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ON THE ASYMPTOTIC BEHAVIOR OF DYNAMIC RENT-SEEKING GAMES

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Abstract. Dynamic rent-seeking games with nonlinear cost functions are analyzed. The local asymptotical stability of the solution is first examined. We show that in the absence of a dominant agent all eigenvalues of the Jacobian are real. Conditions are given for the local asymptotical stability as well as for the local instability of the equilibrium. In the presence of a dominant agent complex eigenvalues are possible. Simple stability conditions are presented for cases when all eigenvalues are real, and the possibility of limit cycles is analyzed in the case of complex eigenvalues.

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1. INTRODUCTION

During the last two decades an increasing attention has been given to the analysis of rent-seeking games. The introductory paper of Tullock [7] has initiated a series of studies on the subject. Decreasing return in rent-seeking technology as well as increasing returns were analyzed in Pérez-Castrillo and Verdier [3], who also examined the possibility of the agent's reaction function to be upward sloping. A systematic approach to establish the existence of an equilibrium has also been offered. Okuguchi [2] has pointed out that rent-seeking games are mathematically equivalent to profit maximizing Cournot oligopolies with hyperbolic price functions. He also proved the existence of a pure symmetric Nash equilibrium. These results have been extended to the nonsymmetric case by Szidarovszky and Okuguchi [4].

The dynamic extension of rent-seeking games has been examined by Szidarovszky and Okuguchi [5] and by Xu and Szidarovszky [8], however the stability of the

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equilibrium was proved under certain convexity and monotonicity conditions, which are not always satisfied.

In this paper we will present a systematic approach to examine the local asymptotical stability of the equilibrium under much more general conditions than in the previously cited studies, and in the case of local instability the possibility of limit cycles will be investigated.

2. THE MATHEMATICAL MODEL

Assume that n agents compete for a rent, which will earn a unit profit for the firm which actually will win the rent. Let x_i denote the effort agent i spends in order to win the rent, and let $c_i(x_i)$ be its cost. The probability of winning the rent for agent i is $\frac{x_i}{\sum_{j=1}^{n} x_j}$, therefore its expected profit is given as

$$\varphi_i(x_1, \ldots, x_n) = \frac{x_i}{\sum_{j=1}^{n} x_j} - c_i(x_i). \tag{1}$$

Let $X_i = \mathcal{R}_+$ be the set of feasible strategies of agent i, then an n-person noncooperative game $\Gamma = \{n, X_1, \ldots, X_n, \varphi_1, \ldots, \varphi_n\}$ is formally defined, and is called a *rent-seeking game.*

For each agent i, denote $Q_i = \sum_{j \neq i} x_j$, then for each $Q_i \geq 0$, its best response can be obtained as

$$x_i(Q_i) = \arg\max_{x_i \geq 0} \left\{ \frac{x_i}{x_i + Q_i} - c_i(x_i) \right\}. \tag{2}$$

Assume that

(i) for all i, c_i is twice continuously differentiable.

Assume in addition that $x_i(Q_i) > 0$, then the first and second order conditions can be written as

$$\frac{Q_i}{(x_i(Q_i) + Q_i)^2} - c_i'(x_i(Q_i)) = 0 \tag{3}$$

and

$$\frac{-2Q_i}{(x_i(Q_i) + Q_i)^3} - c_i''(x_i(Q_i)) < 0. \tag{4}$$

Notice that relation (4) also guarantees that the left-hand side of equation (3) is strictly monotonic, therefore $x_i(Q_i)$ is unique.

