EXISTENCE AND UNIQUENESS OF THE SOLUTION OF THE COUPLED CONDUCTION–RADIATION ENERGY TRANSFER ON DIFFUSE–GRAY SURFACES

Naji Qatanani, Amjad Barham and Qasem Heeh

Abstract. This article gives very significant and up–to–date analytical results on the conductive–radiative heat transfer model containing two conducting and opaque materials which are in contact by radiation through a transparent medium bounded by diffuse–gray surfaces. Some properties of the radiative integral operator will be presented. The main emphasis of this work deals also with the question of existence and uniqueness of weak solution for this problem. The existence of weak solution will be proved by showing that our problem is pseudomonotone and coercive. The uniqueness of the solution will be proved using an idea from the analysis of nonlinear heat conduction.

Full text

References

[1] R. A. Bialecki, *Solving heat radiation problems using the boundary element method*, Topics in Engineering. Southampton: Computational Mechanics Publications 1993. MR1266165. Zbl 0863.73002.

[2] R. Dautray and J. Lions, *Mathematical analysis and numerical methods for science and technology*, Vol.**3**. Spectral theory and applications. Springer–Verlag, 1990. MR1064315(91h:00004a). Zbl 0766.47001.

[3] H. Gajewski, K. Groeger and K. Zacharias, *Nichtlineare Operatorgleichungen und Operatordifferentialgleichungen*, Academie–Verlag, 1974. Zbl 0289.47029.

[4] D. Gilbarg and N. Trudinger, *Elliptic partial differential equations of second order*, Springer–Verlag, 1983. MR0737190(86c:35035). Zbl 0562.35001.

[5] G. Hardy and J. Littlewood, *Inequalities*, Cambridge University Press, 1988. MR0944909 (89d:26016). Zbl 0634.26008.

[6] E. Hennebach, P. Junghanns and G. Vainikko, *Weakly singular integral equations with operators valued kernels and an application to radiation transfer problem*, Integral Equations Oper. Theory, **22** no.1 (1995), 37 – 64. MR1327593(96c:45003). Zbl 0828.45012.

[7] C. Kelley, *Existence and uniqueness of solutions of nonlinear systems of Conductive–Radiative heat transfer equations*, Transp. Theory Stat. Phys , **25** no 2 (1996), 249 – 260. MR1390986(99a:82084). Zbl 0857.45009.

[8] M. Krizek and L. Liu, *On a comparison principle for a quasilinear elliptic boundary value problem of a nonmonotone type*, Appl. Math., **24** no. 1 (1996), 97 – 107. MR1404987(97h:35058). Zbl 0858.35008.

[9] M. Laitinen and T. Tiihonen, *Conductive-radiative heat transfer in grey materials*, Quart. Appl. Math., **59** (2001), 737–768. MR1866555(2002h:35093). Zbl pre01953521.

[10] M. Metzger, *Existence for a time–dependent heat equation with non–local radiation terms*, Math. Methods Appl. Sci., **22**(1999), 1101 – 1119. MR1706102(2000g:35116). Zbl 0933.35104.

[11] M. Modest, *Radiative heat transfer*, McGraw–Hill, 1993.

[12] N. Qatanani, *Use of the multigrid method for heat radiation problem*, J. Appl. Math., **2003** no. 6 (2003), 305 – 317. MR2036974(2004k:65267). Zbl 1024.65117.

[13] N. Qatanani, *Analysis of the heat equation with non–local radiation terms in a non–convex diffuse and grey surfaces*, European Journal of Scientific Research, **15** no. 2 (2006), 245 – 254.

[14] N. Qatanani, *Qualitative analysis of the radiative energy transfer model,* European Journal of Scientific Research, **17** no.3 (2007), 379-391.

[15] N. Qatanani and I. Barghouthi, *Numerical treatment of the two–dimensional heat radiation integral equation*, J. Comput. Anal. Appl., **7** no.3 (2005), 319 – 349. MR2220288(2006k:65392). Zbl 1083.65128.

[16] N. Qatanani and K. Salah, *Error analysis for the finite element approximation of conductive–radiative model*, European Journal of Scientific Research, **11** no.2 (2005), 236 – 245.

[17] N. Qatanani and M. Schulz, *The heat radiation problem: Three–dimensional analysis for arbitrary enclosure geometries*, J. Appl. Math. **2004** no. 4 (2004), 311 – 330. MR2100258(2005f:80003). Zbl 1079.65117.

[18] N. Qatanani and M. Schulz, *Analytical and numerical investigation of the Fredholm integral equation for the heat radiation problem*, Appl. Math. Comput., **175** no.1 (2006), 149 – 170. MR2216333. Zbl 1090.65137.

[19] R. Saldanha and R. Martins, *Existence, uniqueness and construction of the solution of the energy transfer problem in a rigid and nonconvex black body*, Z. Angew. Math. Phys. **42** no.3 (1991), 334-347. Zbl 0742.73005.

[20] R. Saldanha and R. Martins, *An alternative mathematical modelling for coupled conduction/radiation energy transfer phenomenon in a system of N gray bodies surrounded by a vacuum*, Int. J. Non-Linear Mech., **30** no. 4 (1995), 433 – 447. Zbl 0836.35155.

[21] R. Siegel and J. Howell, *Thermal heat transfer*, 3rd edn., Hemisphere, Washington DC. 1992.

[22] E. Sparrow and R. Cess, *Radiation heat transfer*, Hemisphere, augmented edition, 1978.

[23] E. Zeidler, *Nonlinear Functional Analysis and its Applications I: Fixed Point Theorems*, Springer–Verlag, 1986. MR0816732(87f:47083). Zbl 0583.47050.

[24] E. Ziedler, *Nonlinear Functional Analysis and its Applications. II / B: Non–Linear Monotone Operators*, Springer–Verlag, 1990. MR1033498(91b:47002). Zbl 0684.47029.

Naji Qatanani   Amjad Barham
Department of Mathematics Al–Quds University,   Palestine Polytechnic University,
P.O. Box 20002 Abu–Dies, Jerusalem.   Hebron, Palestine.
e-mail: nqatanani@science.alquds.edu   e-mail: amjad@ppu.edu

Qasem Heeh
Department of Mathematics Al–Quds University,
P.O. Box 20002 Abu–Dies, Jerusalem.