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ESTIMATION OF CHANNEL HOLDING TIME DISTRIBUTION IN CELLULAR TELEPHONE SYSTEMS

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Abstract. In estimating the channel holding time distribution in cellular telephone systems, the pdf of the fraction of two non-negative random variables, distance and speed, has to be determined. Instead of the repeated application of numerical integration we show that the pdf of the holding time is the solution of an initial value problem of a non-linear ordinary differential equation, which can be solved very efficiently.

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1.INTRODUCTION.

Many traffic models have been proposed for the performance analysis of cellular telephone systems, mainly relied on simulation models [1]. When we make a traffic model into a mathematical expression, only a few models have a known closed-form solution under special conditions [2], and usually the traffic models cannot be solved in a closed-form. Therefore a simulation or approximating method has to be used to solve the problems.

The channel holding time of a mobile user in cellular telephone systems is important for system design and radio resource allocation. The distribution of this parameter can be computed by a double integral of the moving distance and speed of a mobile in a cell [2]. Although the cumulative distribution functions of the distance and the speed are derived as a closed-form solution, in most cases the double integrals are not solved analytically. In this paper, we propose a method for deriving the cannel holding time distribution and we apply this method to the analysis of cannel holding time for the traffic model of a soft handoff call which has non-closed-form solution. We also perform computer simulations to check the results of the non-closed-form application.

2. TRAFFIC MODEL OF SOFT HANDOFF.

We assume that the origination of a new call is uniformly distributed over a service area. The speed and the direction of the call are uniformly distributed, and they are unchanged during the call. We also assume that the speed and the direction are independent of each other. Soft handoff allows that a mobile station is connected to a new base station before it is disconnected from the old base station during handoff. Thus when a mobile station is in the soft handoff state, two cell sites are connected to use with diversity [3]. Due to this overlapped region, one more boundary exists in a soft handoff traffic model [4]. A circular-shaped cell is

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assumed for the traffic model for a simple analysis. In Fig. 1, the boundary B is the point that the received power level of a mobile is higher than its receiver threshold level, where an incoming soft handoff starts and the outgoing one stops. If the circular cell radius of boundary A is R_{eq}, the radius of outer boundary B of a soft handoff region is defined as R_{so} and it is

$$R_{so} \equiv (1+a)R_{eq}. \tag{2.1}$$

Here a is the soft handoff region factor in the interval $[0,1]$ and it indicates the ratio of overlapped area between cells. Three types of calls exist in the traffic model with a soft handoff scheme, which are a new call, an incoming call and a handoff region call [4]. In this paper, we will discuss only the channel holding time of a new call. The channel holding time of new call in a cell is defined as the time duration from the distant that a channel is occupied by the call to the instant that it is released by either completion of the cell or cross of the cell boundary. Thus it is either the call duration time or the residing time of a mobile in the cell, whichever is smaller. For the new call, the random variable T is defined as the residing time in the cell. In Fig. 1, the moving distance of these calls is illustrated. The cell site is assumed to be located at the center of a cell and is indicated by letter C. A new call (indicated by letter D) is generated at a point within the boundary A. The assigned channel is released when the mobile station is leaving the cell at the cell boundary B indicated by letter E. We assume that the direction of a mobile is uniformly distributed on the interval $[0,2\pi]$. If we define the direction as the angle θ against the vector from the cell site to the mobile, by using symmetry, the random variable θ can be considered in interval $[0,\pi]$ with pdf

$$f_\theta(\theta) = \begin{cases} \frac{1}{\pi}, & \text{for} \quad 0 \le \theta \le \pi \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

$$f_r(r) = \begin{cases} \frac{2r}{R_{eq}^2}, & \text{for} \quad 0 \le r \le R_{eq} \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

The distance from a mobile station to cell boundary B is indicated by letter Z, which and a new random variable W are defined as

$$Z = \sqrt{R_{so}^2 - (r\sin\theta)^2 - r\cos\theta}, \qquad W = r\cos\theta. \tag{2.4}$$

The joint pdf of Z and W can be found as

$$f_{zw}(z,w) = \frac{2}{\pi R_{eq}^2}\frac{z+w}{\sqrt{R_{so}^2-(z+w)^2}} \qquad \text{for} \begin{cases} R_{so}-R_{eq} \le z \le R_{so}+R_{eq}, \\ \frac{-z^2+R_{so}^2-R_{eq}^2}{2z} \le w \le R_{so}-z \end{cases} \tag{2.5}$$

