Electronic Journal: Southwest Journal of Pure and Applied Mathematics
Internet: http://rattler.cameron.edu/swjpam.html
ISSN 1083-0464
Issue 1, July 2002, pp. 6–32.
Submitted: May 12, 2002. Published: July 1 2002.

FINDING ELLIPSES AND HYPERBOLAS TANGENT TO TWO, THREE, OR FOUR GIVEN LINES

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Abstract. Given lines L_j, $j = 1, 2, 3, 4$, in the plane, such that no three of the lines are parallel or are concurrent, we want to find the locus of centers of ellipses tangent to the L_j. In the case when the lines form the boundary of a four sided convex polygon R, let M_1 and M_2 be the midpoints of the diagonals of R. Let L be the line thru M_1 and M_2, let Z be the open line segment connecting M_1 and M_2, let Y be the closed line segment connecting M_1 and M_2, and let X be the open line segment which is the part of L lying inside R. It is well known that if an ellipse E is **inscribed** in R, then the center of E must lie on Z(see [1] and [2]). We prove(Theorem 11) that every point of Z is the center of some ellipse inscribed in R, which implies that the locus of centers of ellipses inscribed in R is precisely equal to Z. In addition, we prove(Theorem 11) that there is a hyperbola tangent to each of the L_j and with center $(h, k) \in R$ if and only if $(h, k) \in X - Y$. More generally, any ellipse tangent to the L_j(and not just inscribed ones) must have its center on L.

1991 A.M.S. (MOS) Subject Classification Codes. 51.

Key Words and Phrases. Ellipse, hyperbola, tangent, quadrilateral.

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