be assumed that the above nine scenarios cover all possibilities. Therefore we propose to select $N = 9$.

Step 2. The occurrence probability p_k for each scenario is then estimated.

The estimation of the individual probabilities

$$p(F_i) \quad \text{and} \quad p(D_j)$$

for $i, j = L, M, H$ are possible by forecasting economic indices that mostly effect these quantities.

Let $x_1, x_2, \dots, x_n$ denote the indices and assume that fuel price and demand expansions can be obtained in some function relations

$$fp = f(x_1, \dots, x_n) \tag{2.1}$$

$$de = g(x_1, \dots, x_n). \tag{2.2}$$

First of all, these functions have to be estimated. Using historical data, nonlinear (or as special case linear) least squares method is suggested. Let

$$x_1^{(t)}, \dots, x_n^{(t)}, fp^{(t)}, de^{(t)}$$

denote historical data set with $t = 1, 2, \dots, T$. The function types can be guessed by graphing the $(x_1^{(t)}, fp^{(t)}), \dots, (x_n^{(t)}, fp^{(t)})$ as well as the $(x_1^{(t)}, de^{(t)}), \dots, (x_n^{(t)}, de^{(t)})$ points. These one-dimensional graphs represent "fp" and "de" as functions of the individual economic indices. In the computer analysis several function types will be examined and the functions with best least squares fit will be accepted. Additive and multiplicative expressions of the single-dimensional $x_i \to fp$ and $x_i \to de$ functions will be first examined. If no satisfactory fit can be established, then multivariable polynomial forms will be considered. The details of linear and nonlinear least squares analysis can be found in all elementary textbooks.

After functions f and g are determined, the economic indices have to be estimated. The result of the forecasting procedure is a probability distribution for each index. Depending on the forecasting methodology, either a discrete or a continuous distribution is provided. Let $\tilde{x}_1, \dots, \tilde{x}_n$ denote these random variables. Based on their distributions, the probability distributions of "fp" and "de" can be obtained in the following way.

If the distributions of $\tilde{x}_i$ are discrete, then the process is as follows. Let x_{i1}, x_{i2}, $\dots$, x_{ir_i} denote the discrete values and $p_{i1}, p_{i2}, \dots, p_{ir_i}$ the occurring probabilities. Then we accept the values

$$fp(j_1, j_2, \dots, j_n) = f(x_{1j_1}, \dots, x_{nj_n}) \tag{2.3}$$

and

$$de(j_1, j_2, \dots, j_n) = g(x_{1j_1}, \dots, x_{nj_n}) \tag{2.4}$$

with occurring probabilities $p_{1j_1} \cdot p_{2j_2} \cdot \dots \cdot p_{nj_n}$. Notice that here we used the independence of the economic indices. This necessary independence has to be taken care by the careful selection of the indices. One possible way of doing it is the use of factor analysis. The details of this methodology can also be found in most advanced statistical textbooks, so the details are omitted.

Let next $[a_L, b_L]$, $[a_M, b_M]$, $[a_H, b_H]$ denote the low, medium, and high fuel-price intervals, and let $[A_L, B_L]$, $[A_M, B_M]$, $[A_H, B_H]$ be the low, medium, and high demand expansion intervals. For $\alpha = L, M, H$ and $\beta = L, M, H$, the probability of α fuel-price with β demand expansion can be computed as

$$P(\alpha, \beta) = \sum_{j_1} \sum_{j_2} \cdots \sum_{j_n} p_{1j_1} p_{2j_2} \dots p_{nj_n}, \tag{2.5}$$

where summation is made for all $(j_1, \dots, j_n)$ such that

$$fp(j_1, \dots, j_n) \in [a_\alpha, b_\alpha]$$

and

$$de(j_1, \dots, j_n) \in [A_\beta, B_\beta].$$

In the case of continuous disributions on the economic indices the distribution function of "fp" and "de" is very difficult to get, since an n-dimensional integral has to be computed. Therefore we suggest stochastic simulation in the following way. Simulate random values of $x_1, \ldots, x_n$, and with each n-tuple, compute the corresponding values of "fp" and "de" by using the already computed f and g functions. If this simulation is repeated N^*-times, then $N^*(fp, de)$ pairs are obtained. With a simple counter in the program it is easy to see how many times the (fp, de) pair belongs to category (α, β) for $\alpha, \beta = L, M, H$. Let $N^*(\alpha, \beta)$ denote these counted values. Then the requested probabilities can be obtained by the relative frequencies:

$$P(\alpha, \beta) \approx \frac{N^*(\alpha, \beta)}{N^*}. \tag{2.6}$$

The final outcome of this step is the sequence of the probability values $P(\alpha, \beta)$ representing the occurring probabilities for each scenario. These probability values are then ordered as $P_1, P_2, \ldots, P_9$ depending which (α, β) pair corresponds to any given scenario.

