S©MR ISSN 1813-3304

СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ
Siberian Electronic Mathematical Reports
http://semr.math.nsc.ru

*Том 2, стр. 192–193 (2005)* УДК 515.16
*Краткие сообщения* MSC 57M25

# AN ALGORITHM OF FINDING PLANAR SURFACES IN THREE-MANIFOLDS

E. SBRODOVA

This paper is devoted to the question: does there exist an algorithm to decide whether or not a given 3-manifold contains a proper essential *planar surface*? By a *planar surface* we mean a punctured disc.

There is an algorithm, due to W. Jaco, to decide whether a 3-manifold admits a proper essential disc, i.e., whether it is boundary reducible. A close result, an algorithm allow us to say whether a manifold contains a proper essential disc with a given boundary, was obtained by W. Haken in 60-th. In 1998 W. Jaco, H. Rubinstein and E. Sedgwick described an algorithm to decide whether or not a given link-manifold contains a proper essential planar surface (a link-manifold is a compact orientable 3-manifold whose boundary consists of tori) [1]. We generalize this result to manifolds with arbitrary boundaries.

A *slope* on the boundary of a 3-manifold $M$ is the isotopy class of a finite set of disjoint simple closed curves $\{\alpha_1, \dots, \alpha_n\}$ in $\partial M$ which are nontrivial and pairwise nonparallel. We say that the boundary of a proper surface $F$ has a slope $\alpha = \{\alpha_1, \dots, \alpha_n\}$ if the boundary components of $F$ are each parallel to one of the curves $\alpha_1, \dots, \alpha_n$.

**Theorem 1.** *There is an algorithm to decide if a compact orientable irreducible and boundary irreducible 3-manifold $M$ contains an essential planar surface $F \subset M$ such that $\partial F$ has a given slope $\alpha$.*

The proof of this theorem is based on the Haken theory of normal surfaces in a 3-manifold with a boundary pattern, see [2].

---

Sbrodova, E., An algorithm of finding planar surfaces in three-manifolds.

The work is supported by INTAS (grant no. 03-51-3663) and Russian Foundation for Basic Research (04-01-96014).
*Communicated by S.V. Matveev October 15, 2005, published October 17, 2005.*

The algorithm in Theorem 1 takes as the input a given slope on the boundary of a manifold. At the same time we are interested in another question: does M contain some planar surface, with arbitrary boundary?

**Theorem 2.** *There is an algorithm to decide if a compact orientable irreducible and boundary irreducible 3-manifold $M$ contains an essential planar surface $F \subset M$.*

The proof of this theorem consists of two steps:

(1) Construct a set $A$ of slopes in $\partial M$ such that if a given manifold $M$ contains a planar slope (one belonging to a planar surface) then also one of the slopes in $A$ must be planar;

(2) For each slope $\alpha \in A$ check if $M$ contains an essential planar surface whose boundary has a slope $\alpha$.

The algorithm in Theorem 1 allows us to do the checking in the last item. For the first item define the notion of the *length* of a curve $\gamma$ (denoted by $l(\gamma)$) as the number of intersection points of $\gamma$ with the edges of the triangulation.

**Lemma.** *(W. Jaco, H. Rubinstein and E. Sedgwick). Let $M$ be an anannular irreducible and boundary irreducible 3-manifold with a fixed triangulation $T$. Then there exists a constant $C = C(M, T)$ such that for any proper essential planar surface $F \subset M$ the length of the shortest boundary curve of $\partial F$ is at most $C$.*

Using this lemma, we can find at least one curve for each slope in $A$.

A certain particular triangulation of manifold allows us to find length estimates for the other curves of planar slopes in $A$.

## References

[1] Jaco, W. and Sedgwick, E., *Decision problems in the space of Dehn fillings*, arXiv:math.GT/9811031.

[2] Matveev, S., *Algoritmic Topology and Classification of 3-Manifolds*, Springer-Verlag, Berlin, Heidelberg, 2003.

Elena Sbrodova
Chelyabinsk State University,
ul. Brat'ev Kashirinykh 129,
454021, Chelyabinsk, Russia
*E-mail address*: sbrodova@csu.ru