PORTUGALIAE MATHEMATICA
Vol. 56 Fasc. 3 – 1999

ON NONHOMOGENEOUS BIHARMONIC EQUATIONS INVOLVING CRITICAL SOBOLEV EXPONENT

M. Guedda

Abstract: In this paper we consider the problem $\Delta^2 u = \lambda\,|u|^{q_c-2}\,u + f$ in Ω, $u = \Delta u = 0$ on $\partial\Omega$, where $q_c = 2N/(N-4)$, $N > 4$, is the limiting Sobolev exponent and Ω is a smooth bounded domain in $\mathbb{R}^N$. Under some restrictions on f and λ, the existence of weak solution u is proved. Moreover $u \geq 0$ for $f \geq 0$ whenever $\lambda \geq 0$.

1 – Introduction

In this article, we show that the problem

$$(1.1) \qquad (P_{\lambda,f}) \qquad \begin{cases} \Delta(\Delta u) = \lambda\,|u|^{q_c-2}\,u + f & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega\,, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $N > 4$, Δ is the Laplacian operator and $q_c = 2N/(N-4)$, has weak solutions in $H^2_\theta(\Omega) = H^2(\Omega) \cap H^1_0(\Omega)$ equipped with the norm

$$\|u\|_{H^2_\theta} = \left(\int_\Omega |\Delta u|^2\right)^{1/2} .$$

To this end we consider the functional

$$(1.2) \quad F_\lambda(u) = \frac{1}{2}\int_\Omega |\Delta u|^2\,dx - \frac{\lambda}{q_c}\int_\Omega |u|^{q_c}\,dx - \int_\Omega f u\,dx\,, \qquad u \in H^2_\theta(\Omega), \quad \lambda > 0\,.$$

Under some suitable conditions, it is proved that (1.1) admits at least two solutions. Our arguments make use of the mountain pass theorem and of the Lions concentration-compactness principle.

Received: November 10, 1997; *Revised*: January 15, 1998.
AMS(MOS) Subject Classification: 35J65, 35J20, 49J45.

Recently, Van der Vorst [10] considered the following problem

$$S = \inf\left\{ \int_\Omega |\Delta u|^2;\ u \in H^2_\theta(\Omega),\ \int_\Omega |u|^{q_c} = 1 \right\} . \tag{1.3}$$

He proved that the infimum in (1.3) is never achieved by a function $u \in H^2_\theta(\Omega)$ when Ω is bounded. In contrast Hadiji, Picard and the author in [7] considered the problem

$$S_\varphi = \inf\left\{ \int_\Omega |\Delta u|^2;\ u \in H^2_\theta(\Omega),\ \int_\Omega |u+\varphi|^{q_c} = 1 \right\} . \tag{1.4}$$

They showed that the infimum in (1.4) is achieved whenever φ is continuous and non identically equal to zero. More precisely it is shown that, for any minimizing sequence (u_m) for (1.4), there exists a subsequence (u_{m_k}) and a function $u \in H^2_\theta(\Omega)$ such that

$$u_{m_k} \rightharpoonup u \ \text{ weakly in } H^2_\theta(\Omega) \quad \text{ and } \quad \|u+\varphi\|_{q_c} = 1 \ .$$

On the other hand, Bernis et al. [1] considered a variant of (1.1) where f is replaced by $\beta\,|u|^{p-2}\,u$, $1 < p < 2$. They proved the existence of at least two positive solutions for β sufficiently small. At this stage, we would like to mention that when $\Omega = \mathbb{R}^N$ P.L. Lions [9] proved that S is achieved only by the function u_ε defined by

$$u_\varepsilon(x) = \frac{\Big[(N-4)\,(N-2)\,N\,(N+2)\,\varepsilon^2\Big]^{\frac{N-4}{8}}}{\Big(\varepsilon + |x-a|^2\Big)^{\frac{N-4}{2}}} , \qquad x \in \mathbb{R}^N \ ,$$

for any $a \in \mathbb{R}^N$ and any $\varepsilon > 0$. This note is organized as follows. In Section 2 we verify that F_λ satisfies the $(\mathrm{PS})_c$ condition. In Section 3 we prove the existence of a local minimizer u of F_λ. Moreover, we show that $u \geq 0$ whenever $f \geq 0$ and $\lambda \geq 0$. Section 4 is devoted to the existence of a second solution to (1.4). The results presented in this paper have been announced in [6].