Step 3. The list of evaluation criteria is next constructed. They may differ from an interest group to another. The goodness of the tenders are measured by these criteria.

The construction of the list of criteria will be done in three major stages.

In the first stage we have to think over the main reasons why the tenders are needed. In other words, we have to construct a list of goals and conditions the tender should satisfy. These goals and conditions are not necessarily quantifiable. For example goal such as "satisfying energy need" or condition to be "environment friendly" should be formulated only verbally. This list simply should serve as a systematic frame in the beginning of the process.

In the second stage we have to divide each goal and condition into verbal subcategories. For example, the "environment friendly" condition might be divided into groups such as air-quality, ground-water quality, noise, etc.

The third stage formulates the measurable criteria, by lising those quantifiable measures that can be used for concrete evaluation of the subcategories. For example, in "air-quality" we might use the CO_2, SO_2, NO_X, etc. content. The corresponding % numbers can be finally used for evaluation. A similar approach is then suggested for each goal and condition.

In practice, the following procedure seems reasonable. Representatives of the different stake-holders (or interest groups) should have the first meeting to determine the list required in the first stage. For each goal and condition we have to identify the group being mainly interested in. After this primary list is completed, then in the second stage (maybe in a later time) the different interest groups should hold separate meetings to construct and get agreements on the further subdivision of the goals and conditions. After an agreement has been reached, the third stage can start. We advise to have at least a couple of days before the third stage starts. The reasons for this break is to give a chance to the participants to think over the results of the previous stages, to have second thoughts on these lists. If any changes or additions are suggested, they have to be incorporated into the final lists. The third stage will need separate meetings for each interest group. They have to go through the lists of the second stage systematically and for each item they have to suggest measures to be used to evaluate the goodness of each tender with respect to these measures. After each interest group has finalized its list, the meetings have to be finished. After two–three days it is advised that the coordinator makes contact (for example in the way of conferences calls) to the leaders of each group in order to check if any new idea or suggestion came up. If there is no suggestion, then the final list of objectives should be considered as the final outcome of the Step 3, otherwise the list should be modified, and the modified list has to be considered final. Let S denote the number of objectives in the final list. Finally we note that a very similar procedure has been successfully applied for finding the best forestry treatment in a Northern Arizona region (see, Eskandari et al. [8]).

Step 4. In constructing the payoff matrices for each (in our case, nine) scenarios, we have to consider two possibilities.

If an objective is completely quantifyable and certain, then the actual data should be put into the payoff matrix. In the case of an uncertain data, expected values should be used in the table. A common method to do so (in absence of a sequence of data in cases of uncertainty) is the following. Ask experts about a most probable value, and the smallest and largest possibilities for that value. Let $x_{\min}$, $x_{\max}$ and x_{prob} denote these values. Then a triangular distribution can be assumed as shown in Figure 1.

