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ULAM-HYERS STABILITY OF FIXED POINT EQUATIONS FOR MULTIVALUED OPERATORS ON KST SPACES

Liliana Guran Manciu

Abstract. In this paper we define the notions of Ulam-Hyers stability on KST spaces and c_w-weakly Picard operator for the multivalued operators case in order to establish a relation between these.

1 Introduction

In 1996, O. Kada, T. Suzuki and W. Takahashi [8] introduced the concept of w-distance on a metric space. Using this new concept they obtained a generalization of Caristi's fixed point theorem, given in 1976 see ([3]).

Latter on, T. Suzuki and W. Takahashi, using the setting of a metric space endowed with a w-distance, gave some fixed point results for the so-called multivalued weakly contractive operators (see[22]).

The Ulam stability of various functional equations have been investigated by many authors (see [1], [2], [5], [6], [7], [11], [15], [18], [19]).

The concept of multivalued weakly Picard operator (briefly MWP operator) was introduced in close connection with the successive approximation method and the data dependence phenomenon for the fixed point set of multivalued operators on complete metric space, by I. A. Rus, A. Petruşel and A. Sântămărian, see [14]. The theory of multivalued weakly Picard operators in L-spaces is presented on [12].

In this paper we define the notions of Ulam-Hyers stability with respect to a w-distance, multivalued c_w-weakly Picard operator and we establish a connection between these notions.

2 Preliminaries

Let (X, d) be a metric space. We will use the following notations:

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$P(X)$ - *the set of all nonempty subsets of* X;

$P_{cl}(X)$ - *the set of all nonempty closed subsets of* X;

$P_{cp}(X)$ - *the set of all nonempty compact subsets of* X;

$D : P(X) \times P(X) \to \mathbb{R}_+$, $D(A, B) = inf\{d(a, b) : a \in A, b \in B\}$ - *the gap functional.*

Let $F : X \to P(X)$ be a multivalued operator and $Y \in X$. Then:

$f : X \to Y$ *is a selection for* $F : X \to P(Y)$ if $f(x) \in F(x)$, for each $x \in X$;

$Graph(F) := \{(x, y) \in X \times Y \mid x \in F(x)\}$- *the graphic of* F;

$Fix(F) := \{x \in X \mid x \in F(x)\}$ - *the set of the fixed points of* F;

$SFix(F) := \{x \in X \mid \{x\} = F(x)\}$ - *the set of the strict fixed points of* F.

We also denote by $\mathbb{N}$ the set of all natural numbers and by $\mathbb{N}^* := \mathbb{N} \setminus \{0\}$.

For the following notations see I.A. Rus [17] and [18], I.A. Rus, A. Petruşel, A. Sîntămărian [14] and A. Petruşel [12].

Definition 1. *Let (X,d) be a metric space and* $F : X \to P_{cl}(X)$ *be a multivalued operator. By definition, F is a multivalued weakly Picard operator (briefly MWP) if for each* $x \in X$ *and each* $y \in F(x)$ *there exists a sequence* $(x_n)_{n\in\mathbb{N}}$ *such that:*

(i) $x_0 = x$, $x_1 = y$;

(ii) $x_{n+1} \in F(x_n)$, *for each* $n \in \mathbb{N}$;

(iii) the sequence $(x_n)_{n\in\mathbb{N}}$ *is convergent and its limit is a fixed point of F.*

Remark 2. *A sequence* $(x_n)_{n\in\mathbb{N}}$ *satisfying the condition (i) and (ii) in the Definition 1 is called a sequence of successive approximations of* F *starting from* $(x, y) \in Graph(F)$.

If $F : X \to P(X)$ is a MWP operator, then we define

$$F^\infty : Graph(F) \to P(Fix(F))$$

by the formula $F^\infty(x, y) := \{z \in Fix(F) \mid$ there exists a sequence of successive approximations of F starting from (x, y) that converges to $z\}$

Definition 3. *Let (X,d) be a metric space and* $F : X \to P(X)$ *be a MWP operator. Then F is called c-multivalued weakly Picard operator (briefly c-MWP operator) if and only if there exists a selection* f^∞ *of* F^∞ *such that*

$$d(x, f^\infty(x, y)) \le cd(x, y), \text{ for all } (x, y) \in Graph(F).$$

For the theory of weakly Picard operators for the multivalued case see [12] and [14].