Notice that if $f \equiv 0$, the result of Section 3 is valid and gives the trivial solution $u = 0$. The method we adopt is closely related to the one of [3].

Before the verification of the $(\mathrm{PS})_c$ condition, let us remark that if v is a solution to (1.1) then $u = \lambda^{\frac{1}{q_c-2}}\,v$ satisfies

$$\begin{cases} \Delta(\Delta u) = |u|^{q_c-2}\,u + g & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega \ , \end{cases} \tag{1.5}$$

where $g = \lambda^{\frac{1}{q_c-2}} f$.

2 – Verification of the $(\mathbf{PS})_c$ condition

Let Ω be a bounded domain in $\mathbb{R}^N$, $N > 4$, and $f \in L^2(\Omega)$. We denote by $F_\lambda \colon H^2_\theta(\Omega) \to \mathbb{R}$ the functional defined by

$$F_\lambda(u) = \frac{1}{2}\int_\Omega |\Delta u|^2\, dx - \frac{\lambda}{q_c}\int_\Omega |u|^{q_c}\, dx - \int_\Omega f u \; dx \; , \tag{2.1}$$

where Δ is the Laplacian operator and λ is a real parameter. We first look for critical points of $F \stackrel{\text{def}}{=} F_1$. We show that F satisfies the Palais–Smale condition in a suitable sublevel strip.

Let S be the best Sobolev embedding constant of $H^2_\theta(\Omega)$ into $L^{q_c}(\Omega)$; that is

$$S = \inf\left\{\int_\Omega |\Delta u|^2;\; u \in H^2_\theta(\Omega),\; \int_\Omega |u|^{q_c} = 1\right\} \tag{2.2}$$

and

$$K = \frac{N^{\frac{q}{q_c}}}{2\,q\;(4\,q_c)^{\frac{q}{q_c}}}\,\|f\|^q_q\,, \qquad q = \frac{q_c}{q_c-1}\;. \tag{2.3}$$

Proposition 2.1. *The functional F satisfies the $(PS)_c$ condition in the sublevel strip $(-\infty,\, \frac{2}{N}\,S^{\frac{N}{4}} - K)$; that is if $\{u_m\}$ is a sequence in $H^2_\theta(\Omega)$ such that*

$$F(u_m) \to c \quad \text{and} \quad dF(u_m) \to 0 \;\; \text{in } H^{-2}_\theta(\Omega)\;, \tag{2.4}$$

where

$$c < \frac{2}{N}\,S^{\frac{N}{4}} - K\;,$$

then $\{u_m\}$ contains a subsequence which converges strongly in $H^2_\theta(\Omega)$.

Proof: Let $\{u_m\}$ be a sequence in $H^2_\theta(\Omega)$ which satisfies (2.4). From (2.4) it is easy to see that $\{u_m\}$ is bounded in $H^2_\theta(\Omega)$; thus there is a subsequence $\{u_{m_k}\}$, and an element u of $H^2_\theta(\Omega)$ such that

$$u_{m_k} \rightharpoonup u \quad \text{weakly in } \; H^2_\theta(\Omega) \tag{2.5}$$

and

$$u_{m_k} \to u \;\; \text{strongly in } L^p(\Omega)\,, \qquad 1 \le p < q_c \;\; \text{and a.e. in } \overline{\Omega}\;. \tag{2.6}$$

The concentration-compactness Lemma of Lions [9] asserts the existence of at most a countable index set J and positive constants $\{\nu_j\}$, $j \in J$ such that

$$|u_{m_k}|^{q_c} \rightharpoonup |u|^{q_c} + \sum_{j\in J} \nu_j\,\delta_{x_j}\;, \tag{2.7}$$

weakly in the sense of measures, and

$$|\Delta u_{m_k}|^2 \to \mu \ , \tag{2.8}$$

for some positive bounded measure μ. Moreover,

$$\mu \ \geq \ |\Delta u|^2 + \sum_{j\in J} S\, \nu_j^{\frac{N-4}{N}}\ \delta_{x_j} \ , \tag{2.9}$$

where

$$x_j \in \overline{\Omega} \quad \text{ and } \quad \nu_j = 0 \ \text{ or } \ \nu_j \geq S^{\frac{N}{4}} \ . \tag{2.10}$$

