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MATHEMATICAL BASIS OF A SYSTEM SUPPORTING INTERCONNECTION DESIGN

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Abstract. A software system is constructed for the computation of characteristic model parameters of multi-conductor strip and microstrip lines which are needed in design of high-speed packaging and Very Large Scale Integration (VLSI) interconnections. New mathematical algorithms are developed which assure high accuracy and an excellent computational efficiency. Three major mathematical improvements are presented in this paper: (1) a new matrix square root algorithm which is used in computing an admittance matrix; (2) a new matrix algorithm based on the theory of M-matrices which provides monotonic global convergence to the exact solution in calculating a diagonally matched load impedance matrix of transmission lines; and (3) a multi-dimensional cubic spline algorithm which is used in computing intermediate values from the database of transmission line parameters. Model parameters of transmission lines are stored in a database, which can be used to determine a proper transmission line structure for a set of desired line electrical characteristics. Databases for three and five conductor transmission line systems were constructed. It was found that five conductor systems can effectively represent the seven, nine or more conductor systems for most practical applications. Use of a special spline interpolation in our algorithms involving interconnection database resulted in a substantial reduction of database size, in comparison to algorithms with linear interpolation without compromising accuracy. Consequently, computer time needed for calculations involving database is also significantly reduced. The expert system has special importance in repeated applications and in cases when several design alternatives have to be considered.

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1. INTRODUCTION.

Unrelenting increase in operating speed and size of integrated circuits and systems require careful analysis of signal transmission through interconnections. Important components in modeling these interconnections are multiple, coupled transmission lines. Transmission line models depict such significant phenomena in high speed circuits as signal delay, reflections at terminations or discontinuities, and noise which are due to electro-magnetic coupling between the adjacent lines (cross-talk) and current surges occurring on power

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Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX and ground lines during the switching of gates [1,2]. Another trend observed in technology is the decreasing level of supply voltage, which is enforced by a dramatic increase in portable equipment applications. Noise analysis in signal transmission becomes more important in the circuits with lower operating voltage because the noise margins and noise immunities do not scale down linearly with the supply voltage. In this situation the inclusion of transmission line phenomena in the analysis and design of circuits and systems is critical. However, such phenomena are quite complicated and their analysis involves sophisticated algorithms, which require the usage of substantial computer resources [2,3].

The most critical features of software support for design are the ease of use and its response time. In most applications several design alternatives should be considered in order to find the most appropriate interconnection structure. Previous experience with design data bases for interconnections [9] indicated that efficiency of techniques is very critical in synthesis which involves repeated calculations. Given some requirements many design alternatives can be generated immediately by applying proper approximation schemes. The systematic use of previously obtained data is the basis of an "expert system,"the fundamentals of which will be also described in this paper.

Currently available interconnection support software is both complicated in use and slow in turn-around time. This paper presents new, fast algorithms for calculating interconnection model quantities such as characteristic impedance and diagonally matched load impedances. These new algorithms are very efficient in terms of computer time and thus are suitable for application in constructing an interconnection design support software.

In selecting a final model for the interconnections there is an additional trade-off between the accuracy and efficiency of computation. Lossy transmission line depicts the transmission related phenomena such as signal delay, cross-talk, and reflections at the terminations and discontinuities with adequate accuracy, but this is obtained at the price of complicated calculations requiring specialized algorithms and computer programs, significant processing time, and involved post-processing analysis. Much more efficient in this respect is a simplified modeling which neglects material losses and uses lossless transmission line models. Analysis of signal transmission with such a simplification yields upper bounds for cross-talk and reflections, which is acceptable in design. The only problem requiring caution is a signal delay which is underestimated in the analysis based on lossless transmission lines. Nevertheless, the noise estimates, which as stated above are becoming most critical, are provided by the lossless line models with sufficient accuracy. In many applications signal delay in interconnections presents only a small fraction of clock cycle. Thus in view of these considerations it was decided to base the system development on modeling interconnections by lossless transmission lines.

2. PROBLEM FORMULATION.

Characterization and analysis of transmission line system begin with a comprehensive modeling of interconnect systems.

2.1 Model of Interconnections.

A simplified model of multiple, parallel interconnections is shown in Figure 2.1. It is assumed for simplicity that the signal sending circuits (sources) are at the near-end ($x = 0$). These circuits are represented by the ideal voltage sources (e_α) with their internal impedances (r_{Gi}); $i = 1, 2, \ldots, N$. The receiving circuits located at $x = D$ are represented by resistors (r_{Li}). An interpretation of those models and receiving circuits built in specific technologies are discussed below.