In [18] are given the definition of Ulam-Hyers stability as follows.

Definition 4. *Let (X,d) be a metric space and* $f : X \to X$ *be an operator. By definition, the fixed point equation*

$$x = f(x) \tag{2.1}$$

is Ulam-Hyers stable if there exists a real number $c_f > 0$ *such that: for each* $\varepsilon > 0$ *and each solution* y^* *of the inequation*

$$d(y, f(y)) \le \varepsilon \tag{2.2}$$

there exists a solution x^* *of the equation 2.1 such that*

$$d(y^*, x^*) \le c_f \varepsilon.$$

Remark 5. *If* f *is a* c*-weakly Picard operator, then the fixed point equation 2.1 is Ulam-Hyers stable.*

3 Main results

First of all let us recall the concept of w-distance which was introduced by O. Kada, T. Suzuki and W. Takahashi (see [8]) as follows.

Definition 6. *Let* (X, d) *be a metric space. Then* $w : X \times X \to [0, \infty)$ *is called a weak distance (briefly* w*-distance) on* X *if the following axioms are satisfied :*

1. $w(x, z) \le w(x, y) + w(y, z)$*, for any* $x, y, z \in X$*;*
2. *for any* $x \in X$*,* $w(x, \cdot) : X \to [0, \infty)$ *is lower semicontinuous;*
3. *for any* $\varepsilon > 0$*, exists* $\delta > 0$ *such that* $w(z, x) \le \delta$ *and* $w(z, y) \le \delta$ *implies* $d(x, y) \le \varepsilon$*.*

By definition, the triple (X, d, w) is a KST-space if X is a nonempty set, $d : X \times X \to \mathbb{R}_+$ is a metric on X and $w : X \times X \to [0, \infty)$ is a w-distance on X.

Let (X, d, w) be a KST space. We say that (X, d, w) is a complete KST space if the metric space (X, d) is complete.

Some examples of w-distance can be find in [8].

Let us denote a c-weakly Picard operator with respect to a w-distance by c_w-weakly Picard operator. Next we define this notion.

Definition 7. *Let (X,d,w) be a* KST *space and* $c_w > 0$ *be a real number.* $F : X \to P(X)$ *is a multivalued* c_w*-weakly Picard operator if there exists a selection* f^∞ *for* F^∞ *such that*

$$w(x, f^\infty(x, y)) \le cw(x, y), \text{ for all } (x, y) \in Graph(F).$$

Theorem 8. *Let* (X, d, w) *be a complete* KST *space and* $F : X \to P(X)$ *be a multivalued weakly* r*-contraction type operators, i.e., there exists* $r \in [0, 1)$ *such that, for every* $x, y \in X$ *and* $u \in F(x)$*, there exists* $v \in F(y)$*, such that* $w(u, v) \le rw(x, y)$*, where* $c := \frac{1}{1-r}$*. Then* F *is a multivalued* c_w*-weakly Picard.*

Proof. We prove that on a KST space a multivalued weakly r-contraction is a multivalued c_w-weakly Picard operator.

For $x_0 \in X$ fixed and $x_1 \in F(x_0)$ there exists $x_2 \in F(x_1)$ such that $w(x_1, x_2) \leq rw(x_0, x_1)$. Inductively, for every $n \in \mathbb{N}$ and every $r \in [0, 1)$, we construct a sequence $(x_n)_{n\in\mathbb{N}} \in X$ such that:

1. $x_{n+1} \in F(x_n)$;
2. $w(x_n, x_{n+1}) \leq r^n w(x_{n-1}, x_n)$.

For every $m, n \in \mathbb{N}$ with $m > n$ we obtain the inequality

$$w(x_n, x_m) \leq \frac{r^n}{1-r} w(x_0, x_1).$$

Since (X, d, w) is a complete KST space the sequence $(x_n)_{n\in\mathbb{N}}$ has a limit. Let $f^\infty(x_0, x_1) = \lim\limits_{n\to\infty} x_n$ be the limit of the sequence, where f^∞ is a selection of the operator F^∞, above defined.

