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MAGNETIC FIELD AND THERMAL RADIATION EFFECTS ON STEADY HYDROMAGNETIC COUETTE FLOW THROUGH A POROUS CHANNEL

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Abstract. This paper investigates effects of thermal radiation and magnetic field on hydromagnetic Couette flow of a highly viscous fluid with temperature-dependent viscosity and thermal conductivity at constant pressure through a porous channel. The influence of the channel permeability is also assessed. The relevant governing partial differential equations have been transformed to non-linear coupled ordinary differential equations by virtue of the steady nature of the flow and are solved numerically using a marching finite difference scheme to give approximate solutions for the velocity and temperature profiles. We highlight the effects of Nahme numbers, magnetic field, radiation and permeability parameters on both profiles. The results obtained are used to give graphical illustrations of the distribution of the flow variables and are discussed.

1 Introduction

Flows in porous media have several applications in geothermal, oil reservoir engineering and astrophysics. [15] investigated on the steady free convection in a porous medium. They also extended their research work into heat dispersion effect on steady convection in an isotropic porous media. [18] studied the heat transfer flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Also,[2] carried out a research on the steady flow in a porous medium with simultaneous free convection heat and mass transfer over a semi-infinite vertical plate. [9] studied a lattice Boltzmann model for convection heat transfer in porous medium. Besides, they further researched into finite differences based lattice Boltzmann simulation of natural convection heat transfer in a horizontal concentric annulus cylinder. [6] examined the effect of surface mass flux on mixed convection along a vertical plate embedded in porous medium. He found out that the velocity and temperature profiles increase as the mixed convection parameter increases, and

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that injection increases the heat transfer for all parameters studied.

Similarly, [11] studied heat transfer inflow through a porous medium bounded by a semi-infinite horizontal plate. He concluded that there is a rise in the rate of heat transfer at the plate with increasing κ (permeability parameter). [14] researched on the suction driven flow and heat transfer in a pipe filled with porous medium. Besides, many researchers studied Couette flow through porous media. [16] investigated non-Darcy flow through porous media in a turbulent plane Couette flow. [3] carried out a research on the stability of hydromagnetic dissipative Couette flow with non-axisymmetric disturbance. [17] also examined a singular perturbation solution for Couette flow over a semi-infinite porous bed. [19] investigated the effect of thermal radiation on a laminar Couette flow and on the theory of non-Newtonian fluid through porous media.

In addition to the above researches, there have been increased interests due to the effects of magnetic fields and some thermophysical properties on the electrically conducting fluid flows. [7] examined the effect of radiation on temperature and velocity in electrodynamics froth flow process. They concluded that the velocity and temperature increase as the radiation increases. [5] also examined the viscous heating of high Prandtl number fluids with temperature-dependent viscosity. They carried out a similar calculation for the simpler case of a homogeneous flow without porous medium. [13], in his work, examined the stability of Couette flow in the presence of an axial magnetic field. [8] investigated the effect of Biot number on thermal criticality in a Couette flow. They concluded that the magnitude of thermal explosion criticality at very large activation energy ($\epsilon = 0$) is lower than that of moderate value of activation energy and thus thermal explosion will occur faster in the former than the latter.

[10] examined the effect of variable viscosity on the transient Couette flow of dusty fluid with heat transfer between parallel plates. He showed some important effects for the variable viscosity and the uniform magnetic field on the transient flow and heat transfer of both the fluid and dust particles. [4] studied on the Couette flow through a porous medium of a high Prandtl number fluid with temperature dependent viscosity. He concluded that, in the steady state, the medium permeability (λ) for both velocity and temperature profiles are positively skew and their skewness increases with (λ). He also affirmed that the lower the permeability of the medium the faster the velocity but it does not significantly influence the temperature profile development. [12] investigated the unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi-infinite moving porous plate with variable suction under the influence of magnetic field in which the temperature varies exponentially with time. [1] considered the problem of MHD free-convection flow with radiation heat transfer in a porous medium. It was shown that the flow field is affected mainly by radiation and convection parameters in addition to magnetic factors.

However, the coupled effects of thermal radiation and magnetic field on the hydromagnetic Couette flow through a porous medium has not been studied. In this manuscript, the effect of thermal radiation and magnetic field of a high viscous fluid with temperature-dependent viscosity and thermal conductivity on hydromagnetic Couette flow through a porous channel is examined using finite difference technique. Different velocity profiles and temperature profiles are shown in figures for different dimensionless groups.