Let now $(\overline{x}_1, \ldots, \overline{x}_n)$ be a positive equilibrium, that satisfies both conditions (3) and (4). That is, with the notation $\overline{Q}_i = \sum_{l \neq i} \overline{x}_l$ and $\overline{s} = \sum_{j=1}^{n} \overline{x}_j$,

$$\overline{Q}_i = c_i'(\overline{x}_i)\overline{s}^2 \tag{5}$$

and

$$c_i''(\overline{x}_i) > \frac{-2\overline{Q}_i}{\overline{s}^3}. \tag{6}$$

Select any arbitrary positive numbers $\overline{x}_1, \dots, \overline{x}_n$, and let $\overline{s} = \sum_{i=1}^{n} \overline{x}_i$ and for all i, $\overline{Q}_i = \sum_{l \neq i} \overline{x}_i$. In addition, select any sequence $\overline{c}_1'', \dots, \overline{c}_n''$ of real numbers that satisfy inequality (6). Then there is a rent-seeking game in which $(\overline{x}_1, \dots, \overline{x}_n)$ is an equilibrium and for all i, $c_i''(\overline{x}_i) = \overline{c}_i''$, since for each i there is a strictly increasing cost function with given first and second derivatives at a single point.

Assume next that the time scale is continuous, and at each time period $t \geq 0$, each agent adjusts its output proportionally to its marginal profit. The resulting dynamic model has the form:

$$\dot{x}_i = k_i \left(\frac{Q_i}{(x_i + Q_i)^2} - c_i'(x_i) \right) \tag{7}$$

where $k_i > 0$ is given for each $i = 1, 2, \dots, n$. Notice that the vector $(\overline{x}_1, \dots, \overline{x}_n)$ is an equilibrium of this dynamic system as the consequence of the first order conditions (5). Assume that the initial vector $\underline{x}(0)$ is selected from a neighborhood of this equilibrium. In the next section the local stability and the dynamic behavior of the solution $\underline{x}(t)$ will be investigated.

3. STABILITY ANALYSIS

In order to analyze the asymptotical behavior of the solution of system (7), the Jacobian has to be first determined. Simple differentiation shows that at the equilibrium

$$\begin{aligned} \underline{J} = \frac{1}{\overline{s}^3} & \begin{pmatrix} k_1(-2\overline{Q}_1 - \overline{s}^3 c_1''(\overline{x}_1)) & k_1(\overline{x}_1 - \overline{Q}_1) & \cdots & k_1(\overline{x}_1 - \overline{Q}_1) \\ k_2(\overline{x}_2 - \overline{Q}_2) & k_2(-2\overline{Q}_2 - \overline{s}^3 c_2''(\overline{x}_2) & \cdots & k_2(\overline{x}_2 - \overline{Q}_2) \\ \vdots & \vdots & \ddots & \vdots \\ k_n(\overline{x}_n - \overline{Q}_n) & k_n(\overline{x}_n - \overline{Q}_n) & \cdots & k_n(-2\overline{Q}_n - \overline{s}^3 c_n''(\overline{x}_n)) \end{pmatrix} \\ &= \frac{1}{\overline{s}^3}(\underline{D} + \underline{a}\underline{1}^t) \end{aligned} \tag{8}$$

with

$$\begin{aligned} \underline{D} &= \operatorname{diag}(k_1(-\overline{s} - \overline{s}^3 c_1''(\overline{x}_1)), \dots, k_n(-\overline{s} - \overline{s}^3 c_n''(\overline{x}_n))), \\ \underline{a} &= (k_1(\overline{x}_1 - \overline{Q}_1), \dots, k_n(\overline{x}_n - \overline{Q}_n))^T, \quad \text{and} \quad \underline{1}^t = (1, \dots, 1). \end{aligned} \tag{9}$$

