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A Method for Finding Common Set of Weights by Multiple Objective programming in Data Envelopment Analysis

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Abstract

This paper presents a multiple objective programming procedure for finding a common set of weights in Data Envelopment Analysis (DEA). DEA is a mathematical programming approach for assessing relative efficiencies within a group of Decision Making Units (DMUs). An important outcome of such an analysis is a set of virtual multipliers or weights accorded to each factor taken into account. These sets of weights are, typically, different for each of the participating DMUs. In this paper, by means of solving only one problem, we can determine common set of weights for all DMUs and their efficiencies. Finally, we employ this method for a group of real data to express its adventages comparing to other methods.

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Keywords: Multiple objective Programming, Mathematical Programming, Data Envelopment Analysis, Efficiency, Goal, Common Set of Weights

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1 Introduction

Data envelopment analysis, as developed by Charnes et al. (1978), does not require any a priori weights for inputs and outputs. In DEA the relative efficiency of a DMU, is determined by assigning weights to the inputs and outputs of DMU that the ratio of the weighted sum of outputs to the weighted sum of inputs is maximized. To this end, factor weights are allowed to vary freely (within the general constraints) in each run of the model. As the models of DEA are run for each DMU separately, the set of weights will typically be different for the various DMUs, and in some cases, it may be considered unacceptable that the same factor is accorded with different weights. A possible answer to this difficulty lies in the specification of a common set of weights. Recently, a possible approach has been put forward in Rall et al. (1991). The purpose of this paper is to introduce a new approach for determining the common set of weights. The second section of the paper, represents a brief disscusion about DEA models. In section 3, the goal mathematical programming model with multiple objectives is briefly discussed. The method for finding a common set of weights is presented in section 4. In section 5 an application of the method for real data is put forward and last section presents the conclusion.

2 Models of DEA

Consider a DMU that consume $X = (X_1, X_2, \ldots, X_m)$ inputs to produce $Y = (Y_1, Y_2, \ldots, Y_s)$ outputs and also, suppose inputs and outputs weights are w_i $(i = 1, \ldots, m)$, μ_r $(r = 1, \ldots, s)$, respectively. We also assume that the input and output components are nonnegative. The efficiency of DMU_p is obtained by solving the following ratio programming problem:

$$
\begin{aligned}
Max \quad & e = \frac{\sum_{r=1}^{s} \mu_r y_{rp}}{\sum_{i=1}^{m} w_i x_{ip}} \\
s.t. \quad & \frac{\sum_{r=1}^{s} \mu_r y_{rj}}{\sum_{i=1}^{m} w_i x_{ij}} \leq 1 \quad , j = 1, \ldots, n, \\
& w_i, \mu_r \geq \epsilon \quad , r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{aligned} \tag{1}
$$

where ϵ is positive non-Archimedean infinitesimal. The first set of constraints are for limiting the evaluation scale. The above mentioned problem is solved separatly for each DMU, and by solving these problems different set of weights are obtained. Instead of solving problem (1) the following linear programming priblem is solved:

$$
\begin{aligned}
Max \quad & e = \sum_{r=1}^{s} u_r y_{rp} \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad , j = 1, \ldots, n, \\
& \sum_{i=1}^{m} v_i x_{ip} = 1, \\
& u_r, \ v_i \geq \epsilon \quad , r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{aligned} \tag{2}
$$

This model is called CCR input orientation multiplier side problem.

3 Goal Multiple Mathematical Programming

Consider the following problem:

$$\begin{array}{ll} Max & (f_1(x), f_2(x), \ldots, f_k(x)) \\ s.t. & \quad x \in X. \end{array} \tag{3}$$

where $f_1, f_2, \ldots, f_k$ are objective functions, and X is the feasible region. Let $b_1, b_2, \ldots, b_k$ be the goals of $f_1, f_2, \ldots, f_k$ we are supposed to meet; therefore problem (3) is called goal multiple objective mathematical programming. If in this problem the weights $w_1, w_2, \ldots, w_k$ are assigned to $f_1, f_2, \ldots, f_k$, then the problem is called weighted problem. By introducing the deviational variables the problem can be written in the following form:

$$\begin{array}{llll} Min & \sum_{j=1}^{k} w_j z_j & & \\ s.t. & f_j(x) + z_j = b_j & , j = 1, \ldots, n, & \\ & x \in X. & & \end{array} \tag{4}$$

Instead of solving(3), the model (4) is solved. In this problem, $\sum_{i=1}^{k} w_i = 1, w_i \geq 0, (i = 1, 2, ..., k)$. If $z_j = 0$, then f_j is reached to the goal b_j. If $z_j \leq 0$, then f_j is higer than the goal.

