S©MR ISSN 1813-3304

СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ

Siberian Electronic Mathematical Reports

http://semr.math.nsc.ru

Том 2, стр. 62–67 (2005) УДК 515.126, 515.128

MSC 54E40, 54D50, 54D65

$\aleph_0$-SPACES AND IMAGES OF SEPARABLE METRIC SPACES

YING GE

ABSTRACT. A space X is an $\aleph_0$-space if and only if X is a sequence-covering and compact-covering image of a separable metric space. It follows that a space X is a k-and-$\aleph_0$-space if and only if X is a sequence-covering and compact-covering, quotient image of a separable metric space.

An investigation of relations between spaces with countable networks and images of separable metric spaces is one of interesting questions on generalized metric spaces. In the past, Michael proved in the past that the space is an $\aleph_0$-space if and only if it is a compact-covering image of a separable metric space [13]. Note that sequence-covering mappings introduced by Siwiec in [14] and sequentially quotient mappings introduced by Boone and Siwiec in [1] were powerful tools to characterize spaces with countable networks. Recently Lin and Yan in [10] proved that a space is an sn-second countable space if and only if it is a sequentially quotient, compact image of a separable metric space, and they also raised the following question.

Question 1. ([10, Question 4.10]. *Is a Fréchet $\aleph_0$-space a closed and sequence-covering image of a separable metric space?*

However, Yan, Lin and Jiang proved that the metrizability is preserved by closed and sequence-covering mappings [18]. Note that S_ω is a *Fréchet* $\aleph_0$-space, which is not even first countable (see [6, Example 1.8.7], for example). So the answer to Question 1 is negative. More precisely, Lin proved that a space is a *Fréchet*, $\aleph_0$-space if and only if it is a closed image of a separable metric space [8]. Considering Question 1, it is natural to pose

YING GE, $\aleph_0$-SPACES AND IMAGES OF SEPARABLE METRIC SPACES.

This project was supported by NSF of the Education Committee of Jiangsu Province in China(No.02KJB110001).

Received March 9, 2005, published May, 24, 2005.

Question 2. *Is an $\aleph_0$-space a sequence-covering and compact-covering image of a separable metric space?*

On the other hand, a space X is a quotient (resp. quotient compact) image of a separable metric space if and only if it is a k-and-$\aleph_0$-space [13] (resp. g-second countable [11]). In regard to these results, Tanaka [17] pointed out that we can add "compact-covering"before "quotient"but it is impossible to add "sequence-covering"before "quotient"for the parenthetic part. However, we do not know whether "sequence-covering"can be added before "quotient"for the nonparenthetic part. That is, we have the following

Question 3. *Is a k-and-$\aleph_0$-space a sequence-covering and compact-covering quotient image of a separable metric space?*

In this paper, we prove that a space X is an $\aleph_0$-space if and only if X is a sequence-covering and compact-covering image of a separable metric space, which give an affirmative answer to Question 2. As a corollary of this result, a space X is a k-and-$\aleph_0$-space if and only if X is a sequence-covering and compact-covering, quotient image of a separable metric space, which answers Question 3 affirmatively. Throughout this paper, all spaces are assumed regular T_1, and all mappings are continuous and onto. N and ω denote the set of all natural numbers and the first infinite ordinal respectively. (β_n) denotes a point of a Tychonoff-product space, the n-th coordinate is β_n. The sequence $\{x_n : n \in \mathrm{N}\}$, the sequence $\{F_n : n \in \mathrm{N}\}$ of subsets and the sequence $\{\mathcal{F}_n : n \in \mathrm{N}\}$ of collections of subsets are abbreviated to $\{x_n\}$, $\{F_n\}$ and $\{\mathcal{F}_n\}$ respectively. Let $\mathcal{F}$ be a collection of subsets of X and let f be a mapping. Then $\bigcup\mathcal{F}$ and $f(\mathcal{F})$ denote $\bigcup\{F : F \in \mathcal{F}\}$ and $\{f(F) : F \in \mathcal{F}\}$ respectively. We say that a sequence $\{x_n\}$ converging to x is eventually in F, if there exists $k \in \mathrm{N}$ such that $\{x_n : n > k\} \bigcup \{x\} \subset F$; is frequently in F if $x \in F$ and for any $k \in \mathrm{N}$ there exists $n > k$ such that $x_n \in F$. For the terms not defined here, please refer to [4].