Since the signs of the real parts of the eigenvalues determine the asymptotical behavior of the solution, we might ignore the positive factor $\frac{1}{\overline{s}^3}$. The characteristic polynomial of $\underline{D} + \underline{a}\underline{1}^T$ can be written as

$$\begin{aligned} \det(\underline{D} - \lambda \underline{I} + \underline{a}\underline{1}^T) &= \det(\underline{D} - \lambda \underline{I}) \det(\underline{I} + (\underline{D} - \lambda \underline{I})^{-1} \underline{a}\underline{1}^T) \\ &= \prod_{i=1}^{n} (k_i(-\overline{s} - \overline{s}^3 c_i''(\overline{x}_i)) - \lambda) \left[1 + \sum_{i=1}^{n} \frac{k_i(\overline{x}_i - \overline{Q}_i)}{k_i(-\overline{s} - \overline{s}^3 c_i''(\overline{x}_i)) - \lambda} \right]. \end{aligned} \tag{10}$$

In order to simplify the notation let $d_i = k_i(-\overline{s} - \overline{s}^3 c_i''(\overline{x}_i))$ and $e_i = k_i(\overline{x}_i - \overline{Q}_i)$ for $i = 1, 2, \ldots, n$, and assume that the different d_i values are $d_1 < d_2 < \cdots < d_r$. Let I_l denote the set of agents with the same d_l value ($l = 1, 2, \ldots, r$), and let m_l denote the number of agents in set I_l. Then the eigenvalues are the solutions of the equation

$$\prod_{l=1}^{r}(d_l - \lambda)^{m_l}\left[1 + \sum_{l=1}^{r}\frac{\gamma_l}{d_l - \lambda}\right] = 0, \tag{11}$$

where $\gamma_l = \sum_{i \in I_l} e_i$. The eigenvalues can be obtained in the following way. If $m_l > 1$, then $\lambda = d_l$ is an eigenvalue with multiplicity $m_l - 1$, and if $m_l = 1$, then the corresponding factor cancels, so in this case $\lambda = d_l$ is not an eigenvalue ($l = 1, 2, \ldots, r$). In this way we have $m_1 + \cdots + m_r - r = n - r$ eigenvalues (including with multiplicities). The other eigenvalues are the solutions of the nonlinear equation

$$1 + \sum_{l=1}^{r}\frac{\gamma_l}{d_l - \lambda} = 0. \tag{12}$$

Since this is equivalent to a polynomial equation of degree r, it has r real (or complex) roots.

The previous derivation implies the following.

Theorem 1. (i) *Assume that with some l, $m_l \geq 2$ and $d_l > 0$. Then the equilibrium is locally unstable.*

(ii) *Suppose that for all l such that $m_l \geq 2$, $d_l < 0$, then let $\lambda_1, \ldots, \lambda_r$ denote the roots of equation*

$$1 + \sum_{l=1}^{r}\frac{\gamma_l}{d_l - \lambda} = 0. \tag{13}$$

If for at least one i, $Re\,\lambda_i > 0$, then the equilibrium is locally unstable. If for all i, $Re\,\lambda_i < 0$, then the equilibrium is locally asymptotically stable.

We note that in the case when for all roots, $\mathrm{Re}\,\lambda_i \leq 0$ and for at least one root $\mathrm{Re}\,\lambda_i = 0$, no conclusion can be given based on only the eigenvalues, since the system is nonlinear.

In order to apply the above theorem, the locations of the roots of equation (13) should be examined. We will consider two major cases.

Case A. Assume that for all l,

$$\gamma_l \leq 0. \tag{14}$$

Notice that if for some l the value of γ_l is zero, then the corresponding term is missing from the left-hand side of equation (13) without changing its structure. Therefore in the following discussion we may assume that inequality (14) is strict for all l. Introduce

$$g(\lambda) = \sum_{l=1}^{r}\frac{\gamma_l}{d_l - \lambda}. \tag{15}$$

It is easy to see that

$$\lim_{\lambda \to d_l + 0} g(\lambda) = \infty, \quad \lim_{\lambda \to d_l - 0} g(\lambda) = -\infty, \tag{16}$$
$$\lim_{\lambda \to \pm\infty} g(\lambda) = 0, \quad \text{and} \quad g'(\lambda) < 0$$

for all λ, except the poles. The graph of $g(\lambda)$ is illustrated in Figure 1, where we assume that the agents are numbered so that $d_1 < d_2 < \cdots < d_r$.