4 A Method for Finding Common Set of Weights

In DEA for calculating the efficiency of different DMU's, different set of weights are obtained, which seems to be unacceptable in reality. So the following model is used to find common set of weights which has some advantages that will be discussed later on. Consider the following problem:

$$\begin{array}{lll} Max & e = \{\frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \ldots, \frac{\sum_{r=1}^{s} u_r y_{rn}}{\sum_{i=1}^{m} v_i x_{in}}\} & \\ s.t. & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 & , j = 1, \ldots, n, \\ & u_r, \ v_i \ \geq \epsilon & , r = 1, \ldots, s, \ i = 1, \ldots, m. \end{array} \tag{5}$$

where U and V are the weights of outputs and inputs, respectively, and ϵ is a positive non-Archimedean infinitesimal.

Problem (5) is a fractional prgramming problem with multiple objectives, in which each objective function has goal equal to 1. As no priority is considered for the objective functions, we take $w_i = 1\,(i = 1, 2, ..., k)$. By introducing n deviational vairables z_j , $(j = 1, 2, \ldots, k)$ we have:

$$
\begin{array}{lll}
Min & \sum_{j=1}^{n} z_j & \\
s.t. & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} + z_j \ = 1 & , \ j = 1, \ \ldots, \ n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 & , \ j = 1, \ \ldots, \ n, \\
& u_r, \ v_i \ \geq \epsilon & , \ r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{array} \tag{6}
$$

If DMU$_p$ is efficient, then $z_p = 0$, othervise $z_p \geq 0$, so z_j is restricted to be non-negative. Therefore problem (6) is changed to the following problem:

$$
\begin{array}{lll}
Min & \sum_{j=1}^{n} z_j & \\
s.t. & \sum_{r=1}^{s} u_r y_{rj} + z_j \sum_{i=1}^{m} v_i x_{ij} - \sum_{i=1}^{m} v_i x_{ij} \ = 0 & , \ j = 1, \ \ldots, \ n, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 & , \ j = 1, \ \ldots, \ n, \\
& z_j \geq 0 & , \ j = 1, \ \ldots, \ n, \\
& u_r, \ v_i \ \geq \epsilon & , \ r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{array} \tag{7}
$$

Note that instead of solving n linear programming DEA models, only one problem is solved and the efficeincy for all DMUs are obtained.

Theorem 1:

If DMU$_p$ is efficient in problem (7), then it is efficient in CCR model.

Proof: Suppose that (U^*, V^*, Z^*) is an optimal solution for problem (7), $\frac{{U^*}^t Y_p}{{V^*}^t X_p} = 1$.

It is evident that (U^*, V^*) is a feasible solution of problem (1) and since $\frac{{U^*}^t Y_p}{{V^*}^t X_p} = 1$, it is optimal.

5 Generalizing The Method for DEA Models

The above method can be generalized to all input and output oriented DEA models. Consider an BCC (Banker, Charnes and Cooper) output oriented model, which is in the following form for DMU$_p$.

$$
\begin{array}{llll}
Min & e = \dfrac{\sum_{i=1}^{m} v_i x_{ip} + v_0}{\sum_{r=1}^{s} u_r y_{rp}} & & \\
s.t. & \dfrac{\sum_{i=1}^{m} v_i x_{ij} + v_0}{\sum_{r=1}^{s} u_r y_{rj}} \geq 1 & , j = 1, \ \ldots, \ n, & \\
 & u_r, \ v_i \ \geq \epsilon & , r = 1, \ldots, s, \ i = 1, \ldots, m. &
\end{array}
\tag{8}
$$

By introducing common weights for all DMUs, the output oriented BCC model with will be in the following form:

$$
\begin{array}{lll}
Min & e = \{\dfrac{\sum_{i=1}^{m} v_i x_{i1} + v_0}{\sum_{r=1}^{s} u_r y_{r1}}, \ldots, \dfrac{\sum_{i=1}^{m} v_i x_{in} + v_0}{\sum_{r=1}^{s} u_r y_{rn}}\} & \\
s.t. & \dfrac{\sum_{i=1}^{m} v_i x_{ij} + v_0}{\sum_{r=1}^{s} u_r y_{rj}} \geq 1 & , j = 1, \ \ldots, \ n, \\
 & u_r, \ v_i \ \geq \epsilon & , r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{array}
\tag{9}
$$