2 Mathematical Analysis

The geometry for the flow consists of two concentric infinite cylindrical surfaces with the outer surface having radius $\mathbf{r_2}$. The hydromagnetic fluid under investigation occupies the annular space of width $\mathbf{b}$ between the cylinders. The system is at rest at first instant and the two surfaces is at constant temperature T_0. At time $t^* = 0$, the outer cylinder is suddenly set in motion and rotate with constant angular velocity ω while the inner cylinder of radius $\mathbf{r_1}$ is stationary in the presence of an applied external transverse magnetic field. The hydromagnetic fluid is considered to have a negligible internally-generated magnetic field and in the course of time, the fluid gradually participates in the motion and viscous dissipation increases the temperature. The electrically conducting fluid is presumed to have a very small electric current within the hydromagnetic fluid flow and the temperature of the two cylinders is maintained at the constant temperature T_0 by cooling. The thickness $\mathbf{b}$ of the fluid layer is assumed to be small compared with the outer cylinder so that the problem is identical to that of Couette flow with a negligible pressure gradient. The permeability is also assumed to be constant.

We reduce the complicated motion of a viscous fluid in a porous solid to that of the motion of a homogeneous fluid with some additional resistance $\mathbf{f_r}$ and the non-homogeneous medium is homogeneous with dynamical properties equal to the local averages of the original fluid continuum. This resistance is modelled as:
$f_r = -\left(\frac{\nu}{\lambda^*}\right) u^*$ where ν is the kinematic viscosity, λ^* the permeability of the porous medium, u^* the local fluid velocity, and the temperature law for the viscosity is taken as:
$\mu = \mu_0 \exp\left\{-\beta\left(T - T_0\right)\right\}$ where μ is the dynamic viscosity, μ_0 the dynamic viscosity at T_0, β a constant, T the local temperature and T_0 the reference temperature. The magnetic field parameter for the MHD flow is modelled as: $M^2 = \frac{\sigma_0 B_0^2 u}{\rho\nu}$ where σ_0 is the magnetic permeability, B_0 the magnetic induction, u the fluid velocity and ρ the fluid density. Following [20], we assumed the radiative heat flux $\frac{\partial q}{\partial z'}$, in the energy equation, as: $\frac{\partial q}{\partial z'} = 4\alpha^2\left(T - T_0\right)$ where α^2 is the radiation absorption coefficient. It is also assumed that the flow we studied takes place in a porous medium and

concerns a high viscosity fluid exhibiting a strong dependence on temperature and constant thermal conductivity. Hall effect, Joule heating and other variable thermo-physical parameters are assumed to be constants. Under the above assumptions, the incompressible hydromagnetic fluid flow, relevant for the problem, is governed by the following equations:

$$\frac{\partial v}{\partial y^*} = 0 \tag{2.1}$$

$$\frac{\partial u^*}{\partial t^*} = \frac{1}{\rho}\frac{\partial}{\partial y^*}\left(\mu\frac{\partial u^*}{\partial y^*}\right) - \frac{\nu}{\lambda^*}u^* + \frac{\sigma_0 B_0^2}{\rho\nu}u^* \tag{2.2}$$

$$\frac{\partial T}{\partial t^*} = \frac{\kappa^*}{\rho C_p}\frac{\partial^2 T}{\partial y^{*2}} + \left(\frac{\mu}{\rho C_p}\right)\left(\frac{\partial u^*}{\partial y^*}\right) - 4\alpha^2\left(T' - T_\infty\right) \tag{2.3}$$

where v is the velocity of flow direction, y^* the local flow direction, κ^* the thermal conductivity, C_p the specific heat capacity at constant pressure, T' the temperature at time t and T_∞ the reference temperature. Subject to the initial and boundary conditions where (2.1) gives $v(y) = v_0$:

$$\left.\begin{array}{lll} t^* < 0, & u^* = 0, & T = T_0 \\ y^* = 0, & u^* = 0, & T = T_0 \\ y^* = b, & u^* = U, & T = T_0 \end{array}\right\} \tag{2.4}$$

where U is the linear velocity of the outer cylinder ($U = r_2\omega$).

