

ABSTRACT. For an arbitrary Riemannian manifold  $X$  and Hermitian vector bundles  $E$  and  $F$  over  $X$  we define the notion of the normal symbol of a pseudodifferential operator  $P$  from  $E$  to  $F$ . The normal symbol of  $P$  is a certain smooth function from the cotangent bundle  $T^*X$  to the homomorphism bundle  $\text{Hom}(E, F)$  and depends on the metric structures, resp. the corresponding connections on  $X$ ,  $E$  and  $F$ . It is shown that by a natural integral formula the pseudodifferential operator  $P$  can be recovered from its symbol. Thus, modulo smoothing operators, resp. smoothing symbols, we receive a linear bijective correspondence between the space of symbols and the space of pseudodifferential operators on  $X$ . This correspondence comprises a natural transformation between appropriate functors. A formula for the asymptotic expansion of the product symbol of two pseudodifferential operators in terms of the symbols of its factors is given. Furthermore an expression for the symbol of the adjoint is derived. Finally the question of invertibility of pseudodifferential operators is considered. For that we use the normal symbol to establish a new and general notion of elliptic pseudodifferential operators on manifolds.