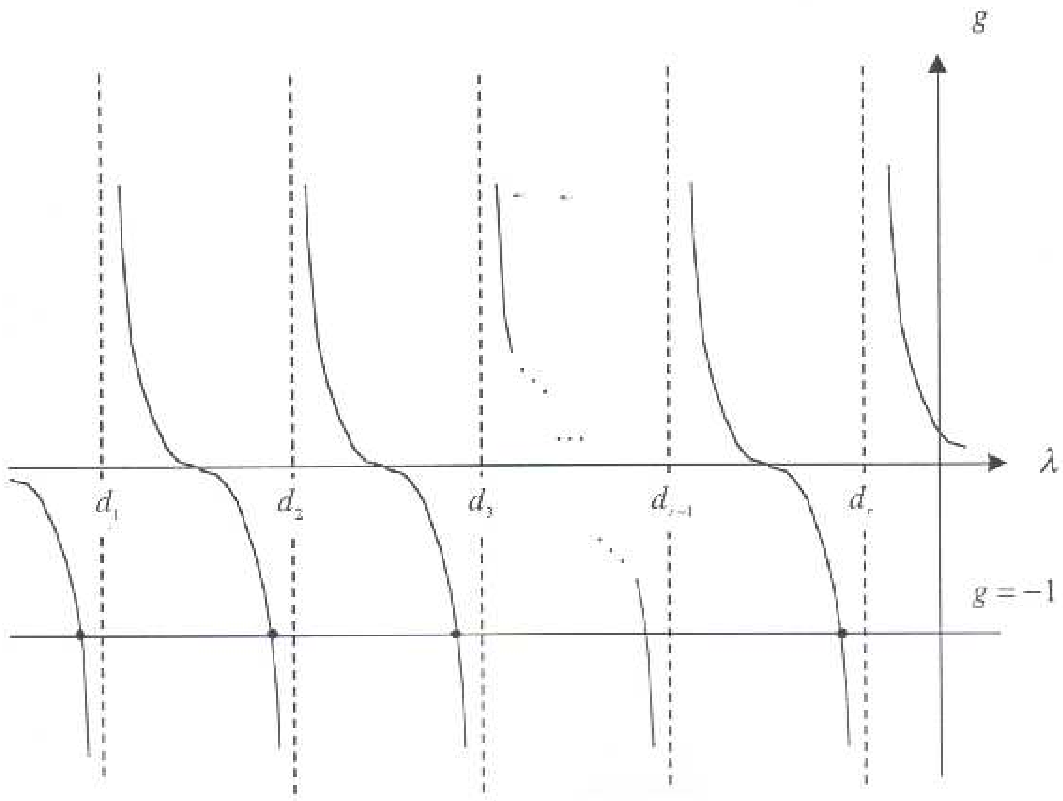

Figure 1. The shape of $g(\lambda)$ in Case A.

Since equation (13) can be written as $g(\lambda) = -1$, we see that all roots are real: one before d_1, and one between each d_i and d_{i+1} $(i = 1, 2, \ldots, r-1)$.

Here we have the following result.

Theorem 2. *Consider the case in which $\gamma_l \leq 0$ for all l.*

(i) *If for all l, $d_l \leq 0$, then all roots are negative implying the local asymptotical stability of the equilibrium.*

(ii) *Assume that $d_r > 0$ and all $d_i < 0$ $(i = 1, 2, \ldots, r-1)$. If $g(0) < -1$, then the equilibrium is locally asymptotical stable and if $g(0) > -1$, then the equilibrium is locally unstable.*

(iii) *If $d_r > 0$ and $d_{r-1} \geq 0$, then the equilibrium is locally unstable regardless of the signs of the other d_i values $(i = 1, 2, \ldots, r-2)$.*

We note again that in the case when the largest root is zero, no conclusion can be given, since the system is nonlinear.