Introducing the following dimensionless parameters:

$$y = \frac{y^*}{b}, \qquad \theta = \beta(T - T_0), \qquad t = \frac{k^* t^*}{\rho C_p b^2},$$
$$U = \frac{u^*}{u}, \qquad \lambda = \frac{\lambda^*}{b^2}.$$

Denoting the Hartmann number,Nahme number and radiation parameter by M, Na_0 and R^2 respectively, we then transform the above governing equations into their dimensionless forms:

$$\frac{d^2u}{dy^2} - \frac{1}{(1-\theta)}\left(\frac{d\theta}{dy}\right)\left(\frac{du}{dy}\right) - [\lambda'(1-\theta) - M^2]u = 0 \tag{2.5}$$

$$\frac{d^2\theta}{dy^2} - \left[R^2 + Na_0\left(\frac{du}{dy}\right)^2\right]\theta + Na_0\left(\frac{du}{dy}\right)^2 = 0 \tag{2.6}$$

where

$$M^2 = \frac{b^2\sigma B_0^2}{\mu_0} \qquad Na_0 = \frac{\mu_0\beta U^2}{\kappa^*} \qquad \text{and} \qquad R^2 = \frac{4\alpha^2 b^2}{\kappa^*}.$$

The initial and boundary conditions are now written as:

$$\begin{aligned} t \leq 0, \quad y = 0, \quad u = 0, \quad \theta = 0; \\ y = 0 \quad u = 0, \quad \theta = 0; \\ y = 1 \quad u = 1, \quad \theta = 0. \end{aligned}$$

3 Method of Solution

The steady non-linear coupled ordinary differential equations (2.5) and (2.6) with the initial and boundary conditions are solved by employing the centred finite difference scheme. This is done by considering a function u as known on a domain D (in the region under investigation) if its values on the grip points on this domain are known. We use these values to calculate an acceptable approximation to the value of u at any other point in D by means of an approximation schemes for derivative of u and θ in terms of these unknown to generate finite differences equations that were discretized. The discretization provided a useful and consistent approximation to the solutions of dimensionless governing equations.

We discretize the governing equations (2.5) and (2.6) based on the steady state conditions. The numerical computation generated linearized system of equations based on our step size and results of these are achieved with the aid of MATLAB application software. Representing the step size by h, the finite difference equations corresponding to the equations are given:

$$\frac{(u_{j+1} + u_{j-1} - 2u_j)}{h^2} - \frac{(u_{j+1} + u_{j-1} - 2u_j)}{h^2}\theta - \frac{(\theta_{j+1} - \theta_j)(u_{j+1} - u_j)}{h^2} - \left[\lambda'(1 - \theta_j) - M^2\right]u_j = 0 \tag{3.1}$$

$$\frac{(\theta_{j+1} - \theta_{j-1}) - 2\theta_j}{h^2} - R^2\theta_j - Na_0\theta_j\frac{(u_{j+1} + u_j)^2}{h^2} + Na_0\frac{(u_{j+1} + u_j)^2}{h^2} \tag{3.2}$$

4 Results and Discussion

Numerical calculations have been performed for the temperature and velocity profiles. The results are presented graphically in figure (1) - (10) for various combinations of the flow parameters. The permeability parameters $\lambda' = 0$ corresponds to free flow; $\lambda' = 2$ corresponds to moderate permeability; $\lambda' \geq 10$ denotes low permeabilities and $\lambda' \leq 1$ represents high permeabilities. The magnetic field effects on this problem are found to proportional directly to the magnetic field strength. Other parameters were studied.

The velocity profiles depend on the permeability and magnetic field parameters. However, the temperature profiles depend on thermal radiation parameter