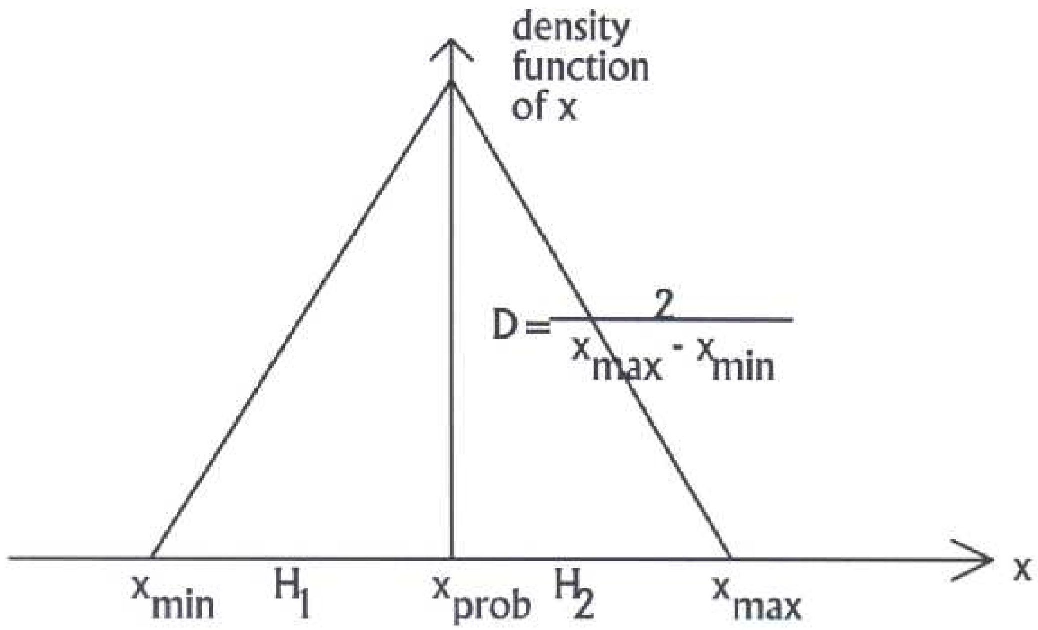

Figure 1. Triangular distribution for uncertain data

The value of the density function at x_{prob} is unknown. Let D denote this value. Then it can be obtained by using the fact that the integral of the density function has to be one. Therefore, using the area of triangles,

$$\frac{(x_{\max} - x_{\min})D}{2} = 1$$

implying that

$$D = \frac{2}{x_{\max} - x_{\min}}. \tag{2.7}$$

So the density function can be written as

$$f(x) = \begin{cases} (x - x_{\min})\frac{D}{H_1} & \text{if } x_{\min} \le x \le x_{\text{prob}} \\ (x_{\max} - x)\frac{D}{H_2} & \text{if } x_{\text{prob}} \le x \le x_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{2.8}$$

where $H_1 = x_{\text{prob}} - x_{\min}$ and $H_2 = x_{\max} - x_{\text{prob}}$. The expected value can be obtained by simple integration:

$$\begin{aligned} \overline{X} &= \int_{x_{\min}}^{x_{\text{prob}}} (x^2 - x \cdot x_{\min})\frac{D}{H_1} dx + \int_{x_{\text{prob}}}^{x_{\max}} (x \cdot x_{\max} - x^2)\frac{D}{H_2} dx \\ &= \frac{D}{H_1}\left[\frac{x^3}{3} - \frac{x^2}{2}x_{\min}\right]_{x_{\min}}^{x_{\text{prob}}} + \frac{D}{H_2}\left[\frac{x^2}{2}x_{\max} - \frac{x^3}{3}\right]_{x_{\text{prob}}}^{x_{\max}} \\ &= \frac{D}{H_1}\left[\frac{x_{\text{prob}}^3 - x_{\min}^3}{3} - x_{\min}\frac{x_{\text{prob}}^2 - x_{\min}^2}{2}\right] \\ &\quad + \frac{D}{H_2}\left[x_{\max}\frac{x_{\max}^2 - x_{\text{prob}}^2}{2} - \frac{x_{\max}^3 - x_{\text{prob}}^3}{3}\right] \\ &= \frac{D}{H_1}\left[\frac{1}{3}x_{\text{prob}}^3 + \frac{1}{6}x_{\min}^3 - \frac{1}{2}x_{\min}x_{\text{prob}}^2\right] \\ &\quad + \frac{D}{H_2}\left[\frac{1}{6}x_{\max}^3 - \frac{1}{2}x_{\max}x_{\text{prob}}^2 + \frac{1}{3}x_{\text{prob}}^3\right] \end{aligned} \tag{2.9}$$

which is very easy to compute. We wish to mention here that $\overline{X}$ equals x_{prob} if and only if

$$x_{\text{prob}} = \frac{x_{\min} + x_{\max}}{2}, \tag{2.10}$$

that is, when the most probable value is the mean of the smallest and largest values. So, in this special case the above calculation can be omitted. If for any of the payoff-matrix elements there is a sequence of (e.g., historical) data, then their average value can be accepted as expectation. In the case of repeated values they have to be considered in the average as many times as they appear. If $x_1, x_2, \dots, x_s$ denote the values with multiplicities $m_1, m_2, \dots, m_s$, then $\overline{X}$ is the weighted average:

$$\overline{X} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_s x_s}{m_1 + m_2 + \cdots + m_s}. \tag{2.11}$$

In the case of unquantifyable objectives, subjective measures should be selected. For example, issues concerning beauty or aesthetics cannot be quantified. In this case a subjective $[0,100]$ scale can be chosen, where 0 corresponds to the possible worst, and 100 corresponds to the possible best outcome. The members of the interest group associated to this objective should make the judgments. It is suggested that after a short discussion, each individual gives his/her assessments, which will be averaged later, and the averaged values will be used in the payoff table.

