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Short Communications

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ON THE SOLVABILITY OF MULTIPOINT BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF NONLINEAR DIFFERENTIAL EQUATIONS WITH FIXED POINTS OF IMPULSES ACTIONS

Abstract. The necessary and sufficient conditions and the effective sufficient conditions are given for the solvability of the multipoint boundary value problems for systems of nonlinear impulsive equations.

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For the nonlinear impulsive system with fixed and finite number of impulses points

$$\frac{dx}{dt} = f(t,x) \ \text{ for a.e. } \ t \in [a,b] \setminus \{\tau_k\}_{k=1}^{m_0}, \tag{1}$$

$$x(\tau_k+) - x(\tau_k-) = I_k(x(\tau_k)) \ \ (k = 1,\dots,m_0), \tag{2}$$

we consider the multipoint boundary value problem of the Caucy–Nicoletti's type

$$x_i(t_i) = \varphi_i(x_1,\dots,x_n) \quad (i = 1,\dots,n), \tag{3}$$

where $t_i \in [a,b]$ $(i = 1,\dots,n)$, $a < \tau_1 < \dots < \tau_{m_0} < b$ (we will assume $\tau_0 = a$ and $\tau_{m_0+1} = b$ if necessary), $f = (f_k)_{k=1}^n \in K([a,b] \times \mathbb{R}^n, \mathbb{R}^n)$, $I_k = (I_{ki})_{i=1}^n : \mathbb{R}^n \to \mathbb{R}^n$ $(k = 1,\dots,m_0;\ i = 1,\dots,n)$ are continuous operators, and $\varphi_i : \mathrm{BV}_s([a,b],\mathbb{R}^n)$ $(i = 1,\dots,n)$ are continuous functionals which are nonlinear, in general.

In this paper, the necessary and sufficient conditions as well as the effective sufficient conditions are given for the solvability and unique solvability of the boundary value problem (1), (2); (3). Analogous problem on the interval $[-a,a]$ have been considered in [3], when the multipoint problem is degenerated into the two-point one, i.e., when $t_i = -\sigma_i a$ $(i = 1, \dots, n)$. The general nonlinear boundary value problems for the impulsive system (1), (2) is considered in [4], where the Conti–Opial type existence and uniqueness theorems are prescribed for the problem.

We realize the results for the boundary condition

$$x_i(t_i) = c_i \quad (i = 1, \dots, n), \tag{4}$$

i.e., when $\varphi_i(x_1, \dots, x_n) \equiv c_i (i = 1, \dots, n)$, where $c_i \in \mathbb{R}^n (i = 1, \dots, n)$ are the constant vectors.

Note, in addition, that analogous results are contained in [5, 6, 8, 9] for the multipoint boundary value problems for systems of ordinary differential equations.

Throughout the paper, the following notation and definitions will be used.

$\mathbb{R} =]-\infty, +\infty[$, $\mathbb{R}_+ = [0, +\infty[$; $[a, b]$ $(a, b \in \mathbb{R})$ is a closed interval.

$\mathbb{R}^{n\times m}$ is the space of all real $n \times m$-matrices $X = (x_{ij})_{i,j=1}^{n,m}$ with the norm

$$\|X\| = \max_{j=1,\dots,m} \sum_{i=1}^{n} |x_{ij}|;$$

$\mathbb{R}_+^{n\times m} = \big\{(x_{ij})_{i,j=1}^{n,m} :\ x_{ij} \geq 0\ (i = 1, \dots, n;\ j = 1, \dots, m)\big\}$.

$\mathbb{R}^n = \mathbb{R}^{n\times 1}$ is the space of all real column n-vectors $x = (x_i)_{i=1}^{n}$; $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$.

$\operatorname{diag}(\lambda_1, \dots, \lambda_n)$ is the diagonal matrix with diagonal elements $\lambda_1, \dots, \lambda_n$; δ_{ij} is the Kronecker symbol, i.e., $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$ $(i, j = 1, \dots, n)$.

θ is the function defined by $\theta(t) = 1$ for $t \geq 0$, and $\theta(t) = -1$ for $t < 0$.

$\overset{b}{\underset{a}{\vee}}(X)$ is the total variation of the matrix-function $X : [a, b] \to \mathbb{R}^{n\times m}$, i.e., the sum of total variations of the latter's components.

$X(t-)$ and $X(t+)$ are the left and the right limit of the matrix-function $X : [a, b] \to \mathbb{R}^{n\times m}$ at the point t (we will assume $X(t) = X(a)$ for $t \leq a$ and $X(t) = X(b)$ for $t \geq b$, if necessary);

$d_1X(t) = X(t) - X(t-)$, $d_2X(t) = X(t+) - X(t)$;

$\|X\|_s = \sup\big\{\|X(t)\| :\ t \in [a, b]\big\}$.