The transmission line equations in terms of voltage waves are

$$\begin{aligned} u(x,t) &= u_+(x,t) + u_-(x,t) \\ i(x,t) &= Y_0 u_+(x,t) - Y_0 u_-(x,t) \end{aligned} \tag{2.1}$$

where u, i are vectors of line voltages and currents respectively, u_+ and u_- are vectors representing respective voltage waves traveling in positive and negative directions of the x-axis, and Y_0 is the characteristic admittance matrix of the interconnection system. The characteristic admittance matrix has positive diagonal and non-positive off-diagonal elements. This matrix is computed using the inductance, L, and capacitance, C, matrices of interconnecting structures. The L, C matrices are determined from the interconnect geometry and material properties using novel approach with the use of expert database. The details are provided below. Some new mathematical algorithms for computing the relevant electrical quantities of interconnects are described further in this paper. At near-end ($x = 0$) we have the following circuit relations in the vector form:

$$Y_G E_G - i_G = Y_G u(0,t) \tag{2.2}$$

where E_G, i_G are respective vectors of voltage sources, and their currents, and Y_G is a diagonal matrix

$$Y_G = \mathrm{diag}(\frac{1}{r_{G1}}, \ldots, \frac{1}{r_{GN}}). \tag{2.3}$$

At the far-end ($x = D$) the circuit relations are

$$Y_L u(D,t) = i(D,t) \tag{2.4}$$

where Y_L is a diagonal matrix

$$Y_L = \mathrm{diag}(\frac{1}{r_{L1}}, \ldots, \frac{1}{r_{LN}}). \tag{2.5}$$

2.2 Coupling Noise Estimators.

Noting that $i_G = i(0,t)$ and utilizing the transmission line equations (2.1) and (2.2) yield

$$u_+(0,t) = \tau_G E_G + \rho_G u_-(0,t) \tag{2.6}$$

where $\tau_G = (Y_0 + Y_G)^{-1} Y_G$ is the matrix of source injection coefficients and $\rho_G = (Y_0 + Y_G)^{-1}(Y_0 - Y_G)$ is the matrix of reflection coefficients at the source end.

Equation (2.6) gives basis for building an estimator of near-end cross-talk. For the limited time ($t \leq 2D\tau_{min}$ where τ_{min} is the smallest modal delay per unit of length), $u_-(0,t) = 0$ and the formula for near-end cross-talk is

$$V_{NE} = \tau_G E_G \tag{2.7}$$

and it permits to obtain the worst case estimate (one line passive with all neighbors active) of cross-talk.

Analogously the application of transmission line equations (2.1) to the far-end ($x = D$) relation yields

$$u_-(D,t) = \rho_L u_+(D,t) \tag{2.8}$$

where $\rho_L = (Y_L + Y_0)^{-1}(Y_0 - Y_L)$ is the matrix of load (far-end) reflection coefficients. Formula (2.8) gives simple estimation of far-end cross talk (V_{FE}) for $D\tau_{min} \leq t \leq 3D\tau_{min}$, because for that time the incoming voltage is $u_+(D,t) = \tau_G E_G$ and thus

$$V_{FE} = (I + \rho_L)\tau_G E_G, \tag{2.9}$$

where I is the identity matrix.

For $t < D\tau_{min}$ there is no voltage at the far-end and thus $V_{FE} = 0$. For other time intervals the formulas are slightly more complicated, because it is necessary to account for reflections.

2.3 Noise Estimation for CMOS and BJT Circuits.

In Complementary Metal-Oxide-Semiconductor (CMOS) circuits commonly used in practice, the far-end can be considered open, i.e. $Y_L = 0$, resulting in perfect reflections because the load consists of small terminating capacitances. This means that the matrix of reflection coefficients at the far-end is the identity matrix:

$$\rho_L = I. \tag{2.10}$$

The near-end in such circuits is terminated by the properly designed drivers forming so called diagonally matched load impedances which bring the entries on the main diagonal of reflection coefficients matrix to zero.

From (2.9) it is deduced that for CMOS circuits the estimate of far-end noise is

$$V_{FE} = 2\tau_G E_G. \tag{2.11}$$

It should be noted that this estimate is valid for limited time $D\tau_{min} \leq t \leq 3D\tau_{min}$, because it does not account for reflections from the near-end which may arrive after the time $3D\tau_{min}$.

The Bipolar Junction Transistors (BJT) circuits are typically terminated by the diagonally matched resistances at the far-end while the near-end circuits are driven by powerful drivers with negligible output impedances such that the near-ends are unterminated. In this case the near-end voltages (including cross-talk) may be determined by the formula (2.7). Far-end may be considered terminated, $\rho_L \cong 0$, and the far-end voltages are approximately equal to the near-end ones but shifted in time. The operation of properly designed BJT circuits is called "first incident switching "as the line voltages reach threshold levels at the first incident.