The above process has to be repeated for each matrix element in order to construct the entire payoff matrix. We strongly recommend that the meeting must continue until the entire table is finalized.

Each of the nine scenarios requires the construction of a payoff matrix. If time permits, several scenarios might be considered in one meeting, however the meetings have to be terminated only at times when a complete payoff matrix is constructed.

The outcome of this step is 9 payoff-matrices for each interest group. We note here certain matrix elements are not affected by the scenarios. These elements do not need to be recalculated. They have to be assessed only once and these values be repeated in each case. For the further analysis we have to assume that each objective function is to be maximized. Therefore all objective values of minimizing objectives should be multiplied by (-1). Therefore, in the final payoff-matrices larger objective value always should reflect better performance.

Step 5. The utility function associated to each objective will transform the objective values into an appropriate scale that can be easily interpreted by the decision makers. The assessment of the utility functions is suggested to be performed by the interest groups (stake holders) which are associated to the particular objective functions. The procedure to be outlined below has to be repeated for each objective.

The first step is to identify a possible worst objective value. Even in cases when an objective function is unbounded, a practical finite minimal value can be identified. Let this value be denoted by $g_{\min}$. Similarly, a possible maximal value has to be then identified, which can be denoted by $g_{\max}$. If U is the utility function associated to this objective, then define

$$U(g_{\min}) = 0 \quad \text{and} \quad U(g_{\max}) = 1. \tag{2.12}$$

To construct the shape of the function in between these extreme values, a bisection-type procedure is suggested. Let

$$g_{\text{mid}} = \frac{g_{\min} + g_{\max}}{2} \tag{2.13}$$

denote the midpoint of the initial interval $[g_{\min}, g_{\max}]$, and ask the interest group to estimate the value $U(g_{\text{mid}})$. In the case of disagreement, the average value has to be selected. Now, we have two subintervals. Repeat the procedure next to each subintervals: estimate the utility function value at both midpoints. Then we will have 4 subintervals. After the same procedure is repeated again, we will have 8 subintervals. So, after k repeats, the number of subintervals becomes 2^k.

The utility function is now given in a finite set of points, and in the second stage, a continuous function has to be constructed. In the literature (see for example, Szidarovszky et al. [1]) three basic types of utility functions are discussed representing risk-seeking, risk-averse, and risk-neutral viewpoints. These types are illustrated in Figures 2, 3, and 4.