Case B. For some l^*,

$$\gamma_{l^*} > 0. \tag{17}$$

We first show that this relation can hold for only one value of l^*. Notice first that there is at most one dominant agent p such that $\overline{x}_p > \overline{Q}_p$. If set I_t does not contain this agent, then

$$\gamma_t = \sum_{i \in I_t} k_i(\overline{x}_i - \overline{Q}_i) \leq 0,$$

therefore agent p must belong to set I_{l^*}. We show next that $d_{l^*} < 0$ necessarily for the set satisfying inequality (17). The second order conditions (6) imply that for the dominant agent p,

$$-\overline{s} - \overline{s}^3 c_p''(\overline{x}_p) < -\overline{s} + 2\overline{Q}_p = \overline{Q}_p - \overline{x}_p,$$

and by multiplying this inequality by k_p we have

$$d_{l^*} = k_p(-\overline{s} - \overline{s}^3 c_p''(\overline{x}_p)) < k_p(\overline{Q}_p - \overline{x}_p) = -e_p.$$

Since for all other agents $i \in I_{l^*}$, $-e_i \geq 0$, we see that

$$d_{l^*} < -\sum_{i \in I_{l^*}} e_i = -\gamma_{l^*} < 0. \tag{18}$$

For the sake of simplicity assume that $l^* = r$. We keep the assumption that $d_1 < d_2 < \cdots < d_{r-1}$ and note that d_r is not necessarily the largest among the d_l values. In this case function g satisfies the following properties:

$$\begin{aligned} &\lim_{\lambda \to d_l + 0} g(\lambda) = \infty, \ \lim_{\lambda \to d_l - 0} g(\lambda) = -\infty \quad (l = 1, 2, \ldots, r-1), \\ &\lim_{\lambda \to d_r + 0} g(\lambda) = -\infty, \ \lim_{\lambda \to d_r - 0} g(\lambda) = \infty, \end{aligned} \tag{19}$$

and

$$\lim_{\lambda \to \pm\infty} g(\lambda) = 0.$$

Notice that no condition can be given on the sign of the derivative $g'(\lambda)$ in this case. The shape of function g is illustrated in Figure 2, where d_r is either the largest, or the smallest among the d_r values, or it is in the middle.