Definition 4. *Let $f : X \longrightarrow Y$ be a mapping.*

(1) f is a sequence-covering mapping [14] *if every convergent sequence of Y is the image of some convergent sequence of X.*

(2) f is a compact-covering mapping [13] *if every compact subset of Y is the image of some compact subset of X.*

(3) f is a pseudo-sequence-covering mapping [2] *if every convergent sequence of Y is the image of some compact subset of X.*

(4) f is a sequentially-quotient mapping [1] *if for every convergent sequence L of Y, there is a convergent sequence S of X such that $f(S)$ is a subsequence of L.*

(5) f is a subsequence-covering mapping [9] *if for every convergent sequence L of Y, there is a compact subset K of X such that $f(K)$ is a subsequence of L.*

(6) f is a quotient mapping [13] *if whenever $f^{-1}(U)$ is open in X, then U is open in Y.*

Remark 5. *We have the following implications from Definition 4.*

(1) Sequence-covering mapping$\Longrightarrow$pseudo-sequence-covering mapping (sequentially-quotient mapping)$\Longrightarrow$subsequence-covering mapping.

(2) Compact-covering mapping$\Longrightarrow$pseudo-sequence-covering mapping.

Definition 6. *Let $\mathcal{F}$ be a cover of a space X.*

(1) $\mathcal{F}$ is a pseudobase of X [13] *if whenever K is compact subset of X and U is an open neighborhood of K, there exists $F \in \mathcal{F}$ such that $K \subset F \subset U$.*

(2) $\mathcal{F}$ is a k-network of X [12] *if whenever K is compact subset of X and U is an open neighborhood of K, there exists a finite $\mathcal{F}' \subset \mathcal{F}$ such that $K \subset \bigcup \mathcal{F}' \subset U$.*

(3) $\mathcal{F}$ is a cs-network of X [5] *if every convergent sequence S converging to a point $x \in U$ with U open in X, then S is eventually in $F \subset U$ for some $F \in \mathcal{F}$.*

(4) $\mathcal{F}$ is a cs-network of X* [3] *if every convergent sequence S converging to a point $x \in U$ with U open in X, then S is frequently in $F \subset U$ for some $F \in \mathcal{F}$.*

Definition 7. *A space X is an $\aleph_0$-space* [13] *if X has a countable pseudobase.*

Remark 8. *We have known that a space X is an $\aleph_0$-space if and only if it has a countable k-network (cs-network, cs*-network) (see* [16, Proposition C]*, for example).*

Now we give the main theorem.

Lemma 9. *(*[6, Lemma3.7.4]*). Let $\mathcal{F}$ be a countable cs*-network of a space X. Then $\mathcal{F}$ is a k-network of X.*

Lemma 10. *Let $f : X \longrightarrow Y$ be a subsequence-covering mapping, while the points in X are G_δ. Then f is sequentially-quotient.*

Proof. Let S be a sequence converging to y in Y. Since f is subsequence-covering, there is a compact subset K in X such that $f(K) = S'$ is a subsequence of S. Put $S' = \{y\} \cup \{y_n : n \in \mathrm{N}\}$, where $\{y_n\}$ converges to y. Pick $x_n \in f^{-1}(y_n) \cap K$, Then $\{x_n\} \subset K$. Note that K is a compact subspace whose points are $G'_\delta s$. Thus K is first countable, hence sequentially compact, so there is a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ that converges to $x \in f^{-1}(y)$. This proves that f is sequentially-quotient. □

The following lemma belongs to Lin [6, Proposition 3.7.14(2)].

Lemma 11. *Let $f : X \longrightarrow Y$ be a sequentially-quotient mapping, and X be an $\aleph_0$-space. Then Y is an $\aleph_0$-space.*

Proof. X is an $\aleph_0$-space, So X has a countable cs-network $\mathcal{B}$ by Remark 8. Put $\mathcal{F} = f(\mathcal{B})$. We only need to prove that $\mathcal{F}$ is a cs^*-network of Y from Remark 8.

Let S be a sequence in Y converging to a point $y \in U$ with U open in Y. Since f is sequentially-quotient, there is a sequence L in X converging to a point $x \in f^{-1}(y) \subset f^{-1}(U)$ such that $f(L)$ is a subsequence of S. Since $\mathcal{B}$ is a cs-network of X, there exists $B \in \mathcal{B}$ such that L is eventually in B and $B \subset f^{-1}(U)$. Thus $f(L)$ is eventually in $f(B) \subset U$, and so S is frequently in $f(B) \subset U$. Note that $f(B) \in \mathcal{F}$. So $\mathcal{F}$ is a cs^*-network of Y. □

Theorem 12. *For a space X, the following are equivalent.*

(1) X is an $\aleph_0$-space.