We introduce n deviational varibles to have.

$$
\begin{array}{lll}
Min & \sum_{j=1}^{n} z_j & \\
s.t. & \dfrac{\sum_{i=1}^{m} v_i x_{ij} + v_0}{\sum_{r=1}^{s} u_r y_{rj}} - z_j \ = 1 & , j = 1, \ \ldots, \ n, \\
 & \dfrac{\sum_{i=1}^{m} v_i x_{ij} + v_0}{\sum_{r=1}^{s} u_r y_{rj}} \geq 1 & , j = 1, \ \ldots, \ n, \\
 & u_r, \ v_i \ \geq \epsilon & , r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{array}
\tag{10}
$$

which is equivalent to the following:

$$
\begin{array}{lll}
Min & \displaystyle\sum_{j=1}^{n} z_j & \\
S.t & \displaystyle\sum_{i=1}^{m} v_i x_{ij} + v_0 - z_j \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} u_r y_{rj} \ = 0 & , j = 1, \ \ldots, \ n, \\
 & \displaystyle\sum_{i=1}^{m} v_i x_{ij} + v_0 - \sum_{r=1}^{s} u_r y_{rj} \geq 0 & , j = 1, \ \ldots, \ n, \\
 & u_r, \ v_i \ \geq \epsilon & , r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{array}
\tag{11}
$$

We note that theorem 1 remains valid in the more general case.

6 Application

Data have been collected for 74 high schools in four cities and surrounding villages. More than 50 items for each school have been chosen. The inputs and outputs are given in table 1.

INPUT :	*OUTPUT* :
1. The educational space. 2. The official staff. 3. The quality of teachers. 4. The educational performance of the last period of the study.	1. The number of students. 2. The educational performance of the period of study. 3. The number of student could be accepted to the governmental universities.

Table 1. Inputs and Outputs for Empirical Study.

The input and output data of this study are presented in table 2.

Running the CCR model for each school resulted in efficiencies and weights shown in table 3.

The common set of weights and the corresponding efficiencies when model (7) applied, are listed in table 4.

7 Conclusion

The model discussed in this paper has the following adventages:

1) By solving only one problem, the common set of weights of DMUs are obtained.

2) All efficiencies can be calculated without solving corresponding separate linear programming problems.

3) The new model is stable for weight variation, which is not true in other DEA models.

4) Some DMUs are efficient in CCR models which are not efficient in this model, which seems to be more realistic.