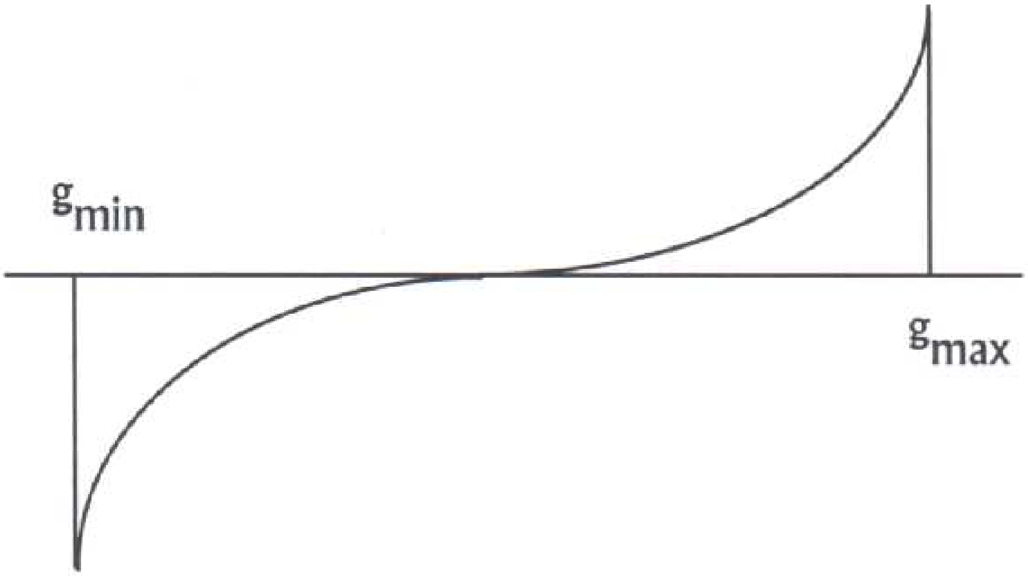

Figure 2. Risk-seeking utility function

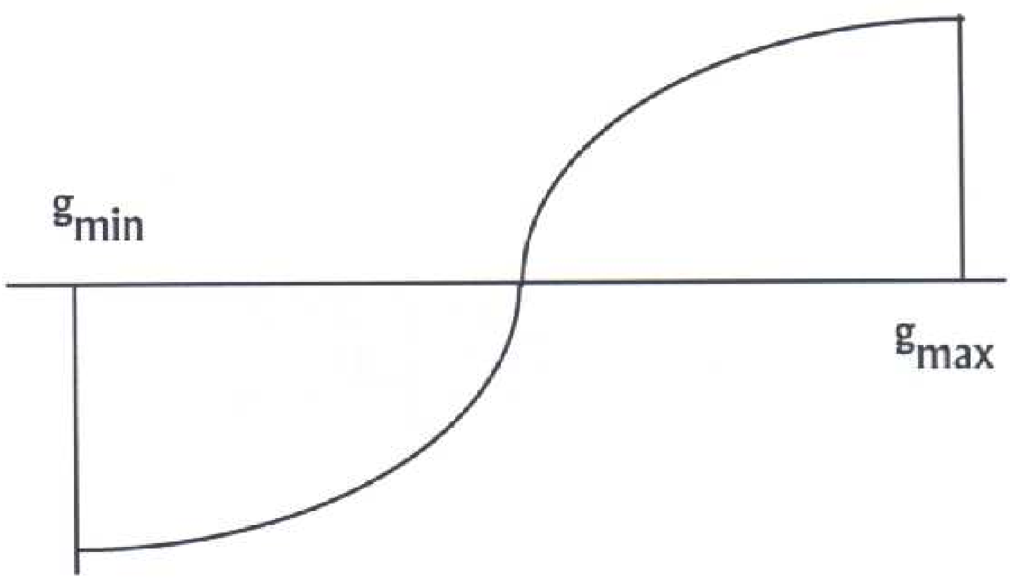

Figure 3. Risk-averse utility function

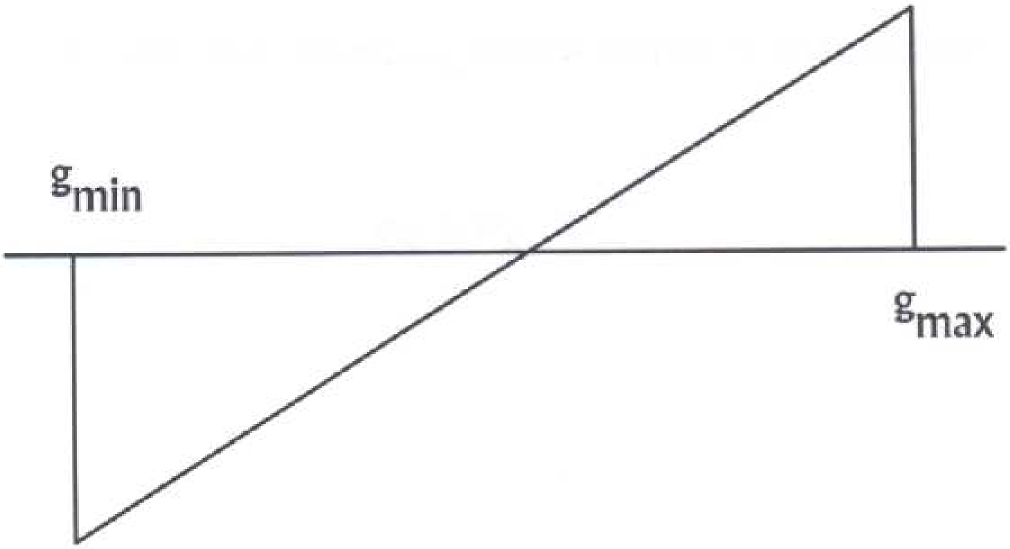

Figure 4. Risk-neutral utility function

In practical applications, however these types are often combined. Therefore we do not propose to fit any of these types to the discrete data. The continuous graph connecting the discrete points have to be the most smooth interpolator connecting the neighbor points. It is well known from the numerical analysis literature that the smoothest interpolator is the natural cubic spline (see for example, Yakowitz and Szidarovszky [9]). The construction of the natural cubic spline can be summarized as follows. Assume, that the independent variable values are $g_1 < g_2 < \cdots < g_n$ and the corresponding function values are $U_1, U_2, \ldots, U_n$. Define first

$$\left.\begin{array}{r} h_k = g_{k+1} - g_k, \\ s_k = \frac{U_{k-1} - U_k}{h_k} \end{array}\right\} \quad (k = 1, 2, \ldots, n-1), \tag{2.14}$$

Select $c_1 = c_n = 0$, and for the unknowns $c_2, \ldots, c_{n-1}$ solve the following tridiagonal system of linear equations

$$c_{k-1}h_{k-1} + 2c_k(h_{k-1} + h_k) + c_{k-1}h_k = 3(s_k - s_{k-1}) \tag{2.15}$$

where $k = 2, 3, \ldots, n-1$. After the values c_k are determined, then define

$$a_k = U_k \tag{2.16}$$

$$b_k = s_k - \frac{h_k}{3}(2c_k + c_{k-1}) \tag{2.17}$$

and

$$d_k = \frac{c_{k+1} - c_k}{3h_k} \quad (k = 1, 2, \dots, n-1). \tag{2.18}$$

And finally, the natural spline in the interval $[g_k, g_{k+1}]$ is given as follows:

$$a_k + b_k \cdot (g - g_k) + c_k \cdot (g - g_k)^2 + d_k \cdot (g - g_k)^3. \tag{2.19}$$

After the natural spline segments are computed for all subintervals of all objective functions, the continuous utility functions are completely constructed. After the procedure is completed, the resulted utility functions (in graphical form) are suggested to be sent back to the interest groups for final examination in order to have second thoughts on their assessed point values. If any changes are suggested by them, then either the resulting utility functions are slightly modified on the graphs, or the entire computation has to be repeated. We strongly believe that the possibility has to be given to the interest groups to see the consequences of their assessments, and to give them an additional chance to make necessary corrections if needed.