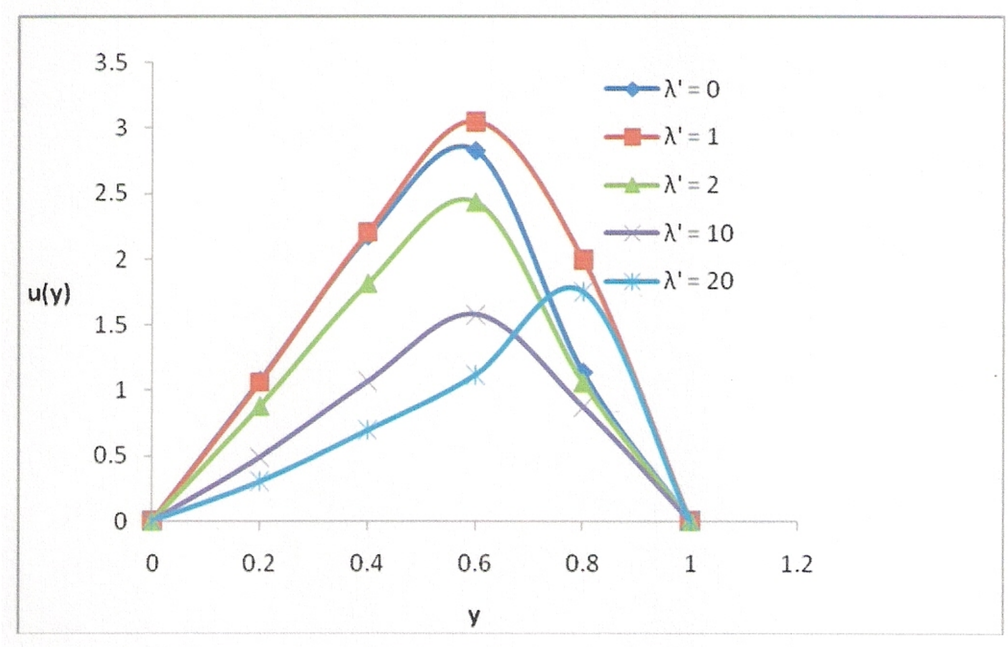

Figure 1: Dimensionless velocity profiles against y at $M = 0$

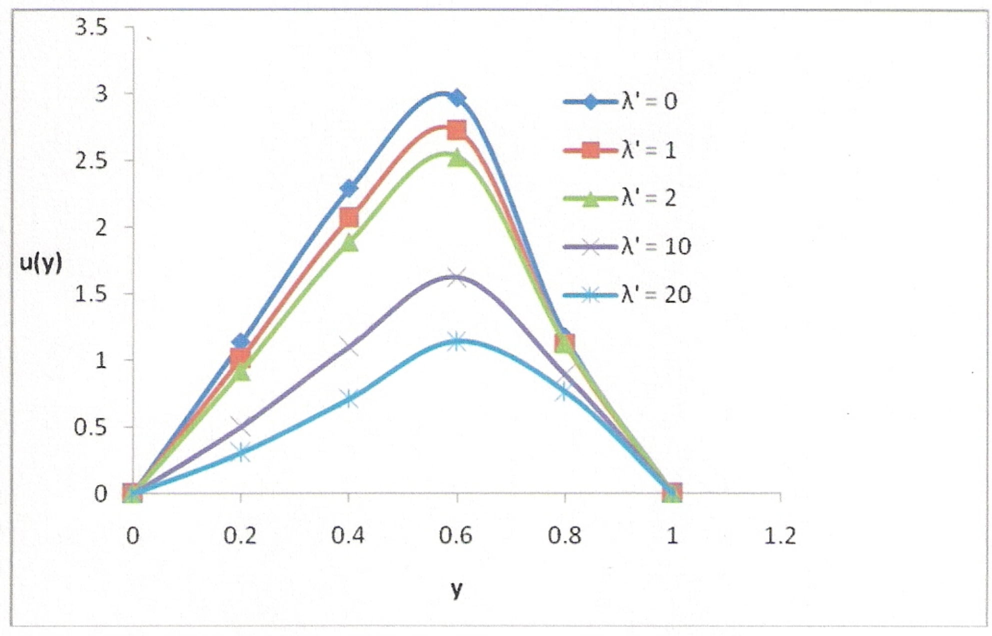

Figure 2: Dimensionless velocity profiles against y at $M = 0.5$

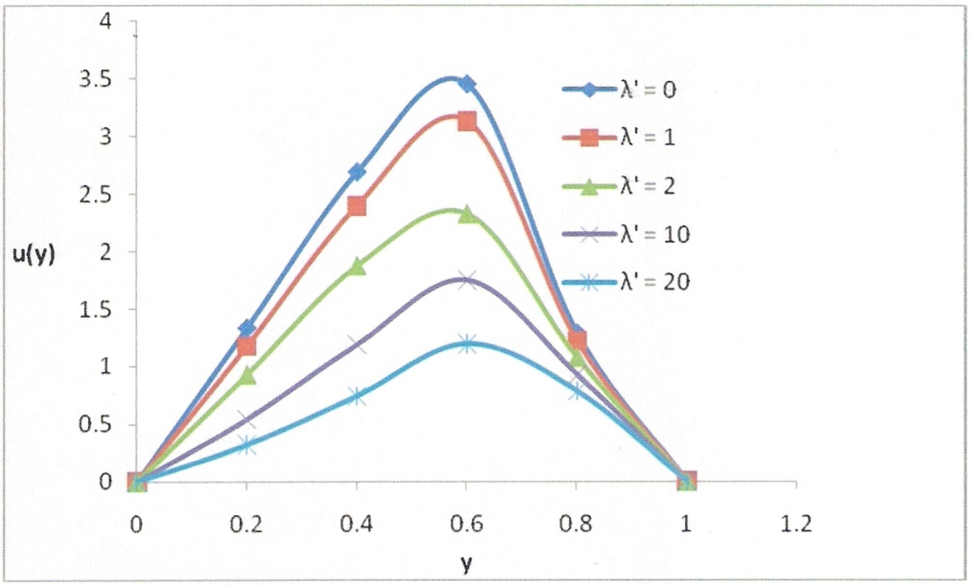

Figure 3: Dimensionless velocity profiles against y at $M = 1$

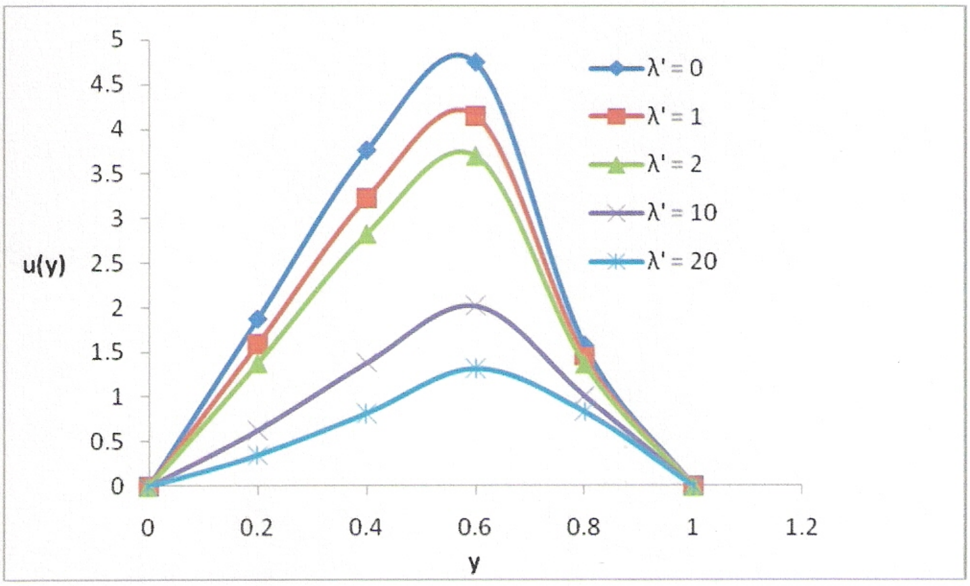

Figure 4: Dimensionless velocity profiles against y at $M = 1.5$

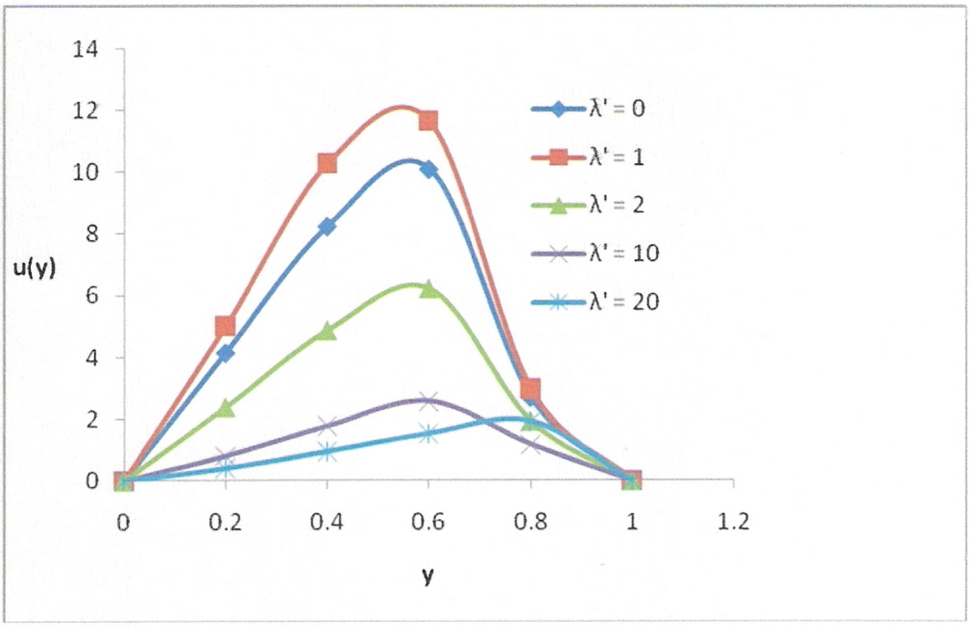

Figure 5: Dimensionless velocity profiles against y at $M = 2$

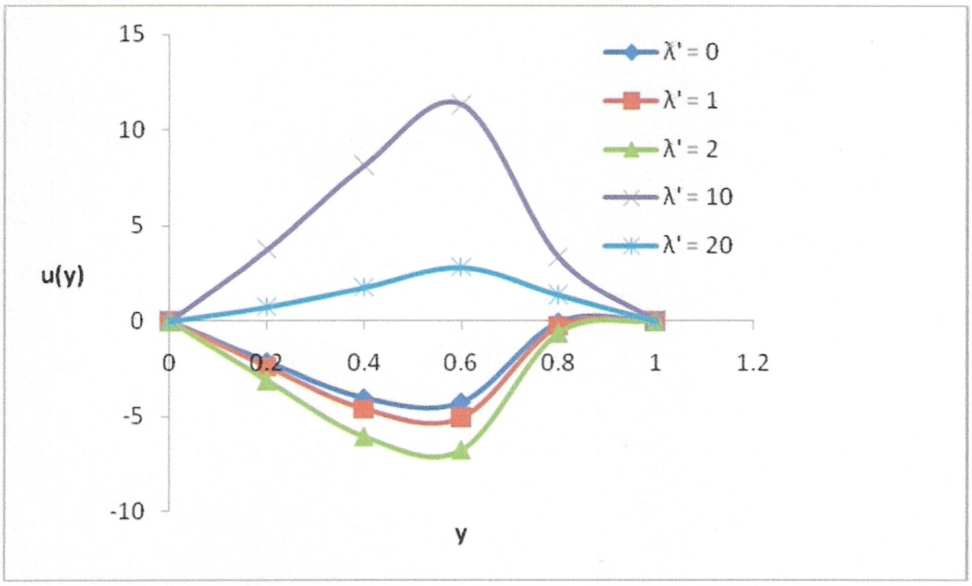

Figure 6: Dimensionless velocity profiles against y at $M = 3$

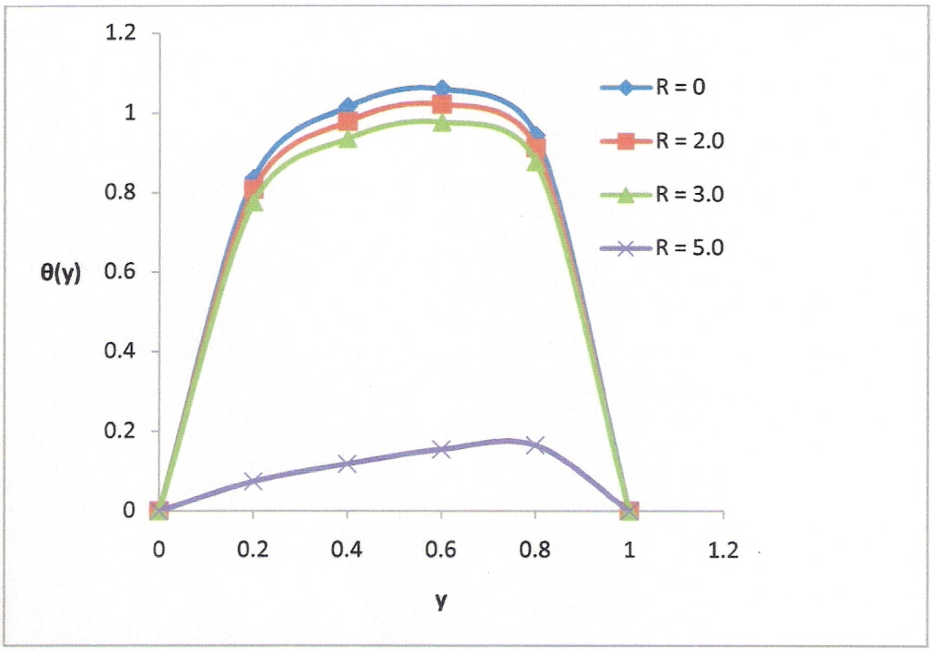

Figure 7: Dimensionless temperature profiles against y at $Na_0 = 10$

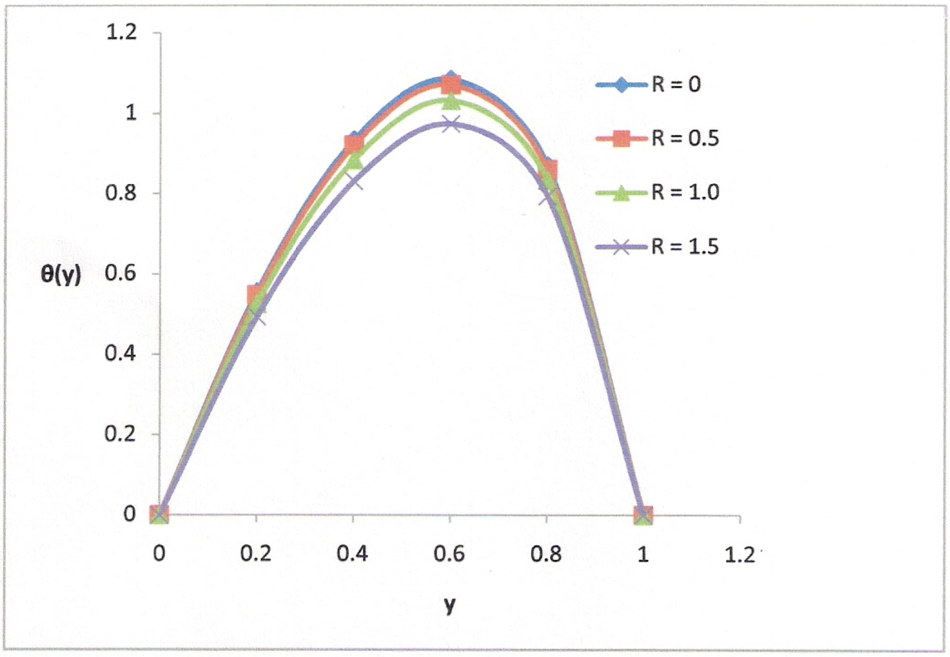

Figure 8: Dimensionless temperature profiles against y at $Na_0 = 10$

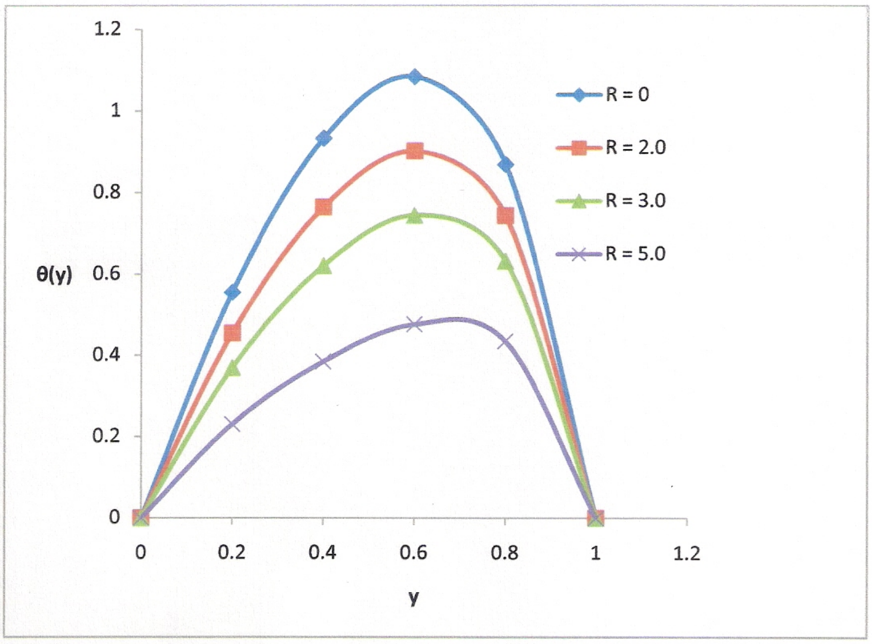

Figure 9: Dimensionless temperature profiles against y at $Na_0 = 100$

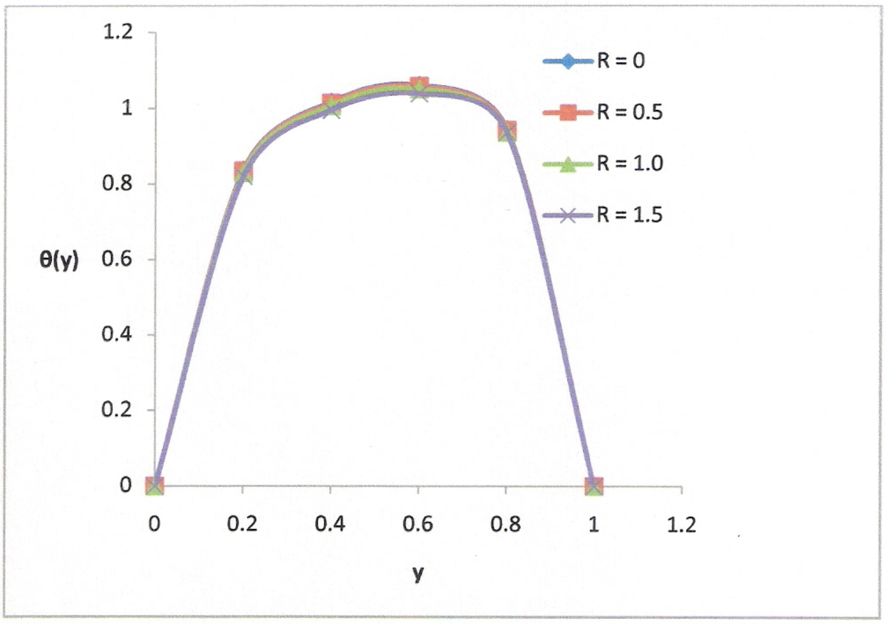