$\mathrm{BV}([a, b], \mathbb{R}^{n\times m})$ is the set of all matrix-functions of bounded variation $X : [a, b] \to \mathbb{R}^{n\times m}$ (i.e., such that $\overset{b}{\underset{a}{\vee}}(X) < +\infty$);

$\mathrm{BV}_s([a, b], \mathbb{R}^n)$ is the normed space $(\mathrm{BV}([a, b], \mathbb{R}^n), \|\cdot\|_s)$;

$\widetilde{C}([a, b], D)$, where $D \subset \mathbb{R}^{n\times m}$, is the set of all absolutely continuous matrix-functions $X : [a, b] \to D$;

$\widetilde{C}_{loc}([a, b] \setminus \{\tau_k\}_{k=1}^{m}, D)$ is the set of all matrix-functions $X : [a, b] \to D$ whose restrictions to an arbitrary closed interval $[c, d]$ from $[a, b] \setminus \{\tau_k\}_{k=1}^{m}$ belong to $\widetilde{C}([c, d], D)$.

If B_1 and B_2 are normed spaces, then an operator $g : B_1 \to B_2$ (nonlinear, in general) is positive homogeneous if

$$g(\lambda x) = \lambda g(x)$$

for every $\lambda \in \mathbb{R}_+$ and $x \in B_1$.

An operator $\varphi : \mathrm{BV}([a, b], \mathbb{R}^n) \to \mathbb{R}^n$ is called nondecreasing if for every $x, y \in \mathrm{BV}([a, b], \mathbb{R}^n)$ such that $x(t) \le y(t)$ for $t \in [a, b]$ the inequality $\varphi(x)(t) \le \varphi(y)(t)$ holds for $t \in [a, b]$.

If $\alpha \in \mathrm{BV}([a, b], \mathbb{R})$ has no more than a finite number of points of discontinuity, and $m \in \{1, 2\}$, then $D_{\alpha m} = \{t_{\alpha m1}, \dots, t_{\alpha m n_{\alpha m}}\}$ $(t_{\alpha m1} < \dots < t_{\alpha m n_{\alpha m}})$ is the set of all points from $[a, b]$ for which $d_m\alpha(t) \ne 0$.

$\mu_{\alpha m} = \max\{d_m\alpha(t) : t \in D_{\alpha m}\}$ $(m = 1, 2)$.

If $\beta \in \mathrm{BV}([a, b], \mathbb{R})$, then

$$\nu_{\alpha m\beta j} = \max\left\{d_j\beta(t_{\alpha ml}) + \sum_{t_{\alpha m\, l+1-m} < \tau < t_{\alpha m\, l+2-m}} d_j\beta(\tau) : l = 1, \dots, n_{\alpha m}\right\}$$

$(j, m = 1, 2)$; here $t_{\alpha 20} = a - 1$, $t_{\alpha 1 n_{\alpha 1}+1} = b + 1$.

By $\nu(t)$ $(a < t \le b)$ we denote a number of points τ_k $(k = 1, \dots, m_0)$ belonging to $[a, t[$.

A matrix-function is said to be continuous, nondecreasing, integrable, etc., if each of its components is such.

$L([a, b], D)$, where $D \subset \mathbb{R}^{n\times m}$, is the set of all measurable and integrable matrix-functions $X : [a, b] \to D$.

If $D_1 \subset R^n$ and $D_2 \subset R^{n\times m}$, then $K([a, b]\times D_1, D_2)$ is the Carathéodory class, i.e., the set of all mappings $F = (f_{kj})_{k,j=1}^{n,m} : [a, b] \times D_1 \to D_2$ such that for each $i \in \{1, \dots, l\}$, $j \in \{1, \dots, m\}$ and $k \in \{1, \dots, n\}$:

a) the function $f_{kj}(\,\cdot\,, x) : [a, b] \to D_2$ is measurable for every $x \in D_1$;

b) the function $f_{kj}(t, \cdot\,) : D_1 \to D_2$ is continuous for almost every $t \in [a, b]$, and

$$\sup\big\{|f_{kj}(\,\cdot\,, x)| : \ x \in D_0\big\} \in L([a, b], \mathbb{R}; g_{ik})$$

for every compact $D_0 \subset D_1$.

By a solution of the impulsive system (1), (2) we understand a continuous from the left vector-function $x \in \widetilde{C}_{loc}([a, b] \setminus \{\tau_k\}_{k=1}^{m_0}, \mathbb{R}^n) \cap \mathrm{BV}_s([a, b], \mathbb{R}^n)$ satisfying both the system (1) for a.e. $t \in [a, b] \setminus \{\tau_k\}_{k=1}^{m_0}$ and the relation (2) for every $k \in \{1, \dots, m_0\}$.