We assert that $\nu_j = 0$ for each j. If not, assume that $\nu_{j_0} \neq 0$, for some j_0. From the hypothesis (2.4),

$$c = \lim_{k\to\infty} F(u_{m_k}) - \frac{1}{2} \Big\langle dF(u_{m_k}), u_{m_k} \Big\rangle \ ,$$
$$c \geq \frac{2}{N} \int_\Omega |u|^{q_c} - \frac{1}{2} \int_\Omega f u + \frac{2}{N}\, S^{\frac{N}{4}} \ .$$

Using the Hölder inequality one has

$$c \ \geq \ \frac{2}{N}\, S^{\frac{N}{4}} - \frac{N^{\frac{q}{q_c}}}{2\, q\ (4\, q_c)^{\frac{q}{q_c}}}\ \|f\|_q^q \ .$$

This contradicts the hypothesis. Consequently $\nu_j = 0$ for each j and

$$\lim_{k\to\infty} \int_\Omega |u_{m_k}|^{q_c} = \int_\Omega |u|^{q_c} \ ,$$

which implies

$$u_{m_k} \to u \quad \text{ strongly in } \ H^2_\theta(\Omega) \ .$$

The proof is complete. ∎

3 – Existence of a solution

In this part we consider the problem of finding solutions to $(P_{\lambda,f})$. We show, under suitable conditions on f and λ, that F_λ has an infimum on a small ball in $H^2_\theta(\Omega)$. We suppose first that $\lambda = 1$, and denote by F the functional F_1. The proof is based on the following lemma.

Lemma 3.1. *There exist constants r and $R>0$ such that if $\|f\|_2 \le R$, then*

$$F(u) \ge 0 \quad \textit{for all} \quad \|u\|_{H^2_\theta(\Omega)} = r \ . \tag{3.1}$$

Proof: Thanks to the Sobolev and Hölder inequalities we have

$$F(u) \ge \frac{1}{2}\int_\Omega |\Delta u|^2 - \frac{1}{q_c}\, S^{-q_c} \Big(\int_\Omega |\Delta u|^2\Big)^{\frac{q_c}{2}} - |\Omega|^{\frac{1}{2}-\frac{1}{q_c}}\, S^{-1}\, \|f\|_2 \Big(\int_\Omega |\Delta u|^2\Big)^{1/2} . \tag{3.2}$$

Inequality (3.2) can be written

$$F(u) \ge h\Big(\|u\|_{H^2_\theta}\Big) \ , \tag{3.3}$$

where

$$h(x) = \frac{1}{2}\, x^2 - \lambda_0\, x^{q_c} - \lambda_1\, x\,, \qquad \lambda_0 = \frac{1}{q_c}\, S^{-q_c} \ \text{ and } \ \lambda_1 = \|f\|_2\, |\Omega|^{\frac{1}{2}-\frac{1}{q_c}}\, S^{-1} \ .$$

Let

$$g(x) = \frac{1}{2}\, x - \lambda_0\, x^{q_c-1} - \lambda_1 \quad \text{ for } \ x \ge 0 \ .$$

There exists $\overline{\lambda} > 0$ such that, if $0 < \lambda_1 \le \overline{\lambda}$, g attains its positive maximum and we get (3.1), with

$$r = \left(\frac{q_c\, S^{q_c}}{2}\right)^{\frac{1}{q_c-1}} \quad \text{ and } \quad R = |\Omega|^{-\frac{1}{2}+\frac{1}{q_c}}\, S\, \overline{\lambda} \ ,$$

thanks to (3.3). ∎

Remark 3.1. Arguing as above we can see that there exists a constant $\alpha > 0$ such that

$$F(u) \ge \alpha, \quad \text{ for all } \ \|u\|_{H^2_\theta} = r \ .$$

Proposition 3.1. *Let R and r be given by Lemma 3.1. Suppose that $f \not\equiv 0$ and*

$$\max\Big(\|f\|_2,\, \|f\|_q\Big) < \min(R', R) \ , \tag{3.4}$$

where

$$R' = \frac{4\, q_c\, S^{\frac{N}{4q}}}{N\, \Big(2\,(q_c-1)\Big)^{\frac{1}{q}}} \ .$$

Then there exists a function $u_1 \in H^2_\theta(\Omega)$ such that

$$F(u_1) = \min_{B_r} F(v) < 0 \ , \tag{3.5}$$

where

$$B_r = \left\{ v \in H^2_\theta,\ \|v\|_{H^2_\theta(\Omega)} < r \right\} ,$$

and u_1 *is a solution to* $(P_{1,f})$. *Moreover,* $u_1 \geq 0$ *whenever* $f \geq 0$.