Figure 10: Dimensionless temperature profiles against y at $Na_0 = 100$

and Nahme number. The effects of Nahme numbers and thermal radiation were examined. We investigated cases where values of: $R = 0, 0.5, 1, 1.5, 2, 3$ and 5; $M = 0, 1, 1.5, 2$ and 3; and $Na_0 = 10$ and 100. Figure (1) - (5) show that increase in magnetic parameter increases the velocity profiles with significant effects of permeability variables. The magnetic field strength has effects on the velocity profiles. This is because the magnetic parameter depicts the ratio of magnetic induction to the viscous force. Hence, increase in the magnetic field parameter reduces the viscosity of the fluid under investigation. Figure (6) shows that at higher moderate permeabilities; the hydromagnetic flow of a high viscous fluid overshoots the boundary.

The velocity profiles for $\lambda' > 0$ are positively skew and their skewness increases with λ' when there is no magnetic field effect on the distribution. On introduction of the magnetic field strength, the velocity profiles have considerable changes for high values of magnetic field parameter and low values of magnetic parameters maintained the same positive skewness with the cases of absence of magnetic field. For high permeability ($\lambda' \leq 1$), velocity profiles exhibit a minimum for moderate permeabilities ($\lambda' = 2$) and increase beyond the value of the free flow ($\lambda' = 0$). For low permeabilities ($\lambda' \geq 10$), the maximum velocity reached increases beyond the value of free flow.

Figures (7) - (8) show that, in all cases of $Na_0 = 10$, the maximum temperature is lower than that of the absence of thermal radiation. Effects of the distributions on the temperature profiles is mesokurtic. It is observed that increase in thermal radiation parameter results in decrease in temperature profiles. In figures (9) - (10) where cases of $Na_0 = 100$ were shown, the same effects were noticed but it is provoked by a wider range of permeability variation. However, the distribution of the temperature profiles, for all values of R, is platykurtic. The kurtosis for temperature