

ABSTRACT. In this paper, we prove the conjectures made in a joint paper of the author with Carlson and Robinson, on the vanishing of cohomology of a finite group G . In particular, we prove that if k is a field of characteristic p , then every non-projective kG -module M in the principal block has nontrivial cohomology in the sense that $H^*(G, M) \neq 0$, if and only if the centralizer in G of every element of order p is p -nilpotent (this was proved for p odd in the above mentioned paper, but the proof here is independent of p). We prove the stronger statement that whether or not these conditions hold, the union of the varieties of the modules in the principal block having no cohomology coincides with the union of the varieties of the elementary abelian p -subgroups whose centralizers are not p -nilpotent (i.e., the nucleus). The proofs involve the new idempotent functor machinery of Rickard.