(2) X is a sequence-covering and compact-covering image of a separable metric space.

(3) X is a sequence-covering image of a separable metric space.

(4) X is a compact-covering image of a separable metric space.

(5) X is a pseudo-sequence-covering image of a separable metric space.

(6) X is a subsequence-covering compact image of a locally separable metric space.

(7) X is a sequentially-quotient image of a separable metric space.

Proof. (2)$\Longrightarrow$(3)$\Longrightarrow$(5)$\Longrightarrow$(6) and (2)$\Longrightarrow$(4)$\Longrightarrow$(5)$\Longrightarrow$(6) by Remark 5. (6)$\Longrightarrow$(7) by Lemma 10 and (7)$\Longrightarrow$(1) by Lemma 11. We prove only (1)$\Longrightarrow$(2).

Let X be an $\aleph_0$-space. Then X has a countable cs-network $\mathcal{F} = \{F_\beta : \beta \in \Lambda\}$, where Λ is a countable index set. Thus $\mathcal{F}$ is also a countable k-network for X from Lemma 9. By the regularity of X, we can assume every element of $\mathcal{F}$ is closed in X. Let Λ_n be a topological space Λ endowed with the discrete topology for every $n \in \mathrm{N}$. Put

$$M = \{b = (\beta_n) \in \Pi_{n\in \mathrm{N}}\Lambda_n : \{F_{\beta_n}\} \textit{ is a network at some point } x_b \textit{ in } X\}.$$

Then M, which is a subspace of Tychonoff product space $\Pi_{n\in\mathrm{N}}\Lambda_n$, is a metric space. Note that every Λ_n is a countable and discrete space. Thus $\Pi_{n\in\mathrm{N}}\Lambda_n$ is separable and so M is a separable metric space. For every $b = (\beta_n) \in M$, $\{F_{\beta_n}\}$ is a network at some point x_b in X. Put $f(b) = x_b$. It is easy to check that $f : M \longrightarrow X$ defined as above is a mapping. It suffices to prove the following two claims.

Claim 1. f is sequence-covering.

For every convergent sequence L of X and every open subset V of X containing L, we call a subfamily $\mathcal{F}'$ of $\mathcal{F}$ to have property $\mathcal{F}(L, V)$ if $\mathcal{F}'$ satisfies the following conditions (1.1)-(1.4).

(1.1) $\mathcal{F}'$ is finite.

(1.2) $\emptyset \neq F \bigcap L \subset F \subset V$ for every $F \in \mathcal{F}'$.

(1.3) For every $z \in L$, there exists a unique $F_z \in \mathcal{F}'$ such that $z \in F_z$.

(1.4) If x is the limit point of L and $x \in F \in \mathcal{F}'$, then $L - F$ is finite.

Obviously, such subfamilies of $\mathcal{F}'$ having property $\mathcal{F}(L, V)$ exist. In fact, we write $L = \{y_m : m \in \omega\}$ be a sequence in X converging to y_0. Since $\mathcal{F}$ is a cs-network of X, there exists $F_0 \in \mathcal{F}$ such that L is eventually in F_0 and $F_0 \subset V$. Since $L - F_0$ is finite, put $L - F_0 = \{y_{n_i} : i = 1, \cdots, k\}$. For every $i \in \{1, \cdots, k\}$, note that $V - ((L \bigcap F_0) \bigcup \{y_{n_j} : j = 1, \cdots, k \textit{ and } j \neq i\})$ is an open neighborhood of y_{n_i} , so there exists $F_i \in \mathcal{F}$ such that $y_{n_i} \in F_i \subset V - ((L \bigcap F_0) \bigcup \{y_{n_j} : j = 1, \cdots, k \textit{ and } j \neq i\})$. Put $\mathcal{F}' = \{F_i : i = 0, 1, \cdots, k\}$, then $\mathcal{F}'$ have property $\mathcal{F}(L, V)$.