The marginal pdf of distance Z is then

$$\begin{aligned} f_z(z) &= \int_{-\infty}^{\infty} f_{zw}(z,w)dw \\ &= \begin{cases} \frac{2}{\pi R_{eq}^2}\sqrt{R_{so}^2 - (\frac{z^2+R_{so}^2-R_{eq}^2}{2z})^2}, & \text{for} \quad R_{so}-R_{eq} \le z \le R_{so}+R_{eq}. \\ 0, & \text{elsewhere} \end{cases} \end{aligned} \tag{2.6}$$

The speed of a mobile is uniformly distributed on the interval $[0, V_{max}]$ with pdf

$$f_v(v) = \begin{cases} \frac{1}{v_{max}}, & \text{for} \quad 0 \le v \le V_{max} \\ 0, & \text{elsewhere} \end{cases} \tag{2.7}$$

Then the residing time T is expressed by

$$T = \frac{Z}{V}. \tag{2.8}$$

The pdf of T is

$$f_T(t) = \int_{-\infty}^{\infty} f_Z(tw) f_V(w)|w|dw$$

$$= \begin{cases} \frac{2}{\pi R_{eq}^2 V_{max}} \int_0^{V_{max}} w\sqrt{R_{so}^2 - (\frac{z^2+R_{so}^2-R_{eq}^2}{2z})^2} dw, & \text{if} \quad \frac{R_{so}-R_{eq}}{V_{max}} \le t \le \frac{R_{so}+R_{eq}}{V_{max}} \\ \frac{2}{\pi R_{eq}^2 V_{max}} \int_0^{(\frac{R_{so}+R_{eq}}{t})} w\sqrt{R_{so}^2 - (\frac{z^2+R_{so}^2-R_{eq}^2}{2z})^2} dw, & \text{if} \quad t \ge \frac{R_{so}+R_{eq}}{V_{max}} \end{cases} \tag{2.9}$$

The above integrals do not have a closed-form solution. In the next section a practical method will be introduced for the efficient computation of $f_T(t)$ for a large set of t values.

3. Proposed integration method.

Assume that Z is a continuous random variable in the interval $[A, B]$, where A and B are positive, and V is a continuous random variable in the interval $[0, V_{max}]$, where $V_{max} > 0$ is given. Let $f_Z(z)$ and $f_V(v)$ denote the density functions of Z and V. From equation (2.8), notice first that the values of T are in the interval $[\frac{A}{V_{max}}, \infty)$, and if $F_T(t)$ denotes the cumulative distribution function of T, then

$$F_T(t) = \begin{cases} 0, & \text{if} \quad z < \frac{A}{V_{max}} \\ P\left\{\frac{Z}{V} \le t\right\} = P\left\{\frac{Z}{t} \le V\right\} & \text{otherwise} \end{cases} \tag{3.1}$$

This probability can be computed by a double integral. The region of integration is illustrated in Fig.2, which shows that we have to consider 2 separated cases. If $t \ge \frac{B}{V_{max}}$, then

$$\begin{aligned} F_T(t) &= \int_A^B \int_{\frac{z}{t}}^{V_{max}} f_Z(z) \cdot f_V(v) dv dz \\ &= \int_A^B f_Z(z)[1 - F_v(\frac{z}{t})]dz \\ &= 1 - \int_A^B f_Z(z) F_V(\frac{z}{t})dz \end{aligned} \tag{3.2}$$

where $F_V(v)$ is the cumulative distribution function of V. If $\frac{A}{V_{max}} \le t \le \frac{B}{V_{max}}$, then

$$\begin{aligned} F_T(t) &= \int_A^{t\cdot V_{max}} \int_{\frac{z}{t}}^{V_{max}} f_Z(z) \cdot f_V(v) dv dz \\ &= \int_A^{t\cdot V_{max}} f_Z(z)[1 - F_V(\frac{z}{t})]dz \\ &= F_Z(t \cdot V_{max}) - \int_A^{t\cdot V_{max}} f_Z(z) F_V(\frac{z}{t})dz \end{aligned} \tag{3.3}$$

where $F_Z(z)$ is the cumulative distribution function of Z. If the random variable V is uniformly distributed in the interval $[O, V_{max}]$, then

$$f_V(v) = \begin{cases} \frac{1}{V_{max}}, & \text{if} \quad 0 \le v \le V_{max} \\ 0, & \text{otherwise} \end{cases} \tag{3.4}$$

$$F_V(v) = \begin{cases} \frac{v}{V_{max}}, & \text{if} \quad 0 \le v \le V_{max} \\ 0, & \text{otherwise} \end{cases} \tag{3.5}$$