The final outcome of this step is a continuous utility function for each interest group and each objective function. The utility functions are given in graphical representations as well as in natural cubic spline forms.

Step 6. In this step the multiattribute utility functions will be constructed. Let $U_1, U_2, \dots, U_s$ denote the individual utility functions associated to the different objective functions. We expect that these utility functions satisfy the mutually utility independence condition, that is, the utility function of any objective does not depend on particular values of the other utilities (for precise definition see, for example, Szidarovszky et al. [1]). In this case the multiattribute utility function is either additive:

$$U = k_1 U_1 + k_2 U_2 + \cdots + k_s U_s \tag{2.20}$$

or multiplicative:

$$kU + 1 = (kk_1 U_1 + 1)(kk_2 U_2 + 1) \cdots (kk_s U_s + 1) \tag{2.21}$$

where $k_1, k_2, \dots k_s$, and k are unknown constants. The main problem is to determine these unknown values. For the proof of the above forms, see for example, Keeney and Raiffa [3].

We will present here two approaches, which are the most popular ones in practical application: least squares estimation, and a systematic assessment procedure.

Assume first that the multiattribute utility function is linear. In the least squares approach we will ask the members of the interest groups to assess the multiattribute utility function values at a set of given values of $U_1, U_2, \dots, U_s$. We suggest to have 5 values: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 for each individual utility function. Altogether we might have 5^s points. If it is too much, we can select 3 points only: 0, $\frac{1}{2}$, and 1, and consider only 3^s points. For each point, say $U_1^{(j)}, \dots, U_s^{(j)}$, let the assessed multiattribute utility value be $U^{(j)}$, for $j = 1, 2, \dots, 3^s$ or 5^s. The unknown k_i values can be then obtained by using standard linear least squares method for relation (2.20).