profiles increases with increase in the thermal radiation parameter. While there is increase in the magnetic field parameter M, the positive skewness of the velocity profiles decreases. At $M = 2$, the skewness of the velocity profiles becomes normal except for very low permeabilities. Furthermore, increase from $M = 1$ to 3 for free flow results in the symmetry of the velocity profile distribution. Moderate and low permeabilities give asymmetrical and positive skewness respectively with the distribution of the velocity profile being platykurtic. At higher magnetic field, different features of velocity distribution fluctuations are noticed for different values of permeabilities. This is obvious when there is free flow and cases of high permeabilities.

5 Concluding Remarks

The problem of hydromagnetic Couette flow of a high viscous fluid through a porous channel in the presence of an applied uniform transverse magnetic field and thermal radiation is investigated. Effects of permeability parameter for the cases of low, moderate and high permeabilities on the numerical solutions were obtained for different magnetic parameters and Nahme number. Temperature and velocity profiles are presented for different Nahme and magnetic field parameters to reveal the coupled effects of thermal radiation and magnetic field were shown, discussed and the following conclusions were made:

1. Increase in thermal radiation of the fluid results to a decrease in the temperature profiles of the hydromagnetic Couette fluid.
2. The permeability of the porous medium and thermal radiation have insignificant effects on the steady hydromagnetic Couette fluid flow.
3. Increase in Nahme number gives correspoding slight increase in the temperature distributions of the fluid flow.
4. Increase in magnetic field leads to an increase in the velocity profiles with significant effects of low and moderate permeability parameters except at high medium permeability with very high magnetic field where increase in magnetic field decreases the velocity profiles.

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