Let $S = \{x_m : m \in \omega\}$ be a sequence in X converging to x_0. We can assume $x_n \neq x_m$ if $n \neq m$. $\mathcal{F}$ is countable, so $\{\mathcal{F}' \subset \mathcal{F} : \mathcal{F}' \textit{ have property } \mathcal{F}(S, X)\}$ is countable by (1.1). Put

$$\{\mathcal{F}' \subset \mathcal{F} : \mathcal{F}' \textit{ have property } \mathcal{F}(S, X)\} = \{\mathcal{F}_n : n \in \mathrm{N}\}.$$

and put $\mathcal{F}_n = \{F_\beta : \beta \in \Delta_n\}$ for every $n \in \mathrm{N}$, where Δ_n is a finite subset of Λ_n. For every $n \in \mathrm{N}$ and every $m \in \omega$, there exists unique $\beta_{nm} \in \Delta_n$, such that $x_m \in F_{\beta_{nm}} \in \mathcal{F}_n$. Put $b_m = (\beta_{nm}) \in \Pi_{n\in\mathrm{N}}\Delta_n$, then

(1.5) $b_m \in M$ and $f(b_m) = x_m$ for every $m \in \omega$.

It suffices to prove that $\{F_{\beta_{nm}} : n \in \mathrm{N}\}$ is a network at x_m in X. Let U be an open neighborhood of x_m. If $x_m = x_0$, put $L = S \bigcap U$, then there exists a subfamily $\mathcal{F}'$ of $\mathcal{F}$ such that $\mathcal{F}'$ has property $\mathcal{F}(L, U)$. Since $S - L$ is finite, put $S - L = \{x_{n_i} : i = 1, \cdots, k\}$. For every $i \in \{1, \cdots, k\}$, note that $X - (L \bigcup \{x_{n_j} : j = 1, \cdots, k \textit{ and } j \neq i\})$ is an open neighborhood of x_{n_i} , so there exists $F_i \in \mathcal{F}$ such that $x_{n_i} \in F_i \subset X - (L \bigcup \{x_{n_j} : j = 1, \cdots, k \textit{ and } j \neq i\})$. Put $\mathcal{F}'' = \mathcal{F}' \bigcup \{F_i : i = 0, 1, \cdots, k\}$, then $\mathcal{F}''$ has property $\mathcal{F}(S, X)$. So there exists $k \in \mathrm{N}$ such that $\mathcal{F}'' = \mathcal{F}_k$. Thus $x_0 \in F_{\beta_{k0}} \in \mathcal{F}'$, hence $x_0 \in F_{\beta_{k0}} \subset U$ by (1.2). If $x_m \neq x_0$, there exist a subfamily $\mathcal{F}'$ of $\mathcal{F}$ such that $\mathcal{F}'$ has property $\mathcal{F}(S - \{x_m\}, X - \{x_m\})$ (note: here $\{x_m\}$ is a one-point set, not a sequence) and $F \in \mathcal{F}$ such that $x_m \in F \in$

$U-(S-\{x_m\})$. Thus $\{F\}\bigcup\mathcal{F}'$ has property $\mathcal{F}(S,X)$; hence, there exists $k\in$ N such that $\{F\}\bigcup\mathcal{F}'=\mathcal{F}_k$, so $x_m\in F_{\beta_{km}}=F\subset U$. Thus $\{F_{\beta_{nm}}:n\in \mathrm{N}\}$ is a network at x_m in X for every $m\in\omega$.

(1.6) $\lim_{n\to\infty}b_n=b_0$

It suffices to prove that $\lim_{n\to\infty}\beta_{kn}=\beta_{k0}$ for every $k\in$ N. For every $k\in$ N, $x_0\in F_{\beta_{k0}}\in\mathcal{F}_k$, where $\beta_{k0}\in\Delta_k$. Since $\mathcal{F}_k$ has property $\mathcal{F}(S,X)$, $S-F_{\beta_{k0}}$ is finite by (1.4), so there exists $n_k\in$ N such that $x_n\in F_{\beta_{k0}}$ for every $n>n_k$. Note that $x_n\in F_{\beta_{kn}}\in\mathcal{F}_k$. Thus $\beta_{kn}=\beta_{k0}$ for every $n>n_k$ by uniqueness in (1.3). So $\lim_{n\to\infty}\beta_{kn}=\beta_{k0}$.

By (1.5) and (1.6), we completes the proof of Claim 1.

Claim 2. f is compact-covering.

Let K is a compact subset of X. We call a subfamily $\mathcal{E}$ of $\mathcal{F}$ to have property $\mathcal{E}(K)$, if $\mathcal{E}$ satisfies the following conditions (2.1) and (2.2).