In the systematic approach we have to ask the stake-holders to assess the multiattribute utility function values at S special points:

$$U_1 = 0, \dots, U_{i-1} = 0,\ U_i = 1,\ U_{i+1} = 0, \dots, U_s = 0 \tag{2.22}$$

for $i = 1, 2, \dots, S$. The multiattribute utility value will correspond to the case, when the ith utility is at maximum level, but all others are at their minima. If $U^{(i)}$ is the assessed multiattribute utility value, then simple substitution shows that

$$k_i = U^{(j)}. \tag{2.23}$$

Consider next the multiplicative form. With keeping the above notation, the least squares method minimizes the least squares fit

$$\sum_j \left[kU^{(j)} + 1 - (kk_1U_1^{(j)} + 1)(kk_2U_2^{(j)} + 1) \cdots (kk_sU_s^{(j)} + 1)\right]^2. \tag{2.24}$$

This unconstrained optimization problem can be solved by standard methods such as gradient-type algorithms. For details, see for example, Hadley [10]. We mention here that several professional software packages, like MATLAB, IMSL, etc. might be used to perform the actual optimization. In the systematic approach substitute points (2.22) into equation (2.21) to see that for $i = 1, 2, \dots, S$,

$$kU^{(j)} + 1 = kk_i + 1, \tag{2.25}$$

since for $j \neq i$, $kk_jU_j + 1 = kk_j \cdot 0 + 1 = 1$, and $kk_iU_i + 1 = kk_i \cdot 1 + 1 = kk_i + 1$. From equation (2.25) we conclude that

$$k_i = U^{(i)} \tag{2.26}$$

similarly to the linear case. After the values k_i are recovered, substitute $U_1 = U_2 = \cdots = U_s = 1$ into the multiplicative formula (2.21) and normalize the multiattribute utility function to 1 to have an additional equation

$$k + 1 = (kk_1 + 1)(kk_2 + 1) \cdots (kk_s + 1) \tag{2.27}$$

for the remaining unknown k. Notice that $k = 0$ is always a solution which reduces the multiplicative form to linear. If $k \neq 0$, then equation (2.27) has to be solved for k. Simple algebra shows that after dividing by k, it can be rewritten as

$$k^{s-1}k_1k_2 \cdots k_s + \cdots + k(k_1k_2 + k_1k_3 + \cdots + k_{s-1}k_s) + (k_1 + k_2 + \cdots + k_s - 1) = 0$$

which is a polynomial equation for k. In the case of multiple solutions, the best solution can be obtained by an additional point selection in the following way. Select for example, $U_1 = U_2 = \cdots = U_s = \frac{1}{2}$ and let the assessed associated multiattribute utility function value be $U^{1/2}$. Then the measure to minimize is the following:

$$\left|kU^{1/2} + 1 - \left(\frac{kk_1}{2} + 1\right)\left(\frac{kk_2}{2} + 1\right) \cdots \left(\frac{kk_s}{2} + 1\right)\right|, \tag{2.28}$$

or by selecting a set of points, then least squares measure (2.24) can be used to select the best value of k from the solution set.

The final outcome of this step is the particular form (2.20) or (2.21) of the multiattribute utility function with the particular values of the coefficient, $k_1, k_2, \dots, k_s$ (or in addition, the value of) k.

Step 7. For each scenario and each tender the corresponding multiattribute utility function values are already computed. Let now consider a given tender. Let $g_1, \dots, g_s$ denote the computed objective function values (as the results of Step 4). Using the individual utility functions (as the results of Step 5) we may easily calculate the sequence of utility values

$$U_1^* = U_1(g_1),\ U_2^* = U_2(g_2), \dots, U_s^* = U_s(g_s),$$

and then using the result of Step 6, the corresponding multiattribute utility function value U^* can be determined by using either equation (2.20) or equation (2.21) depending on which type gave better fit in Step 6.

The outcome of this computation is shown in Table 1, where U_{ij}^* gives the multiattribute utility value of tender i ($i = 1, 2, \dots, p$) under scenario j ($j = 1, 2, \dots, 9$). Finally, for each tender i, we can compute the expected multiattribute utility values as

$$U_i^* = \sum_{j=1}^{9} P_j U_{ij}^*,$$

where the P_j probability values were obtained as the results of Step 2.