Proof: Without loss of generality, we can suppose that $f(a) > 0$ for some $a \in \Omega$.

Let

$$u_\epsilon(x) = \frac{\varepsilon^{\frac{N-4}{4}}\, \phi(x)}{\left(\varepsilon + |x-a|^2\right)^{\frac{N-4}{2}}} , \qquad \varepsilon > 0 ,$$

where $\phi \in C_0^\infty(\Omega)$ is a fixed function such that $0 \leq \phi \leq 1$ and $\phi \equiv 1$ in some neighbourhood of a.

Since

$$\int_\Omega f u_\epsilon \, dx > 0, \qquad \text{for a small } \ \varepsilon ,$$

we can choose $t > 0$ sufficiently small such that

$$F(t\, u_\epsilon) < 0 .$$

Hence

$$\inf_{B_r} F(v) < 0 . \tag{3.6}$$

Let $\{u_m\}$ be a minimizing sequence of (3.6). From (3.4) and Lemma 3.1 we may assume that

$$\|u_m\|_{H^2_\theta} < r_0 < r . \tag{3.7}$$

According to the Ekeland variational principle [5] we may assume

$$\Delta^2 u_m - |u_m|^{q_c} - f \ \to 0 \quad \text{ in } \ H^{-2}_\theta(\Omega) . \tag{3.8}$$

On the other hand, from (2.3) and (3.4), we get

$$\frac{1}{N}\, S^{\frac{N}{4}} - K > 0 . \tag{3.9}$$

We deduce, from (3.8)–(3.9) and Proposition 2.1, that $\{u_m\}$ has a subsequence converging to $u_1 \in H^2_\theta(\Omega)$, and u_1 is a weak solution to $(P_{1,f})$.

Now we suppose that $f \geq 0$. Let $v \in H^2_\theta(\Omega)$ be a solution to the following problem

$$-\Delta v = |\Delta u_1| .$$

As in [10, 11] we get $v > 0$, $v \geq |u_1|$ in Ω,

$$\int_\Omega |\Delta v|^2 = \int_\Omega |\Delta u_1|^2 \quad \text{and} \quad \int_\Omega |v|^{q_c} \geq \int_\Omega |u_1|^{q_c} \ .$$

It then follows that

$$F(v) \leq F(u_1) \quad \text{and} \quad \|v\|_{H^2_\theta} \leq r \ .$$

Consequently F is minimized by a positive function. ∎

This method allows us under suitable conditions on f and λ, to prove the existence of solutions to $(P_{\lambda,f})$.

Theorem 3.1. *Suppose that $f \not\equiv 0$, then there exists $\lambda_f > 0$ such that if the following condition is satisfied*

$$0 < \lambda_f < \lambda^{\frac{1}{q_c-2}} < \min\left(\frac{1}{\|f\|_2}, \frac{1}{\|f\|_q}\right) \min(R', R) \ , \tag{3.10}$$

Problem $(P)_{\lambda,f}$ has at least one solution u_λ. Moreover $u_\lambda \geq 0$ whenever $f \geq 0$.

Proof: For the proof we consider Problem $(P_{1,g})$ where $g = f\,\lambda^{\frac{1}{q_c-2}}$. Condition (3.10) implies that g satisfies (3.4). So the existence follows immediately from Proposition 3.1.

Now suppose, on the contrary, that u_λ exists for any λ such that

$$0 < \lambda^{\frac{1}{q_c-2}} < \min\left(\frac{1}{\|f\|_2}, \frac{1}{\|f\|_q}\right) \min(R', R) \ .$$

Note that, since $\lambda^{-\frac{1}{q_c-2}}\, u_\lambda$ is the solution to $(P_{1,g})$ obtained by (3.5), we have

$$\|u_\lambda\|_{H^2_\theta(\Omega)} \leq r\,\lambda^{\frac{1}{q_c-2}} \ .$$

It follows from this that $\|u_\lambda\|_{H^2_\theta(\Omega)} \to 0$ as $\lambda \downarrow 0$.

