

NEUMANN SYSTEM AND HYPERELLIPTIC AL FUNCTIONS

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Abstract. This article shows that the Neumann dynamical system is described well in terms of the Weierstrass hyperelliptic al functions. The descriptions are very primitive; their proofs are provided only by residual computations but don't require any theta functions.

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