Tender	Scenario				
	1	2	3	$\cdots$	g
1	U_{11}^*	U_{12}^*	U_{13}^*	$\cdots$	U_{19}^*
2	U_{21}^*	U_{22}^*	U_{23}^*	$\cdots$	U_{29}^*
3	U_{31}^*	U_{32}^*	U_{33}^*	$\cdots$	U_{39}^*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
p	U_{p1}^*	U_{p2}^*	U_{3p}^*	$\cdots$	U_{p9}^*

Table 1. Results of Step 7

Repeating the previous steps for each interested group, a final payoff matrix is obtained, in which the rows correspond to the interest groups, the columns to the tenders (decision alternatives), and the (k, l) element U_{kl} of this table gives the final evaluation of tender l by interest group k. This table (given in Table 2) can be considered as the payoff table of a discrete group decision problem, in which the rows correspond to the different members and the columns represent the decision alternatives.

Group	Tenders			
	1	2	$\cdots$	p
1	U_{11}	U_{12}	$\cdots$	U_{1p}^*
2	U_{21}	U_{22}	$\cdots$	U_{2p}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	U_{K1}	U_{K2}	$\cdots$	U_{Kp}

Table 2. Group decision payoff table

The final selection procedure will be outlined in the next section.

FINAL SELECTION PROCEDURE

Notice that the utility function for each group are normalized, so no normalization is needed in the final selection procedure. The final selection is suggested to be based on the following simple algorithm.

The importance of the different interest groups is assumed to be assessed by the final decision makers. Let $\alpha_1, \alpha_2, \ldots, \alpha_K$ denote these factors, where $\alpha_i \geq 0$ for all i, and $\alpha_1 + \alpha_2 + \cdots + \alpha_K = 1$. Then for each tender l, compute the overall evaluation measures

$$U_l^F = \sum_{k=1}^{K} \alpha_k U_{kl}^2, \tag{3.1}$$

and select the tender l which has the largest U_l^F value.

Notice that any other multiobjective procedure, or any social choice of conflict resolution method can be selected for the final decision making. Equation (3.1) maximizes the I_2 distance from the origin, so this method is a special distance-based procedure. The large variety of multiobjective programming methods can be found in for exmaple, Szidarovszky et al. [1], conflict resolution concepts are examined, for example in Myerson [6], and social choice procedures are analyzed, for example in Taylor [11].

REFERENCES

1. Szidarovszky, F., Gershon, M., and Duckstein, L.: *Techniques for Multiobjective Decision Making in Systems Management*, Elsevier, Amsterdam, 1986.
2. Szidarovszky, F. and Molnár, S.: *Game Theory with Engineering Applications* (in Hungarian), Müszaki Könyvkiadó, Budapest, 1986.
3. Kenney, R.L. and Raiffa, H.: *Decisions with Multiple Objectives: Preference and Value Trade-Off*, Wiley, New York, 1976.
4. Fishburn, P.: *Utility Theory for Decision Making*, Wiley, New York, 1970.
5. Selten, R. (ed.): *Game Equilibrium Models*, Vols. 1–4, Springer-Verlag, Berlin, Heidelberg, New York, 1991.
6. Myerson, R.B.: *Game Theory: Analysis of Conflict*, Harvard University Press, Cambridge, MA, 1991.
7. Szidarovszky, F. and Eskandari, A.: A Note on Pareto Equilibria, *Pure Math. and Appl.* **7**, No. 1–2 (1996), 191–195.
8. Eskandari, A., Ffolliott, P., and Szidarovszky, F.: Uncertainty in Multicriterion Watershed Management Problems, *Technology* **332A** (1995), 199–207.
9. Yakowitz, S. and Szidarovszky, F.: *Introduction to Numerical Computations*, (2nd editor, 1989), Macmillan, New York, 1986.
10. Hadley, G.: *Nonlinear and Dynamic Programming*, Addison-Wesley, Reading, MA, 1964.
11. Taylor, A.D.: *Mathematics and Politics*, Springer-Verlag, Berlin, Heidelberg, New York, 1995.
12. Molnár, S., Staub, K., Molnár, M., Conzelmann, G., Koritarov, V., Guziel, K., and Veselka, T.: *Final Audit Report, New Generating Capacity Tenders 97/1 and 97/2*. Prepared for the Hungarian Power Companies Ltd.,

published in Decision and Information Sciences Division, Argonne National Laboratory, USA, 1999.