By using the special form of $F_V(v)$, a more efficient method can be suggested. The above integrals can be further simplified as follows: If $\frac{A}{V_{max}} \le t \le \frac{B}{V_{max}}$, then using integration by parts we have

$$\begin{aligned} F_T(t) &= F_Z(t \cdot V_{max}) - \int_A^{t \cdot V_{max}} \frac{z}{t \cdot V_{max}} f_Z(z) dz \\ &= F_Z(t \cdot V_{max}) - \frac{1}{t \cdot V_{max}} \left\{ [zF_Z(z)]_A^{t \cdot V_{max}} - \int_A^{t \cdot V_{max}} F_Z(z) dz \right\} \\ &= \frac{1}{t \cdot V_{max}} \int_A^{t \cdot V_{max}} F_Z(z) dz. \end{aligned} \tag{3.6}$$

Simple differentiation shows that

$$\begin{aligned} f_T(t) &= -\frac{1}{t^2 V_{max}} \int_A^{tV_{max}} F_Z(z) dz + \frac{1}{tV_{max}} F_Z(tV_{max}) \cdot V_{max} \\ &= -\frac{1}{t} F_T(t) + \frac{1}{t} F_Z(tV_{max}). \end{aligned} \tag{3.7}$$

If $t \ge \frac{B}{V_{max}}$, then from equation (3.2) we have

$$\begin{aligned} F_T(t) &= 1 - \int_A^B f_Z(z) \frac{z}{t \cdot V_{max}} dz \\ &= 1 - \frac{1}{t \cdot V_{max}} E_Z(z). \end{aligned} \tag{3.8}$$

Here the expected value $E_Z(z)$ is a constant that has to be computed only ones. By differentiation,

$$f_T(t) = \frac{1}{t^2 V_{max}} E_Z(z). \tag{3.9}$$

Since $f_T(t) = F_T'(t)$, an ordinary differential equation is obtained for $F_T(t)$ by equality (3.7). However F_Z is not known in a closed-form; therefore, the numerical solution of this differential equation is expensive, since at each step F_Z has to be determined by numerical integration. We can rewrite equation (3.7) as

$$t \cdot f_T(t) = -F_T(t) + F_Z(tV_{max}) \tag{3.10}$$

and further differentiations shows that

$$t \cdot f_T(t) + f_T(t) = -f_T(t) + f_Z(tV_{max}) \cdot V_{max} \tag{3.11}$$

which can be simplified as the initial-value problem

$$f_T'(t) = -\frac{2}{t} f_T(t) + \frac{V_{max}}{t} \cdot f_Z(tV_{max}), \qquad f_T(\frac{A}{V_{max}}) = 0 \tag{3.12}$$

where function f_Z can be easily computed. Notice also that $t > A/V_{max}$; therefore, no singularity occurs, and hence this initial-value problem can be solved very efficiently. For example, the fourth order Runge-Kutta method or any predictor-corrector formulae is useful. For details of such methods see for example [5], [6]. In summary, $f_T(t)$ can be obtained in the following way. In intervals $[0, A/V_{max}]$ and $[B/V_{max}, \infty]$ the function is given in a closed-form, equation (3.1) and (3.8), respectively. In the interval $[A/V_{max}, B/V_{max}]$ the initial-value problem (3.12) has to be solved. Because of roundoff errors, the solution trajectories obtained for different subintervals do not match exactly at the end points. At $t = A/V_{max}$ we have exact match, since the initial value is defined as zero at this point. From equation (3.9) we know that at $t = B/V_{max}$,

$$f_T(t) = \frac{V_{max}}{B^2} E_Z(z). \tag{3.13}$$

Therefore, if $\overline{f_T}$ denotes the computed trajectory in interval $[\frac{A}{V_{max}}, \frac{B}{V_{max}}]$, then the adjusted pdf can be obtained by equation

$$f_{adjusted}(t) = \overline{f_T}(t) \cdot \frac{V_{max} E_Z(z)}{B^2 \overline{f_T}(\frac{B}{V_{max}})}. \tag{3.14}$$

4. Application.

We illustrate now the above equation for the particular case of $R_{eq} = 1Km/Hr$ and $V_{max} = 60Km/Hr$. The solution is obtained by using the fourth-order Runge-Kutta method for solving the initial value problem (3.12), and then for $t > \frac{B}{V_{max}}$ using the closed-form representation (3.9). The pdf of T is illustrated in Figure 4 for different values of a. In order to show the effect of roundoff errors no adjustment was made.

5. Conclusions.

In this paper, we proposed a method for deriving the cannel holding time distribution by a numerical integration approach. Instead of using expensive integration routines repeatedly, for small values of t, the initial value problem of an ordinary differential equation is solved, and for larger values of t, a closed-form solution formula is obtained. The effect of roundoff errors can be eliminated by a special adjustment process making the proposed algorithm not only very effective but also very accurate. Using this methodology, we can determine the cannel holding time of the traffic models which have not a closed-form solution, such as a soft handoff model. The proposed method provides a useful tool for analyzing the traffic model of cellular telephone systems.

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