Sch	I1	I2	I3	I4	O1	O2	O3
S01	6.63	7.25	1.32	2.87	321.00	2.11	79.00
S02	6.63	7.75	1.35	2.50	294.00	2.04	197.00
S03	9.06	10.75	1.27	1.37	338.00	1.35	110.00
S04	7.56	9.50	1.18	2.24	503.00	2.08	81.00
S05	6.71	5.50	1.22	2.92	215.00	2.60	306.00
S06	7.29	12.50	1.34	3.80	337.00	2.84	69.00
S07	5.89	7.25	1.28	1.38	173.00	1.77	38.00
S08	5.89	5.50	1.22	1.73	134.00	2.40	11.00
S09	10.18	7.50	1.38	1.84	322.00	1.72	72.00
S10	8.07	8.25	1.26	2.28	281.00	2.19	30.00
S11	6.80	7.75	1.21	2.35	331.00	2.55	57.00
S12	6.98	7.50	1.53	2.65	267.00	2.59	19.00
S13	10.63	7.25	1.98	4.96	405.00	1.60	8.00
S14	6.00	6.25	1.76	2.43	166.00	1.38	4.00
S15	6.71	5.25	1.08	1.78	94.00	1.39	4.00
S16	6.97	7.50	1.14	1.78	294.00	2.52	91.00
S17	7.02	7.75	1.20	2.49	280.00	2.78	16.00
S18	6.97	7.75	1.09	1.96	254.00	2.07	67.00
S19	5.09	5.25	1.74	2.17	103.00	1.78	0.00
S20	10.04	7.75	1.06	3.49	363.00	2.94	88.00
S21	5.64	6.50	1.79	1.81	155.00	1.46	15.00
S22	6.15	6.50	1.12	3.49	315.00	2.81	59.00
S23	8.65	7.50	1.71	1.75	298.00	2.68	178.00
S24	6.32	7.25	1.78	1.82	194.00	2.67	173.00
S25	7.16	5.25	0.96	1.84	232.00	2.08	42.00
S26	10.38	8.00	1.04	2.36	224.00	2.25	339.00
S27	9.57	9.00	1.65	2.50	485.00	2.35	428.00
S28	9.03	8.00	1.20	2.69	618.00	2.54	155.00
S29	7.37	8.50	1.04	2.63	234.00	2.25	126.00
S30	5.40	4.00	0.91	3.02	205.00	3.31	26.00
S31	10.93	7.25	0.86	2.43	163.00	1.16	9.00
S32	9.19	5.25	0.96	2.57	119.00	3.07	18.00
S33	12.58	8.67	1.52	1.93	669.00	1.73	78.00
S34	7.51	5.58	1.10	2.43	420.00	2.74	97.00
S35	8.34	3.83	1.27	2.00	103.00	1.98	0.00
S36	6.76	3.83	1.30	2.76	196.00	1.96	8.00
S37	6.99	5.58	1.17	1.52	443.00	2.17	32.00
S38	5.74	3.83	1.07	2.33	239.00	2.64	0.00
S39	6.44	4.50	1.22	1.64	342.00	2.22	15.00
S40	10.23	3.58	1.10	2.12	275.00	2.25	8.00
S41	5.49	3.58	1.32	1.54	202.00	1.92	45.00
S42	4.88	3.83	1.45	1.61	167.00	1.65	7.00
S43	8.73	5.17	1.27	1.66	367.00	0.94	60.00
S44	6.12	4.58	1.45	3.04	141.00	3.37	165.00
S45	7.67	4.83	0.92	1.83	307.00	2.50	40.00
S46	13.38	8.25	1.37	2.31	688.00	2.01	342.00
S47	9.35	5.92	1.39	2.29	425.00	1.87	250.00
S48	7.78	6.17	1.77	1.56	359.00	1.44	27.00
S49	8.76	3.83	0.95	2.52	367.00	2.12	0.00
S50	8.86	3.83	1.47	2.16	134.00	2.28	40.00
S51	5.78	2.58	0.99	2.54	97.00	2.55	0.00
S52	7.81	6.42	1.13	1.78	426.00	1.86	27.00
S53	6.84	2.58	1.18	2.41	194.00	1.88	8.00
S54	9.79	6.17	1.15	2.38	394.00	2.08	13.00
S55	10.66	7.42	1.33	1.85	428.00	1.44	20.00
S56	7.69	3.58	1.19	3.36	101.00	3.36	80.00
S57	6.58	5.58	1.12	2.50	458.00	2.46	60.00
S58	9.22	5.92	1.30	2.02	360.00	2.34	32.00
S59	6.33	3.83	1.16	2.37	128.00	2.32	48.00
S60	5.80	2.33	1.16	1.39	116.00	2.10	8.00
S61	7.63	4.25	1.08	2.37	177.00	2.33	8.00
S62	10.07	8.00	1.13	2.68	714.00	2.44	360.00
S63	6.02	5.42	1.20	2.81	334.00	1.96	100.00
S64	8.92	6.92	1.09	2.76	478.00	2.49	38.00
S65	8.01	6.42	1.20	2.58	471.00	2.74	94.00
S66	11.76	8.00	1.15	1.82	598.00	1.79	50.00
S67	6.77	4.17	1.15	2.12	352.00	2.56	110.00
S68	10.56	6.17	1.35	1.76	382.00	1.82	68.00
S69	4.69	3.83	1.46	1.38	68.00	1.23	0.00
S70	4.77	3.58	1.55	2.28	120.00	2.28	45.00
S71	5.72	3.83	1.52	3.25	166.00	3.76	131.00
S72	8.43	7.17	1.39	2.27	140.00	2.29	45.00
S73	7.91	3.58	1.41	3.00	95.00	3.00	42.00
S74	5.57	3.58	1.42	2.71	66.00	3.25	9.00

Table 2. Inputs and outputs of high schools.