Let $n \in \mathbb{N}$ be fixed. Since $(x_m)_{m\in\mathbb{N}}$ converge to the limit $f^\infty(x_0, x_1)$ and $w(x_n, \cdot)$ is lower semicontinuous we have

$$w(x_n, f^\infty(x_0, x_1)) \leq \lim_{m\to\infty} \inf w(x_n, x_m) \leq \frac{r^n}{1-r} w(x_0, x_1).$$

Then, by triangle inequality we obtain
$w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_n) + w(x_n, f^\infty(x_0, x_1))$.
Then $w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_n) + \frac{r^n}{1-r} w(x_0, x_1)$.

If we make $n \to 1$ we have
$w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_1) + \frac{r}{1-r} w(x_0, x_1)$.
Then $w(x_0, f^\infty(x_0, x_1)) \leq \frac{1}{1-r} w(x_0, x_1)$.

Then F is a multivalued c_w-weakly Picard operator with $c = \frac{1}{1-r}$.

Theorem 9. *Let (X, d, w) be a complete KST space and $F : X \to P(X)$ be a multivalued contraction of weakly Kannan type operators, i.e., there exists $\alpha \in [0, \frac{1}{2})$ such that, for every $x, y \in X$ and $u \in F(x)$, there exists $v \in F(y)$, such that $w(u, v) \leq \alpha(D_w(x, F(x)) + D_w(y, F(y)))$, where $D_w(x, T(x)) := \inf\{w(x, y) \mid y \in T(x)\}$ and $c := \frac{1-\alpha}{1-2\alpha}$. Then F is a multivalued c_w-weakly Picard.*

Proof. Next we prove that a multivalued weakly Kannan type operators is a multivalued c_w-weakly Picard operator on a KST space.

For $x_0 \in X$ fixed and $x_1 \in F(x_0)$ there exists $x_2 \in F(x_1)$ such that $w(x_1, x_2) \leq \alpha(w(x_0, x_1) + w(x_1, x_2))$. Inductively, for every $n \in \mathbb{N}$ and some fixed r with $0 \leq r < \frac{1}{2}$ we construct a sequence $(x_n)_{n\in\mathbb{N}} \in X$ such that:

1. $x_{n+1} \in F(x_n)$;

2. $w(x_n, x_{n+1}) \le (\frac{\alpha}{1-\alpha})^n w(x_0, x_1)$.

Put $\lambda = \frac{\alpha}{1-\alpha}$. Then $0 \le \lambda < 1$. For every $m, n \in \mathbb{N}$ with $m > n$ we obtain the inequality

$$w(x_n, x_m) \le \frac{\lambda^n}{1-\lambda} w(x_0, x_1).$$

Since (X, d, w) is a complete KST space the sequence $(x_n)_{n\in\mathbb{N}}$ has a limit. Let $f^\infty(x_0, x_1) = \lim_{n\to\infty} x_n$ be the limit of the sequence, where f^∞ is a selection of the operator F^∞.

Let $n \in \mathbb{N}$ be fixed. Since $(x_m)_{m\in\mathbb{N}}$ converge to the limit $f^\infty(x_0, x_1)$ and $w(x_n, \cdot)$ is lower semicontinuous we have

$$w(x_n, f^\infty(x_0, x_1)) \le \lim_{m\to\infty} \inf w(x_n, x_m) \le \frac{\lambda^n}{1-\lambda} w(x_0, x_1).$$

Then, by triangle inequality we obtain
$w(x_0, f^\infty(x_0, x_1)) \le w(x_0, x_n) + w(x_n, f^\infty(x_0, x_1))$.
Then $w(x_0, f^\infty(x_0, x_1)) \le w(x_0, x_n) + \frac{\lambda^n}{1-\lambda} w(x_0, x_1)$.

If we make $n \to 1$ we have $w(x_0, f^\infty(x_0, x_1)) \le w(x_0, x_1) + \frac{\lambda}{1-\lambda} w(x_0, x_1)$.
Then $w(x_0, f^\infty(x_0, x_1)) \le \frac{1}{1-\lambda} w(x_0, x_1)$.

If we replace $\lambda = \frac{\alpha}{1-\alpha}$ we obtain that $w(x_0, f^\infty(x_0, x_1)) \le \frac{1-\alpha}{1-2\alpha} w(x_0, x_1)$.