3. DETAILS OF MATHEMATICAL ALGORITHMS.

In this section the algorithms for numerical solutions to the basic computational problems are outlined and their applicability is discussed.

3.1 Matrix Square Root Algorithm.

The characteristic admittance matrix of lossless transmission line systems, composed of N-signal conductors and a reference conductor is computed using the formula

$$Y_0 = L^{-1}(LC)^{\frac{1}{2}}.$$

In order to apply this equation to find Y_0, we need a fast method to compute the square root of a matrix.

The definition of square root of and $N \times N$ matrix A can be easily rewritten as a system of N^2 nonlinear equations for the N^2 unknown elements of a square root matrix X. If a_{ij} and x_{ij} denote the (i,j) elements of A and X, respectively, then these equations are as follows:

$$a_{ij} = \sum_{l=1}^{N} X_{il} X_{lj} \qquad (i,j = 1,2,\ldots,N). \tag{3.1}$$

The two most commonly used techniques for solving such equations are the multi-variable Newton method and eigenvalue analysis. It is well known [4] that the Newton method has only local convergence, and therefore a sufficiently good initial approximation is needed to guarantee convergence. Such initial approximations are difficult to obtain. Eigenvalue analysis is a good known method, but it needs more computation than the method suggested below [4]. This computation is particularly difficult and the algorithms may be even ambiguous in case of multiple eigenvalues. The computation speed is essential when real time computation is performed and large size data base construction is needed. In this section an algorithm based on matrix triangularization is introduced, and a closed form is presented for the square roots of triangular matrices.

Assume first that U is an upper triangular matrix with positive diagonal elements.

Theorem 1. *The square root of U is upper triangular, and it can be obtained in a finite, closed form.*

Proof. We are looking for the square root V of matrix U in an upper triangular form. Let u_{ij} and $v_{ij} (i \leq j)$ denote the (i,j) elements of U and V, respectively.

Then equation $U = V^2$ can be rewritten as

$$u_{ij} = \sum_{l=1}^{j} v_{il}v_{lj} \qquad (i \leq j). \tag{3.2}$$

If $i = j$, then

$$u_{ij} = v_{ii}^2,$$

therefore for all i,

$$v_{ii} = \sqrt{u_{ii}}. \tag{3.3}$$

If $j = i + 1$, then

$$u_{ij} = v_{ii}v_{ij} + v_{ij}v_{jj},$$

which implies that

$$v_{ij} = \frac{u_{ij}}{v_{ii} + v_{jj}} \tag{3.4}$$

In the general case, when $j \geq i + 2$,

$$u_{ij} = v_{ii}v_{ij} + \sum_{l=i+1}^{j-1} v_{il}v_{lj} + v_{ij}v_{jj},$$

from which we conclude that

$$v_{ij} = \frac{1}{v_{ii} + v_{jj}}(u_{ij} - \sum_{l=i+1}^{j-l} v_{il}v_{lj}). \tag{3.5}$$

Equations (3.3), (3.4) and (3.5) give a finite algorithm to compute matrix V. Use equation (3.3) with $i = N$ to obtain v_{NN}. Select $i = N - 1$, and use equation (3.3) again with $i = N - 1$ to get $v_{N-1,N-1}$ and then use equation (3.4) with $j = N$ to compute $v_{N-1,N}$. By selecting sequentially $i = N - 2, N - 3, \ldots, 2, 1$, all other rows of V are determined in the following way. The element V_{ii} is obtained from equation (3.3), the element $v_{i,i+1}$ is computed by equation (3.4) with $j = i + 1$, and all other elements of this row are obtained by using equation (3.5) in the order, $j = i + 2, \ldots, N$. Thus the theorem is proven.

It is well known from matrix theory [4] that all real square matrices can be transformed into upper triangular form, and for example, the QR algorithm is very efficient to find this transformation. Assume that $A = T^{-1}UT$, where U is upper triangular and T is invertible. Then

$$X = T^{-1}VT, \tag{3.6}$$

where V is the square root of U, since

$$X^2 = T^{-1}VTT^{-1}VT = T^{-1}V^2T = T^{-1}UT = A.$$

In summary, if A is a square matrix, then in the first step we find U of the triangular transformation of A. The second step is the application of the algorithm presented in the proof of the theorem for matrix U. And finally, X is obtained by using equation (3.6).