Sch	Eff	V_1	V_2	V_3	V_4	U_1	U_2	U_3
S01	0.78	0.001274	0.149605	0.000676	0.150657	0.000100	0.000100	0.000100
S02	0.84	0.000895	0.137220	0.001512	0.150655	0.000100	0.000100	0.000100
S03	0.86	0.001033	0.282509	0.001189	0.000100	0.000100	0.000100	0.728388
S04	0.94	0.001353	0.122213	0.000100	0.101812	0.000100	0.000100	0.102335
S05	1.00	0.000100	0.000100	0.003197	0.148888	0.000100	0.000100	0.000100
S06	0.83	0.001073	0.151195	0.000540	0.123931	0.000100	0.070832	0.000100
S07	0.85	0.000183	0.461621	0.000100	0.028631	0.000100	0.089069	0.519295
S08	0.99	0.000100	0.404785	0.000100	0.041465	0.002722	0.113474	0.348186
S09	0.68	0.000377	0.295474	0.000754	0.000100	0.000100	0.220268	0.377317
S10	0.73	0.000355	0.284557	0.000100	0.032120	0.000100	0.180202	0.224960
S11	0.90	0.000914	0.225218	0.000350	0.088743	0.000100	0.071851	0.131417
S12	0.79	0.000930	0.207597	0.000100	0.097789	0.000100	0.000100	0.119444
S13	0.60	0.001311	0.043840	0.000100	0.014554	0.116497	0.000100	0.000100
S14	0.50	0.001059	0.236961	0.000100	0.111514	0.000100	0.000100	0.135850
S15	0.56	0.000100	0.393804	0.000100	0.032168	0.005745	0.159922	0.326558
S16	1.00	0.000370	0.350033	0.000100	0.055925	0.000100	0.075478	0.294051
S17	0.89	0.000339	0.286270	0.000100	0.044852	0.000100	0.097297	0.227955
S18	0.80	0.000545	0.306505	0.000429	0.037238	0.000100	0.264729	0.230164
S19	0.64	0.000608	0.326452	0.000100	0.081822	0.000100	0.000100	0.268584
S20	0.90	0.000709	0.214461	0.000100	0.000100	0.000100	0.941389	0.000100
S21	0.60	0.000638	0.344731	0.000100	0.086104	0.000100	0.000100	0.283726
S22	0.94	0.001273	0.179464	0.000641	0.147127	0.000100	0.084080	0.000100
S23	1.00	0.000681	0.237236	0.000906	0.043970	0.000100	0.000100	0.353565
S24	1.00	0.000100	0.245570	0.001878	0.107560	0.000100	0.000100	0.175448
S25	0.83	0.000100	0.387852	0.000100	0.031845	0.005403	0.161038	0.320121
S26	1.00	0.000100	0.000100	0.002883	0.000100	0.000100	0.504708	0.200536
S27	1.00	0.000105	0.000100	0.002217	0.104356	0.000100	0.000100	0.000100
S28	0.97	0.000797	0.182719	0.000100	0.060273	0.000100	0.150410	0.102022
S29	0.79	0.000100	0.289422	0.000885	0.022828	0.000100	0.268423	0.209789
S30	1.00	0.000100	0.295136	0.000100	0.160754	0.000100	0.144203	0.000100
S31	0.45	0.000661	0.298195	0.000100	0.000100	0.000100	0.732478	0.151544
S32	0.98	0.000100	0.316281	0.000100	0.000100	0.000100	0.478465	0.209817
S33	1.00	0.001483	0.000100	0.000100	0.039616	0.000100	0.000100	0.259387
S34	0.99	0.000822	0.232256	0.000100	0.017506	0.041103	0.248311	0.150631
S35	0.71	0.000100	0.350881	0.000100	0.000100	0.042709	0.246702	0.261140
S36	0.70	0.001702	0.185033	0.000100	0.008427	0.246116	0.000100	0.000100
S37	1.00	0.001579	0.136910	0.000100	0.117579	0.000100	0.000100	0.116743