(2.1) $\mathcal{E}$ is finite.

(2.2) $K\subset\bigcup\mathcal{E}$.

$\mathcal{F}$ is countable, so $\{\mathcal{E}\subset\mathcal{F}:\mathcal{E}\ \mathit{have\ property}\ \mathcal{E}(K)\}$ is countable by (2.1). Put $\{\mathcal{E}\subset\mathcal{F}:\mathcal{E}\ \mathit{have\ property}\ \mathcal{E}(K)\}=\{\mathcal{E}_n:n\in \mathrm{N}\}$, and put $\mathcal{E}_n=\{F_\beta:\beta\in\Gamma_n\}$ for every $n\in$ N, where Γ_n is a finite subset of Λ_n.

Put $L=\{(\beta_n)\in\Pi_{n\in \mathrm{N}}\Gamma_n:\bigcap_{n\in \mathrm{N}}(K\bigcap F_{\beta_n})\neq\emptyset\}$. Then

(2.3) L is a compact subset of $\Pi_{n\in \mathrm{N}}\Lambda_n$.

Γ_n is finite for every $n\in$ N and so $\Pi_{n\in \mathrm{N}}\Gamma_n$ is compact. Note that $L\subset\Pi_{n\in \mathrm{N}}\Gamma_n$, we only need to prove that L is closed in $\Pi_{n\in \mathrm{N}}\Gamma_n$. Let $b=(\beta_n)\in\Pi_{n\in \mathrm{N}}\Gamma_n-L$. Then $\bigcap_{n\in \mathrm{N}}(K\bigcap F_{\beta_n})=\emptyset$. Note that $K\bigcap F_{\beta_n}$ is closed in K for every $n\in$ N and K is compact, there exists $n_0\in$ N such that $\bigcap_{n\leq n_0}(K\bigcap F_{\beta_n})=\emptyset$. Put $W=\{(\gamma_n)\in\Pi_{n\in \mathrm{N}}\Gamma_n:\gamma_n=\beta_n\ if\ n\leq n_0\}$. Then W is an open neighborhood of b in $\Pi_{n\in \mathrm{N}}\Gamma_n$ and $W\bigcap L=\emptyset$. If not, there exists $c=(\gamma_n)\in W\bigcap L$. Since $c\in L$, we have $\bigcap_{n\in \mathrm{N}}(K\bigcap F_{\gamma_n})\neq\emptyset$, hence $\bigcap_{n\leq n_0}(K\bigcap F_{\gamma_n})\neq\emptyset$. Since $c\in W$, $\bigcap_{n\leq n_0}(K\bigcap F_{\beta_n})=\bigcap_{n\leq n_0}(K\bigcap F_{\gamma_n})\neq\emptyset$. This contradicts that $\bigcap_{n\leq n_0}(K\bigcap F_{\beta_n})=\emptyset$.

(2.4) $L\subset M$ and $f(L)=K$.

Let $b=(\beta_n)\in L$, then $\bigcap_{n\in \mathrm{N}}(K\bigcap F_{\beta_n})\neq\emptyset$. Pick $x\in\bigcap_{n\in \mathrm{N}}(K\bigcap F_{\beta_n})$. If $\{F_{\beta_n}\}$ is a network at x in X, then $b\in M$ and $f(b)=x$, hence $L\subset M$ and $f(L)\subset K$. So we only need to prove that $\{F_{\beta_n}\}$ is a network at x in X. Let V is an open neighborhood of x. There exists a subset W of K such that $x\in W$ with W is open in K and a compact subset $cl_K(W)\subset V$, where $cl_K(W)$ is the closure of W in K. Thus there exists a finite $\mathcal{F}'\subset\mathcal{F}$ such that $cl_K(W)\subset\bigcup\mathcal{F}'\subset V$. Note that compact subset $K-W\subset X-\{x\}$, there exists a finite $\mathcal{F}''\subset\mathcal{F}$ such that $K-W\subset\bigcup\mathcal{F}''\subset X-\{x\}$. Put $\mathcal{F}^*=\mathcal{F}'\bigcup\mathcal{F}''$, then $\mathcal{F}^*$ has property $\mathcal{E}(K)$. So there exists $k\in$ N such that $\mathcal{E}_k=\mathcal{F}^*$. Since $x\in F_{\beta_k}\in\mathcal{E}_k$, $F_{\beta_k}\in\mathcal{F}'$, thus $F_{\beta_k}\subset V$. This proves that $\{F_{\beta_n}\}$ is a network at x in X.