3.2 A New Matrix Algorithm.

In finding diagonally matched load impedance (impedance of drivers in CMOS circuits and far-end terminating impedances in bipolar circuits) the following problem has to be solved. Assume that Y is a given $N \times N$ real matrix with positive diagonal and non-positive off-diagonal elements. It is also assumed that Y^{-1} exists and is non-negative. Find an $N \times N$ real diagonal matrix X with positive diagonal such that matrix

$$\rho = (Y+X)^{-1}(Y-X)$$

has zero diagonal.

Let y_{ij} and r_{ij} denote the (ij) elements of Y and ρ, respectively, and let x_i denote the ith diagonal element of X. Notice first that the defining equation of matrix ρ can be rewritten as

$$(Y+X)\rho = Y - X,$$

therefore this problem is equivalent to the solution of the following system of nonlinear equations:

$$y_{ii} - x_i = \sum_{l=1, l\neq i,j}^{N} y_{il} r_{li} (i = 1, 2, \ldots, N)$$

and

$$y_{ij} = \sum_{l=1, l\neq i,j}^{N} y_{il} r_{lj} + (y_{ii} + x_i) r_{ij} \qquad (i = 1, 2, \ldots, N; j \neq i). \tag{3.7}$$

There are $N + N(N-1) = N^2$ equations for the N^2 unknowns: x_i and $r_{ij} (j \neq i)$. It can be proved that this system has at least one solution, which can be obtained by using the multi-variable Newton method, however the disadvantages of this method being discussed in the previous section still apply. An alternative approach based on diagonalization was proposed in [5], and a method based on optimization was applied in [6].

The proposed new solution method does not require the knowledge of good initial approximation, it always converges monotonically to a solution if starting from the zero matrix, and is computationally more efficient than earlier methods. In practice however we recommend the combination of our method and Newton's method in the following way. Apply our method until the approximation becomes good enough to serve as an initial approximation for a convergent Newton's sequence. Then switch to the Newton's method, which will converge very fast to the solution.

Our algorithm is based on the special mapping $F : R^{N\times N} \to R^{N\times N}$ of real matrices such that $F(A)$ is defined as the diagonal part of A for all matrices. The defining equation for ρ can be rewritten as

$$\rho = (Y+X)^{-1}(Y+X-2X) = I - 2(Y+X)^{-1}X.$$

Therefore

$$F((Y+X)^{-1}X) = \frac{1}{2} I,$$

and consequently,

$$F((Y+X)^{-1}) = \frac{1}{2} X^{-1}.$$

This equation can be rewritten as

$$X = \frac{1}{2}[F((Y+X)^{-1})]^{-1}, \tag{3.8}$$

which is an N-dimensional fixed point problem, since X is diagonal. Consider now the iteration procedure

$$X^{(k+1)} = \frac{1}{2}[F((Y + X^{(k)})^{-1})]^{-1}, \tag{3.9}$$

starting from the initial approximation $X^{(0)} = 0$.

Theorem 2. *The iteration sequence defined by (3.9) is monotonic and bounded, therefore it converges to the smallest non-negative fixed point.*

Proof. The assumptions imply that Y is an M-matrix [7]. Therefore $X^{(1)} \geq 0 = X^{(0)}$. Assume that $X^{(k)} \geq X^{(k-1)}$ with some $k > 0$. The monotonicity of the inverse of M-matrices implies that the mapping defined by the right hand side of equation (3.8) increases in X. Therefore

$$X^{(k+1)} = \frac{1}{2}[F((Y + X^{(k)})^{-1})]^{-1} \geq \frac{1}{2}[F(Y + X^{(k-1)})^{-1})]^{-1} = X^{(k)},$$

hence $X^{(k+1)} \geq X^{(k)}$ for all $k \geq 0$. That is, the iteration sequence is monotonic.

Assume next that X^* is a non-negative fixed point, which exists. We will next prove that for all $k, X^{(k)} \leq X^*$. Obviously $X^{(0)} = 0 \leq X^*$. Assume that for some $k, X^{(k)} \leq X^*$ Then

$$X^{(k+1)} = \frac{1}{2}[F((Y + X^{(k)})^{-1})]^{-1} \leq \frac{1}{2}[F((Y + X^*)^{-1})]^{-1} = X^{\star},$$

therefore $X^{(k)} \leq X^*$ for all $k \geq 0$.

Since the iteration sequence is bounded, it converges to a non-negative diagonal matrix. The continuity of the right hand side of equation (3.8) in X implies that the limit is a fixed point. Since $X^{(k)}$ is bounded from above by all non-negative fixed points, the limit is the smallest non-negative fixed point, which completes the proof.