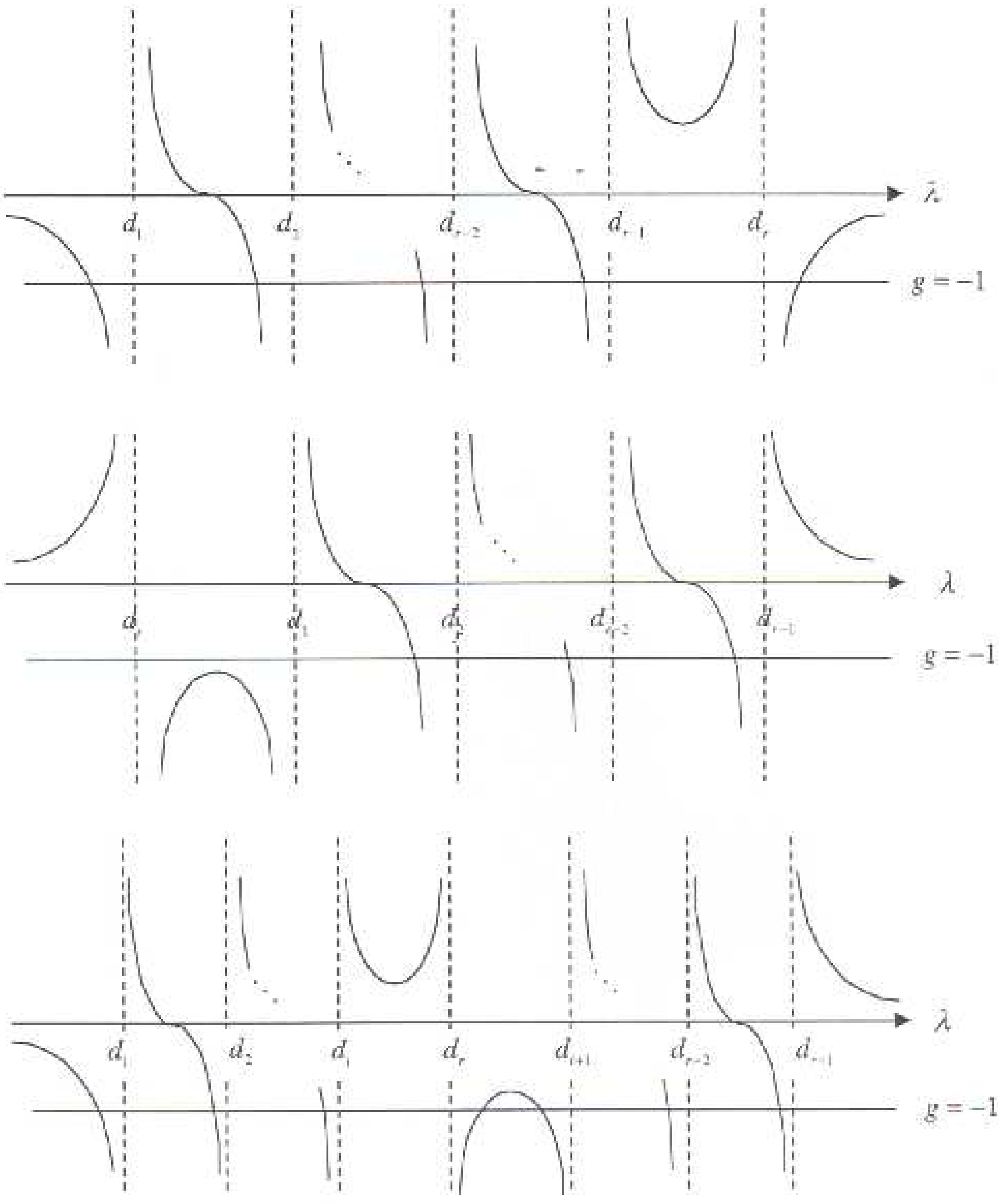

Figure 2. The shape of $g(\lambda)$ in Case B.

The above discussion implies the following

Theorem 3. *Consider the case in which $\gamma_{l^*} > 0$ for some l^*.*

(i) *Assume first that $d_r > d_l$ $(l = 1, 2, \dots, r-1)$. If $d_r \geq 0$, then the equilibrium is unstable. If $d_r < 0$, then in the case of $g(0) > -1$ the equilibrium is locally asymptotically stable, and if $g(0) < -1$, the equilibrium is locally unstable.*

(ii) *Assume now that $d_r < d_l$ $(l = 1, 2, \dots, r-1)$ and there are real roots between d_r and d_1. If $d_{r-2} \geq 0$, then the equilibrium is unstable. If $d_{r-2} < 0 < d_{r-1}$, then in the case of $g(0) < -1$ the equilibrium is locally asymptotically stable, and if $g(0) > -1$, the equilibrium is locally unstable. If $d_{r-1} \leq 0$, then the equilibrium is locally asypmtotically stable.*

(iii) *Assume next that $d_{r-2} < d_r < d_{r-1}$, and there are real roots between d_{r-2} and d_{r-1}. If $d_{r-2} \geq 0$, then the equilibrium is locally unstable. If $d_{r-2} < 0 < d_{r-1}$, then in the case when the roots between d_{r-2} and d_{r-1} are negative, the equilibrium is locally asymptotically stable, and if at least one root between d_{r-2} and d_{r-1} is positive, the equilibrium is locally unstable. If $d_{r-1} \leq 0$, then the equilibrium is locally asymptotically stable.*

