Binary Functions for Theory Change

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The problem of change in corpora of information is indeed interesting. Legislation is under constant modification, new discoveries reshape scientific theories and robots have to update their representation of the world each time a sensor gains new data. The theory of theory change offers a model for these processes under certain idealizations.

A formal language and a logical consequence operation are assumed. Corpora of information are represented as sets of sentences closed under logical consequence, that is, theories. New information is expressed as sentences in the logical language. The notion of change is formalized as functions that take a theory and a sentence to an updated theory. There is a leading principle for these functions: consistency. The result of a change by a consistent sentence should always be a consistent theory.

In 1985 Alchourrón, Gärdenfors and Makinson (henceforth AGM) published the article that became the classical reference in the literature on theory change. They conceived change functions that, under the maxim of consistency, preserve as much as possible of the original theory while accounting for the new information; theories should not be changed beyond necessity. Subset inclusion among theories alone was not enough as a criterion of minimal information loss because, in general, infinitely many theories are incomparable with each other with respect to set inclusion. Hence, it may be impossible to select a single one as the most preservative. As a result AGM functions must commit to a nondeterministic choice or else encode some other criteria for selection. The work of Alchourrón, Gärdenfors and Makinson created a whole new area of research, also referred to as *belief revision*.

At least in two respects the AGM theory is underdefined. One is *the problem of iterated change*. AGM functions model single changes, they take a theory to an updated theory, they perform one single step. But there will be another change after the one just considered that will induce yet another theory. That is, we will have to update the already updated theory. Although the AGM formalism does not forbid the iteration of change functions, it omits any specification of how it should be performed or what the properties of successive change are.

A main concern among researchers studying iterated change is whether there is a unique general model, a single set of properties in the same spirit as the AGM postulates for single changes. Fourteen years after the inception of the AGM theory we find several alternative formalizations differing in their virtues and defects, but whether there could exist such a uniform set of properties of iterated change remains still unknown; perhaps there is no unique regularity to expose.

The other problem not addressed by the AGM theory is *the problem of change in distinct theories*. Although AGM functions provide coherent change operations for single theories separately, these change operations are not necessarily jointly coherent. The change of one theory may be unrelated to the change of another. We have regarded this as a serious limitation of the AGM model and the work in this thesis has been devoted to overcome this limitation.

After showing that AGM functions behave as unary functions (one argument) relative to the theory to be changed the thesis proposes authentic *binary change functions*, that is, functions of

arity two that are defined for every theory and every formula. The thesis argues that they solve the problem of change in distinct theories, and, to some extent, the problem of iterated change. Clearly, binary functions can account for successive change because a theory returned by one application of a function is yet a possible argument of the same function. Consequently binary functions induce a deterministic scheme of iterated change with respect to the arguments of the function, producing a behaviour that has been interpreted as lack of historic memory, which is not always desirable in a model of iterated change.

Inside the AGM framework there exist two examples of binary functions, expansions and full meet functions, but they correspond to limiting cases of acceptable change functions. Outside the AGM tradition, Katsuno y Mendelzon have formalized their *update* operation as a binary function, denoting a quite different kind of change. We show that Katsuno and Mendelzon's axiomatization is incomplete to characterize the update function for infinite propositional languages; that is Katsuno and Mendelzon's original work is just defined for the finite case. We extend their definition stating an appropriate set of postulates, strengthening theirs, and we prove the corresponding representation theorem for possibly infinite propositional languages. In addition, the thesis puts the AGM revision and update operations on an even definitional basis allowing for a better comparison or understanding, when the nature of their difference is still an open question in the philosophical logic literature.

The main result of this thesis is the definition of two new binary functions extending the AGM framework, *iterable AGM functions* and *analytic AGM functions*. We propose them as plausible candidates for changing distinct theories, and we show that they satisfy significant properties of iterated change. We define both functions for possibly infinite languages and in both cases we have provide postulates extending AGM's and prove representation theorems for different formal structures.

Iterable AGM functions posess the following property: the change in one theory depends on the change of the largest theory, the whole language. Or more formally, they are almost constant on the first argument, the theory being revised, (the second argument held fixed).

Analytic AGM functions are more complex. They satisfy a restricted form of monotony on their first argument (the other held fixed) without being almost constant. Analytic functions can be calculated by means of a case analysis, such that if one theory is an extension of another, the cases considered for the first can be lifted to the cases for the second. This seems to be an interesting property for changing distinct theories.

But analytic AGM revisions have also another interest: they establish a formal connection with Katsuno and Mendelzon's update function. Analytic functions provide a new presentation of AGM revision based on the update semantic apparatus establishing in such a way a bridge between the two seemingly incomparable frameworks.

Finally, the thesis compares two conditional logics that provide a logical calculus for theory change, Alchourrón's logic DFT and Boutilier's logic CO. By appealing to their respective consequence relations, the two logics can be used to calculate changes in different theories. We reveal the connection between the two logics showing that in a restricted language the two validate the same conditional sentences.

We provide directions for future work, mainly suggesting the definition of new binary functions for theory change and the definition of a logical calculus for iterable and analytic AGM functions.