Then F is a multivalued c_w-weakly Picard operator with $c = \frac{1-\alpha}{1-2\alpha}$.

Theorem 10. *Let (X, d, w) be a complete KST space and $F : X \to P(X)$ be a multivalued contraction of weakly Reich type operators, i.e., there exists $a, b, c \in \mathbb{R}_+$, with $a+b+c < 1$ such that, for every $x, y \in X$ and $u \in F(x)$, there exists $v \in F(y)$, such that $w(u, v) \le aw(x, y) + bD_w(x, F(x)) + cD_w(y, F(y))$, where $c := \frac{1-c}{1-(a+b+c)}$. Then F is a multivalued c_w-weakly Picard.*

Proof. For $x_0 \in X$ fixed and $x_1 \in F(x_0)$ there exists $x_2 \in F(x_1)$ such that

$$w(x_1, x_2) \le aw(x_0, x_1) + bD_w(x_0, T(x_0)) + cD_w(x_1, T(x_1))$$

$$w(x_1, x_2) \le aw(x_0, x_1) + bw(x_0, x_1) + cw(x_1, x_2))$$

$$w(x_1, x_2) \le \frac{a+b}{1-c} w(x_0, x_1).$$

Inductively, for every $n \in \mathbb{N}$ and $a, b, c \in \mathbb{R}_+$ with $a + b + c < 1$, we construct a sequence $(x_n)_{n\in\mathbb{N}} \in X$ such that:

1. $x_{n+1} \in F(x_n)$;

2. $w(x_n, x_{n+1}) \le (\frac{a+b}{1-c})^n w(x_0, x_1)$.

Put $\beta = \frac{a+b}{1-c}$. Then $0 \leq \beta < 1$. For every $m, n \in \mathbb{N}$ with $m > n$ we obtain the inequality $w(x_n, x_m) \leq \frac{\beta^n}{1-\beta} w(x_0, x_1)$.

Since (X, d, w) is a complete KST space the sequence $(x_n)_{n \in \mathbb{N}}$ has a limit. Let $f^\infty(x_0, x_1) = \lim\limits_{n \to \infty} x_n$ be the limit of the sequence, where f^∞ is a selection of the operator F^∞.

Let $n \in \mathbb{N}$ be fixed. Since $(x_m)_{m \in \mathbb{N}}$ converge to the limit $f^\infty(x_0, x_1)$ and $w(x_n, \cdot)$ is lower semicontinuous we have

$$w(x_n, f^\infty(x_0, x_1)) \leq \lim_{m \to \infty} \inf w(x_n, x_m) \leq \frac{\beta^n}{1-\beta} w(x_0, x_1).$$

Then, by triangle inequality we obtain
$w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_n) + w(x_n, f^\infty(x_0, x_1))$.
Then $w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_n) + \frac{\beta^n}{1-\beta} w(x_0, x_1)$.

If we make $n \to 1$ we have $w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_1) + \frac{\beta}{1-\beta} w(x_0, x_1)$.
Then $w(x_0, f^\infty(x_0, x_1)) \leq \frac{1}{1-\beta} w(x_0, x_1)$.
If we replace $\beta = \frac{a+b}{1-c}$ we obtain that $w(x_0, f^\infty(x_0, x_1)) \leq \frac{1-c}{1-(a+b+c)} w(x_0, x_1)$.

Then F is a multivalued c_w-weakly Picard operator with $c = \frac{1-c}{1-(a+b+c)}$.

Theorem 11. *Let (X, d, w) be a complete KST space and $F : X \to P(X)$ be a multivalued contraction of weakly Čiriċ type operators, i.e., there exists $q \in [0, 1)$ such that, for every $x, y \in X$ and $u \in F(x)$, there exists $v \in F(y)$, such that $w(u, v) \leq q \max\{w(x, y), D_w(x, F(x)), D_w(y, F(y)), \frac{1}{2} D_w(x, F(y))\}$ where $c := \frac{1}{1-q}$. Then F is a multivalued c_w-weakly Picard.*

Proof. Let $x_0 \in X$ be fixed. For $x_1 \in F(x_0)$ there exists $x_2 \in F(x_1)$ such that:

1. $w(x_1, x_2) \leq q w(x_0, x_1)$
2. $w(x_1, x_2) \leq q w(x_0, x_1)$
3. $w(x_1, x_2) \leq q w(x_1, x_2)$
4. $w(x_1, x_2) \leq \frac{q}{2} w(x_0, x_2)$
   $w(x_1, x_2) \leq \frac{q}{2} (w(x_0, x_1) + w(x_1, x_2)$
   $w(x_1, x_2) \leq \frac{q}{2-q} (w(x_0, x_1)$

Then $w(x_1, x_2) \leq \max\{q, \frac{q}{2-q}\} w(x_0, x_1)$. Since $q > \frac{q}{2-q}$, for every $q \in [0, 1)$, then $w(x_1, x_2) \leq q w(x_0, x_1)$.

On this way, inductively we construct a sequence $(x_n)_{n \in \mathbb{N}} \in X$ such that:

1. $x_{n+1} \in F(x_n)$;
2. $w(x_n, x_{n+1}) \leq q^n w(x_0, x_1)$.

For every $m, n \in \mathbb{N}$ with $m > n$ we obtain the inequality

$$w(x_n, x_m) \leq \frac{q^n}{1-q} w(x_0, x_1).$$

Since (X, d, w) is a complete KST space the sequence $(x_n)_{n\in\mathbb{N}}$ has a limit. Let $f^\infty(x_0, x_1) = \lim_{n\to\infty} x_n$ be the limit of the sequence, where f^∞ is a selection of the operator F^∞.

Let $n \in \mathbb{N}$ be fixed. Since $(x_m)_{m\in\mathbb{N}}$ converge to the limit $f^\infty(x_0, x_1)$ and $w(x_n, \cdot)$ is lower semicontinuous we have

$$w(x_n, f^\infty(x_0, x_1)) \leq \lim_{m\to\infty} \inf w(x_n, x_m) \leq \frac{q^n}{1-q} w(x_0, x_1).$$

Then, by triangle inequality we obtain
$w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_n) + w(x_n, f^\infty(x_0, x_1))$.
Then $w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_n) + \frac{q^n}{1-q} w(x_0, x_1)$.

If we make $n \to 1$ we have $w(x_0, f^\infty(x_0, x_1)) \leq w(x_0, x_1) + \frac{q}{1-q} w(x_0, x_1)$.
Then $w(x_0, f^\infty(x_0, x_1)) \leq \frac{1}{1-q} w(x_0, x_1)$.

Then F is a c_w-weakly Picard operator with $c = \frac{1}{1-q}$. □

On the other hand we define Ulam-Hyers w-stability of fixed point equations for multivalued operators as follows.

Definition 12. *Let (X,d,w) be a KST space and $F : X \to P(X)$ be a multivalued operator. By definition, the fixed point equation*

$$x \in F(x) \tag{3.1}$$

is Ulam-Hyers stable with respect to a w-disance if there exists a real number $c > 0$ such that, for each $\varepsilon > 0$ and each solution u^* of the inequation

$$D_w(u, F(u)) \leq \varepsilon, \tag{3.2}$$

there exists a solution x^* of the equation 3.1 such that

$$w(u^*, x^*) \leq c\varepsilon.$$

Theorem 13. *If F is a multivalued c_w-weakly Picard operator, then the fixed point equation 3.1 is Ulam-Hyers stable with respect to a w-distance.*

Proof. Let $\varepsilon > 0$ and let u^* be a solution of the inequation 3.2.
Let $y \in F(u^*)$ be such that $D_w(u^*, F(u^*)) = w(u^*, y)$.

We take a solution of equation 3.1 such that $x^* := f^\infty(u^*, y)$.

Then we have $w(u^*, x^*) = w(u^*, f^\infty(u^*, y)) \leq c(u^*, y) \leq \varepsilon$. □

Remark 14. *From Theorem 13 it follows that for each example of multivalued c_w-weakly Picard operator we have an example of equation 3.1 which is Ulam-Hyers stable with respect to a w-distance.*

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Liliana Guran Manciu
Department of Pharmaceutical Sciences,
Faculty of Medicine, Pharmacy and Dentistry,
Vasile Goldiş Western University of Arad,
Revoluţiei Avenue, no. 94-96, 310025, Arad, Romania.
E-mail: gliliana.math@gmail.com

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