S38	0.95	0.000523	0.313624	0.000100	0.025511	0.040098	0.120500	0.245089
S39	0.99	0.000832	0.318273	0.000100	0.042972	0.065075	0.000100	0.262378
S40	0.95	0.001705	0.212490	0.000100	0.000100	0.263489	0.000100	0.026215
S41	0.91	0.000608	0.392601	0.000807	0.045178	0.043592	0.000100	0.386872
S42	0.78	0.000480	0.422576	0.000100	0.062896	0.019748	0.000100	0.383408
S43	0.80	0.002153	0.000100	0.000100	0.000100	0.117303	0.000100	0.236472
S44	0.98	0.000100	0.231171	0.001119	0.038539	0.000100	0.159776	0.175003
S45	1.00	0.000422	0.346639	0.000100	0.038956	0.000100	0.214986	0.274829
S46	1.00	0.000725	0.000100	0.001466	0.021035	0.000100	0.000100	0.310646
S47	0.90	0.000662	0.075771	0.001927	0.000100	0.168699	0.000100	0.000100
S48	0.76	0.002049	0.014018	0.000100	0.055986	0.000100	0.000100	0.361302
S49	1.00	0.001674	0.181970	0.000100	0.008223	0.242197	0.000100	0.000100
S50	0.76	0.000100	0.311194	0.001044	0.000100	0.035306	0.162080	0.289645
S51	1.00	0.000188	0.384995	0.000100	0.000100	0.235469	0.347675	0.018785
S52	0.84	0.000789	0.271425	0.000100	0.000100	0.000100	0.426992	0.289930
S53	1.00	0.002400	0.283836	0.000100	0.001291	0.384034	0.000100	0.000100
S54	0.77	0.000861	0.205359	0.000100	0.000100	0.058911	0.218157	0.161621
S55	0.73	0.001423	0.079752	0.000100	0.000100	0.043945	0.101416	0.290798
S56	1.00	0.000100	0.256735	0.001591	0.000100	0.116412	0.489194	0.000100
S57	1.00	0.001957	0.039734	0.000100	0.151836	0.000100	0.000100	0.000100
S58	0.83	0.000668	0.252634	0.000100	0.000100	0.063811	0.120734	0.229881
S59	0.79	0.000100	0.325330	0.000403	0.000100	0.043933	0.224396	0.240846
S60	1.00	0.000100	0.470286	0.000100	0.065601	0.015854	0.000100	0.419035
S61	0.78	0.000100	0.328081	0.000100	0.000100	0.040521	0.230410	0.243958
S62	1.00	0.000655	0.000100	0.001478	0.099188	0.000100	0.000100	0.000100
S63	0.87	0.001404	0.164779	0.000745	0.165956	0.000100	0.000100	0.000100
S64	0.87	0.000530	0.245726	0.000100	0.000100	0.000100	0.597116	0.125928
S65	0.96	0.000862	0.197041	0.000100	0.064501	0.000100	0.166135	0.109823
S66	1.00	0.001664	0.000100	0.000100	0.019138	0.000100	0.194212	0.302636
S67	1.00	0.000689	0.262407	0.000781	0.058663	0.022042	0.000100	0.240952
S68	0.78	0.000597	0.268111	0.000999	0.000100	0.058699	0.000100	0.361723
S69	0.62	0.000100	0.497289	0.000100	0.069428	0.016503	0.000100	0.442776
S70	0.82	0.000389	0.338490	0.000100	0.050009	0.016519	0.000100	0.307967
S71	1.00	0.000100	0.185280	0.002189	0.174675	0.000100	0.000100	0.000100
S72	0.72	0.000100	0.305850	0.000100	0.025446	0.005298	0.119898	0.255880
S73	0.85	0.000100	0.278900	0.000100	0.000100	0.070131	0.172620	0.168248
S74	0.98	0.000100	0.299050	0.000100	0.041335	0.011738	0.000100	0.268487