Passing to the limit in $(P_{\lambda,f})$ we deduce that $f \equiv 0$, which yields to a contradiction. ∎

4 – Existence of a second solution

In this section we shall show, under additional conditions that $(P_{\lambda,f})$ possesses a second solution. Here we use the mountain pass theorem without the Palais–Smale condition [2, 8]. As in the preceding section, we first deal with the case $\lambda = 1$.

Assume that condition (3.4) is satisfied and that $f > 0$ in some neighbourhood of a. Set

$$v_\varepsilon = \frac{u_\varepsilon}{\|u_\varepsilon\|_{q_c}} \ .$$

The main result of this section is the following.

Theorem 4.1. *There exists $t_0 > 0$ such that if f satisfies*

$$\|f\|_q^q < \frac{t_0}{K_1} \int_\Omega f v_\varepsilon \, dx, \qquad \text{for small enough} \ \ \varepsilon > 0 \ , \tag{4.1}$$

where

$$K_1 = \frac{N^{\frac{q}{q_c}}}{2\, q\; (4\, q_c)^{\frac{q}{q_c}}} \ ,$$

then $(P_{1,f})$ has at least two distinct solutions.

Proof: The proof relies on a variant of the mountain pass theorem without the (PS) condition. We have, for ε sufficiently small (see [4]),

$$\|\Delta v_\varepsilon\|_2^2 \ = \ S + \mathrm{O}(\varepsilon^{\frac{N-4}{2}}) \ . \tag{4.2}$$

Set

$$h(t) \ = \ F(t\, v_\varepsilon) \ = \ \frac{1}{2}\, t^2\, X_\varepsilon - \frac{1}{q_c}\, t^{q_c} - t \int_\Omega f v_\varepsilon \, dx \qquad \text{for} \ \ t \geq 0 \ ,$$

where $X_\varepsilon = \|\Delta v_\varepsilon\|_2^2$.

Since $h(t)$ goes to $-\infty$ as t goes to $+\infty$, $\sup_{t\geq 0} h(t)$ is achieved at some $t_\varepsilon \geq 0$. Remark 3.1 asserts that $t_\varepsilon > 0$, and we deduce

$$h'(t_\varepsilon) = t_\varepsilon\, (X_\varepsilon - t_\varepsilon^{q_c-2}) - \int_\Omega f v_\varepsilon \, dx = 0 \qquad \text{and} \qquad h''(t_\varepsilon) \leq 0 \ , \tag{4.3}$$

thus

$$\left(\frac{1}{q_c - 1}\right)^{\frac{1}{q_c-2}} X_\varepsilon^{\frac{1}{q_c-2}} \ \leq \ t_\varepsilon \ \leq \ X_\varepsilon^{\frac{1}{q_c-2}} \ . \tag{4.4}$$

Let $t_0 = \frac{1}{2}\, (\frac{1}{q_c-1})^{\frac{1}{q_c-2}}\, S^{\frac{1}{q_c-2}}$. We deduce from (4.2) and (4.4) that, for ε_0 small,

$$t_0 < t_\varepsilon \qquad \text{for} \ \ \varepsilon \in (0, \varepsilon_0) \ . \tag{4.5}$$

Thus

$$\sup_{t\geq 0} h(t) \ = \ \sup_{t\geq t_0} h(t) \ .$$

On the other hand, since the function $t \longrightarrow \frac{1}{2}\,t^2 X_\varepsilon - \frac{1}{q_c}\,t^{q_c}$ is increasing on the interval $[0, X_\varepsilon^{\frac{1}{q_c-2}}]$, we get

$$h(t_\varepsilon) \;\le\; \frac{2}{N}\,S^{\frac{N}{4}} - t_\varepsilon \int_\Omega f v_\varepsilon\,dx + \mathrm{O}(\varepsilon^{\frac{N-4}{2}})\ ,$$

thanks to (4.2). Hence

$$h(t_\varepsilon) \;\le\; \frac{2}{N}\,S^{\frac{N}{4}} - t_0 \int_\Omega f v_\varepsilon\,dx + \mathrm{O}(\varepsilon^{\frac{N-4}{2}})\ . \tag{4.6}$$

Consequently if we let

$$t_0 \int_\Omega f v_\varepsilon\,dx \;>\; K_1\,\|f\|_q^q\ , \tag{4.7}$$

we deduce that

$$\sup_{t\ge 0} F(t v_\varepsilon) \;<\; \frac{2}{N}\,S^{\frac{N}{4}} - K\ . \tag{4.8}$$

