Algebraic combinatorics in Vienna

The researchers in Vienna are *Christian Krattenthaler, Markus Fulmek*, and *Maria Prohaska*. The investigations mainly concentrated on the combinatorics of tableaux and nonintersecting lattice paths and can be roughly divided into four topics: (I) Counting nonintersecting lattice paths with a given total number of turns and the computation of Hilbert series for determinantal and pfaffian rings, (II) Classical group characters, (III) Tableau algorithms and bijections, (IV) Multiple basic hypergeometric series. Finally, in another direction, research was undertaken in collaboration with the Strasbourg group on (V) Permutation statistics.

ad **(I) Counting nonintersecting lattice paths with a given total number of turns and the computation of Hilbert series for determinantal and pfaffian rings**. As shown by [R1, R2, R4, R5, R8], the computation of Hilbert series for determinantal and pfaffian rings boils down to counting families of nonintersecting lattice paths with given starting and end points and a given total number of turns in certain regions. Here the advances have been significant. All the enumeration problems that come from determinantal rings, rings generated by minors of symmetric matrices, one-sided ladder determinantal rings, and pfaffian rings could be solved by developing a combinatorial theory for "non-crossing two-rowed arrays". See the accompanying article [P8] for more details. The forth-coming papers [P9, P11] will contain a full account of these results.

ad **(II) Classical group characters**. Here the main interest was on the combinatorics of formulas for classical group characters. In [P2] the Gessel–Viennot methodology of nonintersecting lattice paths was extended to provide combinatorial proofs of all the Jacobi–Trudi type determinant formulas for (irreducible) symplectic and orthogonal characters, and the symplectic and odd orthogonal Giambelli identities. Only the even Giambelli identity could not be given a combinatorial proof. As a by-product, in [P3] for the first time Proctor's [R11] and Sundaram's [R12] odd orthogonal tableaux, which are two very different combinatorial descriptions of odd orthogonal characters, could be related to each other by a bijection. Besides, classical combinatorial methods and tableau descriptions coming from algebraic geometry [R9] were successfully combined in [P10] to obtain a number of new identities for classical group characters.

ad **(III) Tableau algorithms and bijections**. Here, a new algorithm, a "modified jeu de taquin", was found and applied to provide beautiful bijective proofs for the hook-content formula for Schur functions [P6] and similar formulas for super Schur functions [P7].

ad **(IV) Multiple basic hypergeometric series**. Inspired by Burge [R3], in [P4] we introduce new objects into the theory of partitions, "cylindric partitions". Again, there is a close relationship with nonintersecting lattice paths of a particular type. We extend almost all of Burge's results (which, in our language, concern two-rowed cylindric partitions) to cylindric partitions with an arbitrary number of rows. It is then shown that cylindric partitions can be used to derive multiple basic hypergeometric series associated with root systems. In particular, $\tilde{A}_r$ basic hypergeometric summations of Milne [R10] could be given new and elementary proofs. Besides, several new $\tilde{A}_r$ basic hypergeometric summation and transformation formulas were found. Finally, in [P5] new A_r extensions of Heine's ${}_2\phi_1$-transformations [R7, (1.4.1), (1.4.5), (1.4.6)] were found.

ad **(V) Permutation statistics.** In [P1] generalizations of the classical statistics "maj" and "inv" (the major index and the number of inversions) on words are introduced that depend on a graph on the underlying alphabet and the behaviour of each letter at the end of a word. The question of characterizing those graphs that lead to equidistributed "maj" and "inv" is then posed and answered. This work extends a previous result of Foata and Zeilberger [R6] who considered the same problem under the assumption that all letters have the same behaviour at the end of a word.

Papers

P1. D. Foata and C. Krattenthaler, *Graphical major indices II*, in preparation.

P2. M. Fulmek and C. Krattenthaler, *Lattice path proofs for determinant formulas for symplectic and orthogonal characters*, preprint.

P3. M. Fulmek and C. Krattenthaler, *Bijections between odd orthogonal tableaux*, preprint.

P4. I. M. Gessel and C. Krattenthaler, *Cylindric partitions*, preprint.

P5. R. A. Gustafson and C. Krattenthaler, *Heine transformations for a new kind of basic hypergeometric series in $U(n)$*, preprint.

P6. C. Krattenthaler, *An involution principle-free bijective proof of Stanley's hook-content formula*, preprint.

P7. C. Krattenthaler, *A bijective proof of the hook-content formula for super Schur functions and a modified jeu de taquin*, preprint.

P8. C. Krattenthaler, *Counting nonintersecting lattice paths with turns*, Seminaire Lotharingien, item "Back Issues" **34** (1995), ibid.

P9. C. Krattenthaler, *Non-crossing two-rowed arrays*, in preparation.

P10. C. Krattenthaler, *Classical group characters of rectangular and nearly rectangular shape*, in preparation.

P11. C. Krattenthaler and M. Prohaska, *A remarkable formula for counting nonintersecting lattice paths in a ladder with respect to turns*, in preparation.

References

R1. S. S. Abhyankar, *Enumerative combinatorics of Young tableaux*, Marcel Dekker, New York, Basel, 1988.

R2. S. S. Abhyankar and D. M. Kulkarni, *On Hilbertian ideals*, Linear Alg. Appl. **116** (1989), 53–76.

R3. W. H. Burge, *Restricted partition pairs*, J. Combin. Theory Ser. A **63** (1993), 210–222.

R4. A. Conca, *Symmetric ladders*, Nagoya Math. J. **136** (1994), 35–56.

R5. A. Conca and J. Herzog, *On the Hilbert function of determinantal rings and their canonical module*, Proc. Amer. Math. Soc. **122** (1994), 677–681.

R6. D. Foata and D. Zeilberger, *Graphical major indices*, preprint.

R7. G. Gasper and M. Rahman, *Basic hypergeometric series*, Encyclopedia of Mathematics And Its Applications 35, Cambridge University Press, Cambridge, 1990.

R8. J. Herzog and N. V. Trung, *Gröbner bases and multiplicity of determinantal and Pfaffian ideals*, Adv. in Math. **96** (1992), 1–37.

R9. P. Littelmann, *A generalization of the Littlewood–Richardson rule*, J. Algebra **130** (1990), 328–368.

R10. S. C. Milne, *Classical partition functions and the $U(n+1)$ Rogers–Selberg identity*, Discrete Math. **99** (1992), 199—246.

R11. R. A. Proctor, *Young tableaux, Gelfand patterns, and branching rules for classical groups*, J. Algebra **164** (1994), 299–360.

R12. S. Sundaram, *Orthogonal tableaux and an insertion algorithm for $SO(2n+1)$*, J. Combin. Theory Ser. A **53** (1990), 239–256.