3.3 A Special Multi-Dimensional Cubic Spline Algorithm.

The practical way to reduce the memory requirement (influenced by the database size) in constructing the interconnect database must consider a good approximation schemes between the nodes. In the literature the most commonly used methods of function approximations are the interpolating polynomials, the least squares fits, and splines. In our case the first two methods cannot be applied. Interpolating polynomials have very high degree if the number of nodes is large, as it is in our case. It is well known that interpolation polynomials do not necessarily converge to the target functions and they are subject to error cummulation. Least squares polynomials do not interpolate, therefore there are always errors at the nodes. Cubic splines offer several advantages in this application. In each segment the splines are cubic polynomials, they are differentiable and smooth functions which interpolate at all nodes. For more details, see, for example [8]. The differentiality and smoothness are required in numerical simulation with the use of database. The basis for multi-dimensional spline functions is the construction of a special one-dimensional spline, which is first discussed. Consider four points $(x_{i-1}, f_{i-1}), (x_i, f_i), (x_{i+1}, f_{i+1})$ and (x_{i+2}, f_{i+2}). In segment $[x_i, x_{i+1}]$

$$s_i(x) = A_i\omega^3 + B_i\omega^2 + C_i\omega + D_i \tag{3.10}$$

where A_i, B_i, C_i, D_i are unknown coefficients and

$$\omega = \frac{x - x_i}{h_i}$$

is a new variable with $h_i = x_{i+1} - x_i$. We do not require that the step sizes $h_{i-1} = x_i - x_{i-1}, h_i = x_{i+1} - x_i$, and $h_{i+1} = x_{i+2} - x_{i-1}$ are the same as in most publications.

The following conditions are assumed:

$$
\begin{aligned}
s_i(x_i) &= f_i \\
s_i(x_{i+1}) &= f_{i+1} \\
s_i'(x_i) &= p_i'(x_i) \\
s_i'(x_{i+1}) &= p_{i+1}'(x_{i+1})
\end{aligned} \tag{3.11}
$$

where p_i and p_{i+1} are the quadratic interpolation polynomials fitting the points $(x_{i-1}, f_{i-1}), (x_i, f_i)$, (x_{i+1}, f_{i+1}), and $(x_i, f_i), (x_{i+1}, f_{i+1}), (x_{i+2}, f_{i+2})$, respectively. The first and second conditions mean that s_i interpolates at the two end-points of the segment, the third and fourth equations imply that

$$s_i'(x_i) = s_{i-1}'(x_i)$$

and

$$s_i'(x_{i+1}) = s_{i+1}'(x_{i+1}),$$

that is, the spline is differentiable at the nodes. Between the nodes it is obviously differentiable, since it is a cubic polynomial. Conditions (3.11) can be rewritten as

$$
\begin{aligned}
&D_i = f_i \\
&A_i + B_i + C_i + D_i = f_{i+1} \\
&\frac{1}{h_i} C_i = -\frac{h_i}{h_{i-1}(h_{i-1}+h_i)} f_{i-1} + \frac{-h_{i-1}+h_i}{h_{i-1}h_i} f_i + \frac{h_{i-1}}{h_i(h_{i-1}+h_i)} f_{i+1} \\
&\frac{1}{h_i}(3A_i + 2B_i + C_i) = -\frac{h_{i+1}}{h_i(h_i+h_{i+1})} f_i + \frac{-h_i+h_{i+1}}{h_i h_{i+1}} f_{i+1} + \frac{h_i}{h_{i+1}(h_i+h_{i+1})} f_{i+2}
\end{aligned}
$$

where we use the well known Lagrange form of interpolation polynomials [4]. We have four linear equations for four unknown coefficients. After solving for A_i, B_i, C_i, D_i substitute the results into the cubic polynomial form of $s_i(x)$ to obtain the formula

$$s_i(x) = f_{i-1}P_{i-1}(\omega) + f_i P_i(\omega) + f_{i+1}P_{i+1}(\omega) + f_{i+2}P_{i+2}(\omega), \tag{3.12}$$

where

$$
\begin{aligned}
P_{i-1}(\omega) &= \alpha_i\omega^3 - 2\alpha_i\omega^2 + \alpha_i\omega \\
P_i(\omega) &= (2 + \beta_i + \gamma_i)\omega^3 + (-3 - 2\beta_i - \gamma_i)\omega^2 + \beta_i\omega + 1 \\
P_{i+1}(\omega) &= (-2 + \delta_i + \eta_i)\omega^3 + (3 - 2\delta_i - \eta_i)\omega^2 + \delta_i\omega \\
P_{i+2}(\omega) &= \phi_i\omega^3 - \phi_i\omega^2
\end{aligned} \tag{3.13}
$$

with

$$
\begin{aligned}
&\alpha_i = \frac{h_i^2}{h_{i-1}(h_{i-1}+h_i)}, \beta_i = \frac{-h_{i-1}+h_i}{h_{i-1}}, \gamma_i = -\frac{h_{i+1}}{h_i+h_{i+1}}, \\
&\delta_i = \frac{h_{i-1}}{h_{i-1}+h_i}, \eta_i = \frac{-h_i+h_{i+1}}{h_{i+1}}, \quad \text{and} \quad \phi_i = \frac{h_i^2}{h_{i+1}(h_i+h_{i+1})}.
\end{aligned}
$$