Note that there exists t_1 large enough such that

$$F(t_1\,v_\varepsilon) < 0 \quad \text{and} \quad \|t_1\,v_\varepsilon\|_{H^2_\theta} > r\ , \tag{4.9}$$

where r is given by Lemma 3.1. Hence

$$\alpha \le c_2 = \inf_{\gamma\in\Gamma}\max_{s\in[0,1]} F(\gamma(s)) < \frac{2}{N}\,S^{\frac{N}{4}} - K\ ,$$

where

$$\Gamma = \Big\{\gamma \in C\Big([0,1], H^2_\theta(\Omega)\Big) :\ \gamma(0) = 0,\ \gamma(1) = t_1\,v_\varepsilon\Big\}\ ,$$

provided ε is small enough. Then, according to the mountain pass theorem without the (PS) condition, there exists a sequence $\{u_m\}$ in $H^2_\theta(\Omega)$ such that

$$F(u_m) \to c_2 \quad \text{and} \quad dF(u_m) \to 0\ \text{ in } H^{-2}_\theta(\Omega)\ .$$

Since $c_2 < \frac{2}{N}\,S^{\frac{N}{4}} - K$, we deduce from Proposition 2.1 that there exists u_2 such that $c_2 = F(u_2)$ and u_2 is a weak solution to $(P_{1,f})$.

This solution is distinct from u_1 since $c_1 < 0 < c_2$. So the proof is complete. ∎

Finally, by using Theorem 4.1, we deduce the

Corollary 4.1. *Assume* (3.10). *If*

$$\lambda^{\frac{q-1}{q_c-2}} \;<\; \frac{t_0}{K_1\,\|f\|_q^q} \int_\Omega f v_\varepsilon\,dx\ ,$$

for ε small enough, then problem $(P_{\lambda,f})$ has at least two solutions.

ACKNOWLEDGEMENTS – The author thanks C. Picard and M. Kirane for their useful discussions and for informing him of Ref. [1] and the referee for careful examination of the paper and valuable remarks. This work was partially supported by C.C.I. Kénitra (Maroc).

REFERENCES

[1] Bernis, F., Garcia-Azorero, J. and Peral, I. – Existence and multiplicity of nontrivial solutions in semilinear critical problems of fourth order, *Advances in Dif. Equat.,* 1(2) (1996), 219–240.

[2] Brezis, H. and Nirenberg, L. – Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents, *Comm. Pure Appl. Math.,* 36 (1983), 437–477.

[3] Chabrowski, J. – On multiple solutions for the nonhomogeneous p-Laplacian with a critical Sobolev exponent, *Diff. and Integ. Equat.,* 8(4), (1995), 705–716.

[4] Edmuns, E., Fortunato, D. and Jannelli, E. – Critical exponents, critical dimensions and the biharmonic operator, *Arch. Rational Mech. Anal.,* 112 (1990), 269–289.

[5] Ekeland, I. – On the variational principle, *J. Math. Anal. Appl.,* 47 (1974), 324–353.

[6] Guedda, M. – *A Note on Nonhomogeneous Biharmonic Equations Involving Critical Sobolev Exponent*, report LAMIFA, Univ. de Picardie, Fac. de Maths et d'Info, Amiens.

[7] Guedda, M., Hadiji, R. and Picard, C. – in preparation.

[8] Guedda, M. and Veron, L. – Quasilinear elliptic equations involving critical Sobolev exponents, *Nonlinear Anal. The. Meth. and Appl.,* 13(8) (1989), 879–902.

[9] Lions, P.L. – The concentration-compactness principle in the calculus of variations, the limit case, Parts 1 and 2, *Riv. Mat. Iberoamericana,* 1 (1985), 45–121, 145–201.

[10] Van Der Vorst, R.C.A.M. – Best constant for the embedding of the space $H^2 \cap H_0^2(\Omega)$ into $L^{\frac{2N}{N-4}}(\Omega)$, *Diff. and Integ. Equat.,* 6(2) (1993), 259–276.

[11] Van Der Vorst, R.C.A.M. – Fourth order elliptic equations with critical growth, *C.R.A.S.,* 320(I) (1995), 295–299.

M. Guedda,
LAMFA, Faculté de Mathématiques et d'Informatique, Université de Picardie Jules Verne,
33, rue Saint-Leu, 80039 Amiens – FRANCE