(iv) *Assume finally that $d_{i-1} < d_r < d_i$ with some $i \leq r-2$, and there are real roots between d_{i-1} and d_i. If $d_{r-2} \geq 0$, then the equilibrium is locally unstable. If $d_{r-2} < 0 < d_{r-1}$ then in the case of $g(0) < -1$ the equilibrium is locally asymptotically stable, and if $g(0) > -1$, the equilibrium is locally unstable. If $d_{r-1} \leq 0$, then the equilibrium is locally asymptotically stable.*

Notice that the theorem does not apply for cases when the largest real root is zero or there are no roots in the intervals indicated. In every case there are always $(r-2)$ real roots, therefore either all roots are real, or there is exactly one complex conjugate pair of roots. The next example shows that both possibilities might occur.

Example 1. Consider the special case when $r = 2$, $k_1 = k_2 = 1$, $d_2 = -1$, $d_1 = 0$. Let $2\overline{x}_2 - \overline{s} = \gamma > 0$ be a parameter. From equation (18) we must have $\gamma < 1$. Furthermore

$$2\overline{s}_1 - m_1\overline{s} = 2(\overline{s} - \overline{x}_2) - m_1\overline{s} = -(m_1 - 2)\overline{s} - 2\overline{x}_2 = -(m_1 - 1)\overline{s} - \gamma.$$

By introducing the notation $\alpha = (m_1 - 1)\overline{s}$ we see that α can take on any nonnegative value. Then equation (13) has the form

$$\frac{-\alpha - \gamma}{0 - \lambda} + \frac{\gamma}{-1 - \lambda} = -1,$$

which simplies as

$$\lambda^2 + \lambda(1 + \alpha) + (\alpha + \gamma) = 0.$$

The discriminant is

$$D = (1 + \alpha)^2 - 4(\alpha + \gamma) = (1 - \alpha)^2 - 4\gamma$$

showing that the cases of both real and complex roots are possible.

1. The Possibility of Limit Cycles.

In the case of locally unstable equilibrium we can demonstrate the existence of limit cycle motion by applying the Hopf bifurcation theorem (see, for example, Guckenheimer and Holmes, 1983). We need to determine the region in the parameter space where pure complex eigenvalues exist. Letting $\lambda = i\alpha$, equation (13) has the form

$$\sum_{l=1}^{r-1} \frac{\gamma_l}{d_l - i\alpha} + \frac{\gamma_r}{d_r - i\alpha} = -1, \tag{20}$$

where $\gamma_l < 0$ $(i = 1, 2, \ldots, r-1)$, and $\gamma_r > 0$.

By equating the real and imaginary parts we have

$$\sum_{l=1}^{r-1} \frac{\gamma_l d_l}{d_l^2 + \alpha^2} + \frac{\gamma_r d_r}{d_r^2 + \alpha^2} = -1$$

and

$$\sum_{l=1}^{r-1} \frac{\gamma_l i\alpha}{d_l^2 + \alpha^2} + \frac{\gamma_r i\alpha}{d_r^2 + \alpha^2} = 0. \tag{21}$$

Since we assume that $\alpha \neq 0$, the second equation implies that

$$\gamma_r = -(d_r^2 + \alpha^2) \sum_{l=1}^{r-1} \frac{\gamma_l}{d_l^2 + \alpha^2} \tag{22}$$

and combining this relation with the first equation of (21) leads to a simple equation for α^2:

$$\sum_{l=1}^{r-1} \frac{\gamma_l (d_l - d_r)}{d_l^2 + \alpha^2} = -1. \tag{23}$$

Notice that this is a polynomial equation of degree $r-1$ for α^2, which can be solved by routine methods (see for example, Szidarovszky and Yakowitz [6]). Notice that in case (i) of Theorem 3, the left-hand side is always positive, so no real root exists. This observation coincides with the corresponding results when we say that all roots of equation (13) are real, so no pure complex root exists. The existence of real roots of equation (23) depends on the signs and the orders of magnitudes of the differences $d_l - d_r$. The following example illustrates the possibility of real roots and hence the possibility of limit cycles.