For simple calculation we notice that

$$\delta_i = -\gamma_{i-1}\frac{h_{i-1}}{h_i}, \eta_i = \beta_{i+1}\frac{h_i}{h_{i+1}}, \alpha_i = \gamma_{i-1}\frac{h_i}{h_{i-1}},$$

and

$$\phi_i = -\gamma_i(\frac{h_i}{h_{i+1}})^2.$$

Based on equations (3.12) and (3.13) multi-dimensional splines are constructed in the same way as B-splines are defined [8]. Consider and n-dimensional net, where in coordinate $k(k = 1, 2, \ldots, n)$ the nodes are $x_{k1}, x_{k2}, \ldots, x_{kn_k}$. Define $h_{ki} = x_{k,i+1} - x_{ki}$ and assume that for a point $(x_1, \ldots, x_n), x_{ki_k} < x_k \le x_{k,i_k+1}$ for all k. Let $\omega_k = \frac{x_k - x_{ki_k}}{h_{ki_k}}$, and define polynomials $P_{k,i_k-1}, P_{k,i_k}, P_{k,i_k+1}$, and P_{k,i_k+2} similarly to equation (3.13) where $\alpha_i, \beta_i, \gamma_i, \delta_i, \eta_i$, and ϕ_i are computed by using step sizes in direction x_k. Then the spline segment is given as

$$S_{i_1 i_2 \ldots i_n}(x_1, \ldots, x_n) = \sum_{l_1=i_1-1}^{i_1+2} \sum_{l_2=i_2-1}^{i_2+2} \cdots \sum_{l_n=i_n-1}^{i_n+2} P_{1l_1}(\omega_1) P_{2l_2}(\omega_2) \ldots P_{nl_n}(\omega_n) f_{l_1 l_2 \ldots l_n} \tag{3.14}$$

where $f_{l_1 l_2 \ldots l_n}$ is the function value at the node $(x_{1l_1}, x_{2l_{2\ldots,}} x_{nl_n})$.

4. IMPLEMENTATION AND CASE STUDIES.

We now turn to the problem of implementation of the new algorithms for computing capacitance and inductance matrices of multi-conductor transmission line systems and other quantities relevant to signal transmission on packaging interconnections. The algorithms described in Section 3 have been implemented in a prototype design support system, named DIME (DIagonally Matched load impedance Estimator), written in C-language. The system DIME is currently running on SPARC work stations.

4.1 System Structure.

The software package consists of two main parts: routine computation of characteristic quantities in signal transmission, and analysis/design synthesis routine. Figure 4.1 shows the block diagram of the entire system. The routine for computation of transmission lines characteristic quantities includes a graphic editor accepting the pictorial input of the interconnecting structure. The program can accept both ASCII data and graphical representations of an interconnecting system cross section as its input data for analysis. The graphic editor captures the cross section of the structure. The geometry of cross section is specified in terms of grid point numbers, where geometrical resolution is controlled by the number of grid points used to represent the minimum feature of a transmission line system. The input data from the graphic editor is used to compute transmission line characteristics such as capacitance, inductance and diagonally matched load impedance matrices. The results are stored in a database for analysis or synthesis of interconnections.

Using the outputs from the parameter computation routine, a database of transmission line characteristics is generated. The order of complexity of the database is $O(n^m)$ where n is the number of the database entries (nodes) generated for interconnection technology, and m is the number of conductor dimensions representing a given transmission line geometry. For a homogeneous strip line structure, $m = 3$, since the following three independent variables are used: normalized conductor width, normalized conductor thickness, and normalized inter-conductor separation. The stated quantities are normalized with respect to the thickness of dielectric layer supporting the conductors.

The databases similar to the one reported in [9] are used to obtain desired transmission line characteristics and their physical dimensions. The database entries are used to calculate the quantities not found directly in the database using an interpolation routine. A set of conductor dimensions (thickness, width, and inter-conductor separation) is a key to access generated stripline databases; whereas the thicknesses and dielectric constants of dielectrics are additional (secondary) quantities added to the access key data for computation of actual dimensions of designed structure.