Table 3. Efficiencies and weights by CCR model.

Sch	E(J)	Z(J)
S01	0.67	0.33
S02	0.74	0.26
S03	0.63	0.37
S04	0.81	0.19
S05	0.87	0.13
S06	0.69	0.31
S07	0.76	0.24
S08	0.90	0.10
S09	0.63	0.37
S10	0.71	0.29
S11	0.87	0.13
S12	0.75	0.25
S13	0.34	0.66
S14	0.42	0.58
S15	0.52	0.48
S16	1.00	0.00
S17	0.87	0.13
S18	0.79	0.21
S19	0.56	0.44
S20	0.76	0.24
S21	0.51	0.49
S22	0.78	0.22
S23	0.95	0.05
S24	0.95	0.05
S25	0.83	0.17
S26	0.81	0.19
S27	0.87	0.13
S28	0.90	0.10
S29	0.74	0.26
S30	1.00	0.00
S31	0.36	0.64
S32	0.86	0.14
S33	0.68	0.32
S34	0.98	0.02
S35	0.63	0.37
S36	0.59	0.41
S37	1.00	0.00
S38	0.92	0.08
S39	0.95	0.05
S40	0.74	0.26
S41	0.84	0.16
S42	0.68	0.32
S43	0.49	0.51
S44	0.94	0.06
S45	1.00	0.00
S46	0.81	0.19
S47	0.75	0.25
S48	0.60	0.40
S49	0.72	0.28
S50	0.69	0.31
S51	0.81	0.19
S52	0.80	0.20
S53	0.63	0.37
S54	0.69	0.31
S55	0.56	0.44
S56	0.84	0.16
S57	0.91	0.09
S58	0.81	0.19
S59	0.76	0.24
S60	0.91	0.09
S61	0.74	0.26
S62	0.97	0.03
S63	0.69	0.31
S64	0.80	0.20
S65	0.93	0.07
S66	0.74	0.26
S67	1.00	0.00
S68	0.69	0.31
S69	0.51	0.49
S70	0.75	0.25
S71	1.00	0.00
S72	0.68	0.32
S73	0.76	0.24
S74	0.91	0.09

Table 4. The common set of weights and the corresponding efficiencies.

References

1. Ali, A. I., and L. M. Seiford: Translation Invarience in Data Envelopment Analysis, Operations Research Letters, Vol. 9, No. 5, (1990), 403-405.
2. Banker, R. D., A. Charnes, and W. W. Cooper: Some Methods for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis, Management Science, Vol. 30, No. 9, (1984), 1078-1092.
3. Charnes, A., W. W. Cooper, A. Y. Lewin, and L. M. Seiford: Data Envelopment Analysis: Theory, Methodology, and Applications, Boston: Kluwer Academic Publishers, (1994).
4. Charnes, A., W. W. Cooper, and E. Rhodes: Measuring the Efficiency of Decision Making Units, European Journal of Operational Research, Vol. 2, No. 6, (1978), 429-444.
5. Charnes, A., W. W. Cooper, R. M. Thrall: Classifying and Characterizing Efficiencies and Inefficiencies in Data Envelopment Analysis, Operations Research Letters, Vol. 5, No. 3, (1986), 105-110.
6. Charnes, A., W. W. Cooper, Wei QL and Huang ZM: Cone-Ratio Data Envelopment Analysis and Multi-Objective Programming, Int. J. Syst. Sci., Vol. 20, (1989), 1099-1118.
7. Dyson, R. G., and E. Thanassoulis: Reducing Weight Flexibility in Data Envelopment Analysis, Journal of Operational Research Society, Vol. 39, (1988), 563-576.
8. Golany, B., and Y. Roll: Alternate Methods of Treating Factor Weights in DEA, OMEGA Int. J. of Mgmt Sci., Vol. 21, No. 1, (1993), 99-109.
9. Hosseinzadeh Lotfi, F., S. Mehrabian and M. R. Alirezaee: Determining the Comparative Efficiency of High Schools; An Application of Data Envelopment Analysis, Journal of Sciences, Islamic Azad University, Vol. 7, No. 23, (1997), 403-420.
10. Roll, Y., W. Cook and B. Golany: Controlling factor Weights in Data Envelopment Analysis, IIE Trans., Vol. 23, No.1, (1991), 2-9.
11. Roll, Y. and B. Golany: Alternate Methods of Treating Factor Weights in DEA, Omega, Vol. 21, No. 1, (1993), 99-109.
12. Thompson, R. G., L. N. Langemeier, C. T. Lee, E. Lee and R. M. Thrall: The Role of Multiplier Bounds in Efficiency Analysis With Application to Kansas Farming, J. Economet., Vol. 46, (1990), 93-108.
13. Thompson, R. G., W. Lesso, R. R. Pettit, C. Samson, L. H. Sterm, F. D. Singleton Jr., B. A. Smith and R. M. Thrall: Systems Task Force Comparative Site Evaluations for the Texas SSC Project: Vol. E and Appendices, Woodlands, TX: Hoston Area Research Center, (1985).
14. Thompson, R. G., G. Russell, F. D. Singleton Jr., R. M. Thrall and A. Barton: Comparative Site Evaluation for Locating a High-Energy Physics Lab in Texas, Interface, Vol. 16, No. 6, (1986), 35-49.