Quite a number of issues on the theory of systems of differential equations with impulsive effect (both linear and nonlinear) have been studied sufficiently well (for a survey of the results on impulsive systems see e.g. [1, 2, 7, 10, 11, 13, 14], and references therein). But the above-mentioned works do not contain the results analogous to those obtained in [5, 6, 8, 9] for ordinary differential equations.

Using the theory of the so-called generalized ordinary differential equations (see e.g. [1, 2, 12] and references therein), we extend these results to the systems of impulsive equations.

To establish the results dealing with the boundary value problems for the impulsive system (1), (2) we use the following concept.

It is easy to show that the vector-function x is a solution of the impulsive system (1), (2) if and only if it is a solution of the following system of generalized ordinary differential equations (see e.g. [8, 9, 13] and references therein):

$$dx(t) = dA(t) \cdot f(t, x(t)),$$

where

$$A(t) \equiv \operatorname{diag}(a_{11}(t), \dots, a_{nn}(t)),$$
$$a_{ii}(t) = \begin{cases} t & \text{for } a \le t \le \tau_1, \\ t + k & \text{for } \tau_k < t \le \tau_{k+1} \ \ (k = 1, \dots, m_0; \ \ i = 1, \dots, n); \end{cases}$$
$$f(\tau_k, x) \equiv I_k(x) \ \ (k = 1, \dots, m_0).$$

It is evident that the matrix-function A is continuous from the left, $d_2A(t) = 0$ if $t \in [a, b] \setminus \{\tau_k\}_{k=1}^{m_0}$ and $d_2A(\tau_k) = 1$ $(k = 1, \dots, m_0)$.

Definition. Let $t_1, \dots, t_n \in [a, b]$ and $a < \tau_1 < \dots < \tau_{m_0} < b$. We say that the triplet $(P, \{H_k\}_{k=1}^{m_0}, \varphi_0)$ consisting of a matrix-function $P = (p_{il})_{i,l=1}^n \in L([a, b], \mathbb{R}^{n\times n})$, a finite sequence of constant matrices $H_k = (h_{kil})_{i,l=1}^n \in R^{n\times n}$ $(k = 1, \dots, m_0)$ and a positive homogeneous nondecreasing continuous operator $\varphi_0 = (\varphi_{0i})_{i=1}^n : \mathrm{BV}_s([a, b], \mathbb{R}_+^n) \to \mathbb{R}_+^n$ belongs to the set $U(t_1, \dots, t_n; \tau_1, \dots, \tau_{m_0})$ if $p_{il}(t) \ge 0$ for a.e. $t \in [a, b]$ $(i \ne l;\ i, l = 1, \dots, n)$, $h_{kil} \ge 0$ $(i \ne l;\ i, l = 1, \dots, n;\ k = 1, \dots, m_0)$, and the system

$$x_i'(t) \operatorname{sgn}(t - t_i) \le \sum_{l=1}^n p_{il}(t) x_l(t) \ \text{ for a.e. } t \in [a, b] \setminus \{\tau_k\}_{k=1}^{m_0} \ \ (i = 1, \dots, n),$$

$$(x_i(\tau_k+) - x_i(\tau_k-)) \operatorname{sgn}(\tau_k - t_i) \le \sum_{l=1}^n h_{kil} x_l(\tau_k) \ \ (i = 1, \dots, n;\ k = 1, \dots, m_0)$$

has no nontrivial nonnegative solution satisfying the condition

$$x_i(t_i) \le \varphi_{0i}(x_1, \dots, x_n) \ \ (i = 1, \dots, n).$$

The set analogous to $U(t_1, \dots, t_n; \tau_1, \dots, \tau_{m_0})$ has been introduced by I. Kiguradze for ordinary differential equations (see [5, 6]).

Theorem 1. *The problem* (1), (2); (3) *is solvable if and only if there exist continuous from the left vector-functions* $\alpha_m = (\alpha_{mi})_{i=1}^n \in \widetilde{C}_{loc}([a, b] \setminus \{\tau_k\}_{k=1}^{m_0}, \mathbb{R}^n) \cap \mathrm{BV}_s([a, b], \mathbb{R}^n)$ $(m = 1, 2)$ *such that the conditions*