Database generation sequence is shown in Figure 4.2. It consists of three parts which are performed repeatedly for all the combinations of dimension values during database generation: node selection for each dimension of transmission line structure, calculation of characteristics for each combination of dimension value selected, and insertion into database.

4.2 Case Studies.

The program package has been applied to many cases of multi-conductor strip and micro-strip line structures with a specific epoxy-glass (glass fabric soaked with an epoxy-resin, trade name FR-4) and alumina (Al_2O_3) as their dielectric layers. The data reported here are for structures with FR-4. We assume that the width of all conductors in a transmission line system is the same. Figure 4.3 shows the diagonally matched load impedance, Z_{dm}, curves of the center line with fixed conductor thickness of three parallel transmssion strip line systems, where the relative dielectric constant of FR-4 is 4.4. The curves show monotonous decrease as the width of the conductor increases and the interconductor separation decreases. The same types of curves are shown in Figure 4.4, where fixed width (curves a) and fixed separation (curves b) are used. The diagonally matched load impedance values decrease as the thickness increases (curves b) and increase as the separation increases monotonically (curves a). The impedance curves of other lines of the three conductor transmission line system are slightly higher than the ones presented here.

Cases with five parallel conductors are illustrated in Figure 4.5, where the center line impedances with fixed conductor thickness (Figure 4.5a), and fixed conductor width and fixed inter-conductor separation (Figure 4.5b) are used. The shape of the curves are very similar to the three conductor cases, but the values of the diagonally matched load impedance are lower than those of the three conductor cases. These differences in diagonally matched load impedances are due to couplings from the additional (fourth and fifth) conductors.

As the number of conductors in the interconnecting system increases, the diagonally matched load impedance values decrease. The differences in the impedance values between N conductor systems and $N+1$ conductor systems decreases as N increases. Figure 4.6 shows the changes in diagonally matched load impedance due to the increase in the number of conductors. The curves were calculated for the conductor width of 1 mil, inter-conductor separation of 1 mil, and 20 mil in dielectric layer thickness, respectively. As shown in Figure 4.6, the impedance value levels off beyond 5 conductors. The same computation was carried out for many different conductor and dielectric dimensions, and similar behavior was always observed.

As an extreme case, systems with different heights were examined. The biggest difference in impedance between 5 and 7 conductor system does not exceed 15%. The differences between 7 and 9 conductor system do not exceed 6% in magnitude for FR-4 dielectric layers as shown in Figure 4.7. As the number of conductors increases, the additional conductors are located far from the conductor under consideration, resulting in relatively smaller contribution in impedance than nearby ones. In this study it was found that modeling with a 5 conductor system can adequately represent most practical cases of multi-conductor transmission line systems in diagonally matched load impedance computation. Databases for 3600 dimension sets for 3 and 5 conductor systems were established using the program. Dimensions not found in the database entries are interpolated using our three-dimensional cubic spline algorithm.

Several cases of multi-conductor microstrip line systems have been examined with several layers of dielectric material. Figure 4.8 shows an example of a three conductor system with two layers of dielectric, where the first dielectric layer represents FR-4 and the second one represent anti-corrosion coating material such as polyamide with a dielectric constant $\varepsilon = 3.0$. The system can handle structures with up to four layers of different dielectrics.

The developed expert system was applied to a three conductor structure described in [10]. The system produces results which are about 5% different from those reported in [10]. Figure 4.9 presents the diagonally matched load impedance of the center line of the three conductor system: Figure 4.9a shows the impedance curves for a fixed conductor thickness; and Figure 4.9b includes curves for a fixed conductor width and curves for a fixed inter-conductor separation. These values are slightly higher than those for strip line cases shown earlier in Figure 4.3.

Microstrip line cases with five conductors are illustrated in Figure 4.10: Figure 4.10a presents the cases with a fixed conductor thickness; Figure 4.10b shows cases with a fixed conductor width and cases with a fixed inter-conductor separation; and cases with the second dielectric thickness are presented in Figure 4.10c. The diagonally matched load impedances of the center conductor of the five conductor system are slightly lower than those for the three conductor system, but are higher than those for the strip line cases shown in Figure 4.5. This is due to the impedance contribution from two additional parallel conductors of the five conductor system. Notice that in Figure 4.10c the impedance drops sharply as the thickness begins to increase and rises slightly as the thickness becomes large. This implies that the thickness of a thin coating layer should be controlled precisely, whereas the thick layer can be used for crude process environment. As the number of conductors increases, the diagonal load impedance value of the center conductor decreases.