(2.5) $K\subset f(L)$.

Let $x\in K$. For every $n\in$ N, there exists $\beta_n\in\Gamma_n$ such that $x\in F_{\beta_n}$. Put $b=(\beta_n)$, then $b\in L$. Furthermore, $f(b)=x$ by the proof of (2.4). So $K\subset f(L)$.

By (2.3), (2.4) and (2.5), we completes the proof of Claim 2. $\square$

Now we answer Question 3. Recall a space X is a k-space if $F\subset X$ is closed in X iff $F\bigcap K$ is closed in K for every compact subset K of X.

Lemma 13. *([7, Lemma1.4.2]). Let $f:X\longrightarrow Y$ be a mapping and let Y be a k-space. If f is compact-covering, then f is quotient.*

Corollary 14. *For a space X, the following are equivalent.*

(1) X is a k-and-$\aleph_0$-space.

(2) X is a sequence-covering and compact-covering quotient image of a separable metric space.

Proof. We only need to prove that (1)$\Longrightarrow$(2).

Let X be a k-and-$\aleph_0$-space. X is an $\aleph_0$-space, so there exists a sequence-covering and compact-covering mapping $f : M \longrightarrow X$ from Theorem 12, where M is a separable metric space. Note that X is a k-space. So f is also quotient from Lemma 13. Thus X is a sequence-covering and compact-covering quotient image of a separable metric space. □

References

[1] J.R. Boone, F. Siwiec, *Sequentially quotient mappings*, Czech. Math. J., **26**(1976), 174-182.

[2] Y. Ikeda, C. Liu, Y. Tanaka, *Quotient compact images of metric spaces*, and related matters, Topology Appl., **122** (2002), 237-252.

[3] Z. Gao, *$\aleph$-space is invariant under perfect mappings*, Questions Answers in Gen. Topology, **5** (1987), 271-279.

[4] G. Gruenhage, *Generalized metric spaces*, In: K.Kunen and J.E.Vaughan eds., Handbook of Set-Theoretic Topology, Amsterdan: North-Holland, 423-501, 1984.

[5] J.A. Guthrie, *A characterization of $\aleph_0$-spaces*, Gen. Top. Appl., **1** (1971), 105-110.

[6] S. Lin, *Generalized metric spaces and mappings*, Chinese Science Press, Beijing, 1995. (Chinese)

[7] S. Lin, *Point-countable covers and sequence-covering mappings*, Chinese Science Press. Beijing, 2002. (Chinese)

[8] S. Lin, *metric spaces and topologies on function spaces*, Chinese Science Press. Beijing, 2004. (Chinese)

[9] S. Lin, C. Liu, M. Dai, *Images on locally separable metric spaces*, Acta. Math. Sinica, **13** (1997), 1-8.

[10] S. Lin, P. Yan, *Sequence-Covering Maps of Metric Spaces*, Top. Appl., **109** (2001), 301-314.

[11] C. Liu, M. Dai, *Spaces with a locally countable weak base*, Math. Japonica, **41** (1995), 261-267.

[12] P. O'Meara, *On paracompactness in function spaces with the compact-open topology*, Proc. Amer. Math. Soc., **29** (1971), 183-189.

[13] E. Michael, *$\aleph_0$-spaces*, J. Math. Mech., **15** (1966), 983-1002.

[14] F. Siwiec, *Sequence-covering and countably bi-quotient mappings*, General Topology Appl., **1** (1971), 143-154.

[15] Y. Tanaka, *σ-hereditarily closure-preserving k-networks and g-metrizability*, Proc. Amer. Math. Soc., **112** (1991), 283-290.

[16] Y. Tanaka, *Theory of k-networks*, Questions Answers in Gen. Topology, **12** (1994), 139-164.

[17] Y. Tanaka, *Theory of k-networks II*, Questions Answers in Gen. Topology, **19** (2001), 27-46.

[18] P. Yan, S. Lin, S. Jiag, *Metrizability is preserved by closed and sequence-covering maps*, In: 2000 Summer Conference on Topology and its Appl, June 26∼29, Maimi University, Oxford, Ohio, USA.

Ying Ge
Department of Mathematics,
Suzhou University,
Suzhou, 215006, P.R.China
E-mail address: geying@pub.sz.jsinfo.net