Example 2. Consider now the case of $r = 3$, $\gamma_1 = -a$, $\gamma_2 = -b$ (where a and b are positive parameters), $\gamma_3 = \gamma > 0$ is the bifurcation variable, and $d_1 = -2$, $d_2 = 1$, $d_3 = -1$. Then equation (20) has the form

$$\frac{-a}{-2-\lambda} + \frac{-b}{1-\lambda} + \frac{\gamma}{-1-\lambda} = -1. \tag{24}$$

If $\lambda = i\alpha$, then

$$\frac{-a}{-2-i\alpha} + \frac{-b}{1-i\alpha} + \frac{\gamma}{-1-i\alpha} = -1.$$

Equating the real and imaginary parts,

$$\frac{2a}{4+\alpha^2}+\frac{-b}{1+\alpha^2}+\frac{-\gamma}{1+\alpha^2}=-1$$

and

$$\frac{-ai\alpha}{4+\alpha^2}+\frac{-bi\alpha}{1+\alpha^2}+\frac{\gamma i\alpha}{1+\alpha^2}=0.$$

Notice that we are interested in the case of $\alpha \neq 0$. So solving both equations for $\frac{\gamma}{1+\alpha^2}$, we get

$$\frac{\gamma}{1+\alpha^2}=\frac{2a}{4+\alpha^2}-\frac{b}{1+\alpha^2}+1=\frac{a}{4+\alpha^2}+\frac{b}{1+\alpha^2}. \tag{25}$$

A fourth order equation is obtained for α:

$$\alpha^4+\alpha^2(5+a-2b)+(4+a-8b)=0. \tag{26}$$

Notice that this equation will have positive roots if $4+a-8b<0$. In order to satisfy relation (18) we have to guarantee that $\gamma<1$, that is,

$$\frac{\gamma}{1+\alpha^2}=\frac{a}{4+\alpha^2}+\frac{b}{1+\alpha^2}<\frac{1}{1+\alpha^2},$$

which can be rewritten as

$$\alpha^2(a+b-1)<4-a-4b.$$

For example, by selecting $a=0.15$, $b=0.75$, this inequality is satisfied for all $\alpha^2 \geq 0$, furthermore from equation (26) we have

$$\alpha^2 \approx 0.451 \quad \text{and} \quad \alpha^2 \approx -4.101,$$

from which we see that the real roots are

$$\alpha_{1,2} \approx \pm 0.672.$$

Substituting these values into equation (25) we see that $\gamma \approx 0.799$. Differentiating equation (24) with respect to γ leads to the derivative

$$\frac{d\lambda}{d\gamma}=\frac{\frac{1}{1+\lambda}}{\frac{-a}{(-2-\lambda)^2}+\frac{-b}{(1-\lambda)^2}+\frac{\gamma}{(1+\lambda)^2}},$$

which has the approximating value $(0.468+0.719i)$ at $\lambda=i\alpha$ with the above values of α and γ. Hence the real part is nonzero showing the existence of a limit cycle.

CONCLUSIONS

In this paper nonlinear rent-seeking games were analyzed where the cost functions were assumed to be nonlinear. A dynamic adjustment process in which each agent adjusts its output proportionally to its marginal profit was examined. The local asymptotical stability of the equilibrium was first examined. In the absence of a dominant agent it turns out that all eigenvalues are real. Conditions for the local asymptotical stability and also for the local instability of the equilibrium were then derived. In the presence of a dominant agent complex eigenvalues are also possible. Simple conditions were presented for the stability and also for the instability of the equilibrium in the case of real eigenvalues. If complex eigenvalues are possible, then we have shown the possibility of limit cycles via the use of the Hopf bifurcation theorem.

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