Abstract. A Hopf module is an A-module for an algebra A as well as a C-comodule for a coalgebra C, satisfying a suitable compatibility condition between the module and comodule structures. To formulate the compatibility condition one needs some kind of interaction between A and C. The most classical case arises when $A = C =: H$ is a bialgebra. Many subsequent variants of this were unified independently by Doi and Koppinen; in their version an auxiliary bialgebra H, over which A is a comodule algebra and C a module coalgebra, governs the compatibility. Another very general type of interaction between A and C is an entwining map as studied by Brzeziński — without an auxiliary bialgebra.

Every Doi-Koppinen datum induces an entwining structure, so Brzeziński's notion of an entwined module includes that of a Doi-Koppinen Hopf module. This paper investigates whether the inclusion is proper.

By work of Tambara, every entwining structure can be obtained from a suitable Doi-Koppinen datum whenever the algebra under consideration is finite dimensional.

We show by examples that this need not be true in general.