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HIGHER *-DERIVATIONS BETWEEN UNITAL C*-ALGEBRAS

M. Eshaghi Gordji, R. Farokhzad Rostami and S. A. R. Hosseinioun

Abstract. Let $\mathcal{A}, \mathcal{B}$ be two unital C^*-algebras. We prove that every sequence of mappings from $\mathcal{A}$ into $\mathcal{B}$, $H = \{h_0, h_1, ..., h_m, ...\}$, which satisfy $h_m(3^n uy) = \sum_{i+j=m} h_i(3^n u)h_j(y)$ for each $m \in \mathbb{N}_0$, for all $u \in U(\mathcal{A})$, all $y \in \mathcal{A}$, and all $n = 0, 1, 2, ...$, is a higher derivation. Also, for a unital C^*-algebra $\mathcal{A}$ of real rank zero, every sequence of continuous mappings from $\mathcal{A}$ into $\mathcal{B}$, $H = \{h_0, h_1, ..., h_m, ...\}$, is a higher derivation when $h_m(3^n uy) = \sum_{i+j=m} h_i(3^n u)h_j(y)$ holds for all $u \in I_1(\mathcal{A}_{sa})$, all $y \in \mathcal{A}$, all $n = 0, 1, 2, ...$ and for each $m \in \mathbb{N}_0$. Furthermore, by using the fixed points methods, we investigate the Hyers–Ulam–Rassias stability of higher $*-$derivations between unital C^*-algebras.

Full text

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M. Eshaghi Gordji
Department of Mathematics, Semnan University,
P.O. Box 35195-363, Semnan, Iran.
e-mail: madjid.eshaghi@gmail.com

R. Farokhzad Rostami
Department of Mathematics,
Shahid Beheshti University,
Tehran, Iran.

S. A. R. Hosseinioun
Department of Mathematics,
Shahid Beheshti University,
Tehran, Iran.