COMMENT ON 'COUNTING NONINTERSECTING LATTICE PATHS WITH TURNS' BY C. KRATTENTHALER

Martin Rubey

Institut für Mathematik der Universität Wien,
Strudlhofgasse 4, A-1090 Wien, Austria.
e-mail: a9104910@unet.univie.ac.at

The argument in the proof of Theorem 4 on pp. 11/12 that a family of two-rowed arrays with associated permutation not the identity permutation must contain a crossing point contains an error: the inequality $A_1^{(\sigma(i+1))} - 1 \le A_1^{(\sigma(i))}$ on page 12 is not true in general.

The paragraph at the bottom of page 11 to the top of page 12 must be replaced by:

"Finally we have to show that, given a pair $(\mathcal{P}, \sigma)$, $\mathcal{P} = (P_1, \dots, P_r)$, $\sigma \ne \mathrm{id}$, there exist neighbouring two-rowed arrays having a crossing point.

If $\sigma \ne \mathrm{id}$ then there must exist an i with $\sigma(i+1) < i+1$. Without loss of generality we may assume that i is minimal with this property. Then all two-rowed arrays P_j with $j \le i$ have nonnegative type, whereas P_{i+1} has negative type.

Let us momentarily define $a_0^{(i+1)} := A_1^{(\sigma(i+1))} - 1$. (Recall that we denoted the entries of P_j by $a_\ell^{(j)}$ and $b_\ell^{(j)}$, respectively, see (22).) Furthermore set $s_{i+1} = 0$. We show next that, either we find two neighbouring arrays containing a crossing point, or for any $j \in \{\sigma(i+1), \dots, i\}$ there is an index $s_j \in \{1, 2, \dots, k_j\}$ such that

$$a_{s_j}^{(j)} \le a_{s_{j+1}}^{(j+1)} \quad \text{and} \quad b_{s_j}^{(j)} \ge b_{s_{j+1}}^{(j+1)}. \tag{C1}$$

We do a reverse induction on j. Suppose we have already found indices $s_i, s_{i-1}, \dots, s_{j+1}$ such that (C1) is satisfied. Since P_j has nonnegative type, the element $a_1^{(j)}$ exists. If $a_{s_{j+1}}^{(j+1)} < a_1^{(j)}$, then because of (13) and (C1) we have also

$$b_0^{(j)} = A_2^{(\sigma(j))} \le A_2^{(j)} \le A_2^{(\sigma(i+1))} < b_0^{(i+1)} \le b_{s_{j+1}}^{(j+1)},$$

and thus $(a_1^{(j)}, b_{s_{j+1}}^{(j+1)})$ is a crossing point of P_j and P_{j+1}.

On the other hand, if $a_{s_{j+1}}^{(j+1)} \ge a_1^{(j)}$ then let s_j be maximal such that $a_{s_j}^{(j)} \le a_{s_{j+1}}^{(j+1)}$. We already know that $s_j \ge 1$. Because of (C1) we also have

$$E_1^{(j)} \ge E_1^{(\sigma(i+1))} \ge A_1^{(\sigma(i+1))} - 1 = a_0^{(i+1)} \ge a_{s_{j+1}}^{(j+1)},$$

and thus $s_j \le k_j$.

Since P_j has nonnegative type, the element $b^{(j)}_{s_j}$ exists. If $b^{(j)}_{s_j} < b^{(j+1)}_{s_{j+1}}$, then by construction of s_j we have $a^{(j+1)}_{s_{j+1}} < a^{(j)}_{s_j+1}$, and thus $(a^{(j)}_{s_j+1}, b^{(j+1)}_{s_{j+1}})$ is a crossing point of P_j and P_{j+1}. If, on the other hand, $b^{(j)}_{s_j} \ge b^{(j+1)}_{s_{j+1}}$, then we have found s_j such that (C1) holds.

For $j = \sigma(i+1)$ though, this leads to a contradiction. For, the type of P_j is $\sigma(j) - j \ge 0$, and therefore

$$A^{(\sigma(j))}_1 \le a^{(j)}_1 + j - \sigma(j) \le a^{(j)}_{s_j} + j - \sigma(j) \le a^{(i+1)}_0 + j - \sigma(j) = A^{(j)}_1 - 1 + j - \sigma(j),$$

where we used (C1) for the last inequality. But on the other hand, by (13) we have $A^{(\sigma(j))}_1 \ge A^{(j)}_1 + j - \sigma(j)$, a contradiction."

Institut für Mathematik der Universität Wien, Strudlhofgasse 4, A-1090 Wien, Austria.