Figure 4.11 describes the changes of the diagonally matched load impedance of the center conductor as the number of conductors increases. Significant changes are observed among two, three, four, and five conductor systems, but the differences become negligible for the cases beyond five conductor cases. It implies that five conductor systems can effectively represent seven, and nine conductor systems, as in strip line cases. The impedance difference between seven and nine conductor systems does not exceed 3% as shown in Figure 4.12. This impedance difference can be interpreted in terms of near-end and far-end noise defined in the equations (2.7) and (2.9).

Figure 4.13 shows the changes in noise levels as the number of conductors increases. This is computed by applying 5V step pulses to all parallel conductors except the center conductor where the noise levels are measured. Here again, very little changes in noise level are observed beyond seven conductors. It is our conclusion that seven, nine, and eleven conductor systems can be represented by five conductor systems.

4.3 Accuracy of Computation and Database Size.

Enhancing computational accuracy and efficiency where the major goals in developing the new mathematical algorithms. These goals were achieved using new efficient methods for solving field problems to generate database, and a cubic spline algorithm for very fast interpolation between data values allowing the interactive use of the system in the interconnection design process.

Figure 4.14 shows a strip line structure with two conductors. The conductor dimensions are the same as those used by Scheinfein and Palusinski [11]. As shown in Figure 4.14a, the normalized thickness τ of the conductor is 0.25, the normalized width ω is 0.5, and the inter-conductor separation σ is 0.5, and the height h is 1.0. The computed results presented in Figure 4.14b show that they are very close to the measured values.

A study on accuracy and database size reduction was carried out for two interpolation schemes: linear and spline interpolation. Interpolation with three-dimensional cubic spline algorithm can generate diagonally matched load impedances with much higher accuracy than linear interpolation. Figure 4.15 shows an accuracy comparison between spline interpolation (solid line) and linear interpolation (dotted line). The difference between the exact computation and the cubic spline interpolation is negligible when 8 interpolation nodes are used. About 10% difference between the exact computation and the linear interpolation was observed when the same number of interpolation nodes were used.

To reduce the size of the database by reducing the number of interpolation nodes, some nodes were dropped from the database and spline interpolation. In order to provide the rationale for database size reduction an accuracy degradation resulting from the size reduction was examined. Figure 4.16 includes three cases of different node selections: 12 nodes (solid line), 6 nodes (dotted line) and 4 nodes (broken line). The impedance values were computed for fixed conductor width and fixed inter-conductor separation (differences are not well visible in this scale). For better illustration of consequences of size reduction the differences in Z_{dm} are plotted in Figure 4.17. The differences between the case of 12 nodes and of 6 nodes are negligible: the differences do not exceed 1.5%, which is acceptable for most practical applications. More substantial differences between the case of 12 nodes and that of 4 nodes were observed in some thickness ranges. Larger differences were observed when the impedance values change significanly; the smaller the impedance values the smaller the differences between the interpolated results and the exact calculation results. From the above observations, it was concluded that the number of spline nodes of each dimension can be reduced by half, in comparison to a case with linear interpolation without significant accuracy degradation.

Substantial database size reduction can be achieved using spline interpolation without significant accuracy degradation. With 50% of spline nodes for each of three dimensions, the database size can be reduced to 12.5% of the original size. The database size is even a more crucial factor in building an expert system for designing interconnects with layer dielectrics, since the number of dimensions increases.

5. Conclusion.

With the new development in microelectronics and rapidly expanding applications in portable equipment, high speed digital circuits with low operating voltage are gaining in importance. Such circuits require accurate estimation of noise because their noise margin (noise immunity) are getting reduced. Such circuits are very susceptible to signal perturbations and signal integrity is a principal requirement in their design. This paper presents three major mathematical improvements in calculating interconnection characteristics essential for the estimation of noise in high speed digital circuits. A new matrix square root algorithm is more efficient than other algorithms known in the literature. A new efficient matrix algorithm based on the theory M-matrices was implemented to determine diagonally matched load impedance. A special multidimensional cubic spline algorithm gives a very accurate interpolation scheme for interconnect database and also allows usage of database with significantly reduced size. These algorithms are successfully exploited in a system supporting interconnection design.

The optimum size of the database can be determined by finding the best location of the nodes to be used for spline large-scale interpolation with a given error tolerance. This otimization problem is nonlinear, non-convex, and therefore only heuristic algorithms can be developed for its solution. In our next paper, this problem will be addressed in detail and a particular solution method will be introduced. Database of 3 and 5 conductor systems are constructed using the new program, each comprising 1800 sets of transmission line characteristics. The computation results obtained from the new algorithms were verified using available measured data. The comparisons of calculated and measured values were satisfactory.

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