FRACTIONAL ORDER DIFFERENTIAL INCLUSIONS ON THE HALF-LINE

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Abstract. We are concerned with the existence of bounded solutions of a boundary value problem on an unbounded domain for fractional order differential inclusions involving the Caputo fractional derivative. Our results are based on the fixed point theorem of Bohnnenblust-Karlin combined with the diagonalization method.

Full text

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