$$\alpha_1(t) \le \alpha_2(t) \quad \textit{for } t \in [a, b]$$

and

$$(-1)^j\Big(f_i\big(t,x_1,\dots,x_{i-1},\alpha_{ji}(t),x_{i+1},\dots,x_n\big)-\alpha'_{ji}(t)\Big)\operatorname{sgn}(t-t_i)\le 0$$
$$\text{for almost every } t\in[a,b],\ \ t\ne t_i,\ \ t\notin\{\tau_1,\dots,\tau_{m_0}\},$$
$$\alpha_1(t)\le (x_l)_{l=1}^n\le\alpha_2(t)\ \ (j=1,2;\ \ i=1,\dots,n),$$
$$(-1)^m\Big(x_i+I_{ki}(x_1,\dots,x_n)-\alpha_{mi}(\tau_k+)\Big)\le 0\ \text{ for }\ \tau_k\ge t_i,$$
$$\alpha_1(\tau_k)\le (x_l)_{l=1}^n\le\alpha_2(\tau_k)\ \ (m=1,2;\ \ i=1,\dots,n;\ \ k=1,\dots,m_0)$$

hold, and the inequalities

$$\alpha_{1i}(t_i)\le\varphi_i(x_l,\dots,x_n)\le\alpha_{2i}(t_i)\ \ (i=1,\dots,n)$$

are fulfilled on the set $\big\{(x_l)_{l=1}^n\in\widetilde{C}_{loc}([a,b]\setminus\{\tau_k\}_{k=1}^{m_0},R^n)\cap\mathrm{BV}_s([a,b],\mathbb{R}^n)$, $\alpha_1(t)\le(x_l)_{l=1}^n\le\alpha_2(t)$ *for* $t\in[a,b]\big\}$.

Theorem 2. *Let the conditions*

$$f_i(t,x_1,\dots,x_n)\operatorname{sgn}[(t-t_i)x_i]\le\sum_{l=1}^n p_{il}(t)|x_l|+q_i(t)$$
$$\text{for almost every } t\in[a,b],\ \ t\ne t_i,\ \ t\notin\{\tau_1,\dots,\tau_{m_0}\}\ \ (i=1,\dots,n),\tag{5}$$

where $q_i\in L([a,b],\mathbb{R}_+)$ $(i=1,\dots,n)$, *and*

$$I_{ki}(x_1,\dots,x_n)\theta(\tau_k-t_i)\operatorname{sgn}x_i\le$$
$$\le\sum_{l=1}^n h_{kil}|x_l|+q_i(\tau_k)\ \ (k=1,\dots,m_0;\ \ i=1,\dots,n)\tag{6}$$

be fulfilled on R^n, *and the inequalities*

$$|\varphi_i(x_1,\dots,x_n)|\le\varphi_{0i}\big(|x_1|,\dots,|x_n|\big)+\gamma\ \ (i=1,\dots,n)$$

be fulfilled on the set $\widetilde{C}_{loc}([a,b]\setminus\{\tau_k\}_{k=1}^{m_0},\mathbb{R}^n)\cap\mathrm{BV}_s([a,b],\mathbb{R}^n)$, *where* $\gamma\in\mathbb{R}_+$. *Let, moreover,*

$$\Big((p_{il})_{i,l=1}^n,\big\{(h_{kil})_{i,l=1}^n\big\}_{k=1}^{m_0};(\varphi_{0i})_{i=1}^n\Big)\in U(t_1,\dots,t_n;\tau_1,\dots,\tau_{m_0}),\tag{7}$$

where $q_i\in L([a,b],\mathbb{R}_+)$ $(i=1,\dots,n)$, $\gamma\in\mathbb{R}_+$. *Then the problem* (1), (2)*;* (3) *is solvable.*

Corollary 1. *Let the conditions* (5) *and* (6) *be fulfilled on* $\mathbb{R}^n$, *where* $p_{ii}\in L^\mu([a,b],\mathbb{R})$, $p_{il}\in L^\mu([a,b],\mathbb{R}_+)$ $(i\ne l;\ i,l=1,\dots,n)$, $q_i\in L([a,b],\mathbb{R}_+)$ $(i=1,\dots,n)$, $h_{kil}=\alpha_{kl}p_{il}(\tau_k)$ $(k=1,\dots,m_0;\ i,l=1,\dots,n)$, $1\le\mu\le+\infty$, $\alpha_{ik}\in\mathbb{R}_+$ $(i=1,\dots,n;\ k=1,\dots,m_0)$. *Let, moreover, the inequalities*

$$\big|\varphi_i(x_1,\dots,x_n)\big|\le\sum_{k=l}^n\left(\gamma_{1il}\|x_l\|_{L^\nu}+\gamma_{2il}\Big[\sum_{k=1}^{m_0}|x_l(\tau_l)|^\nu\Big]^{\frac{1}{\nu}}\right)+\gamma$$

be fulfilled on the set $\widetilde{C}_{loc}([a,b]\setminus\{\tau_k\}_{k=1}^{m_0},\mathbb{R}^n)\cap\mathrm{BV}_s([a,b],\mathbb{R}^n)$ $(i=1,\dots,n)$ *and let*

$$r(\mathcal{H}_0)<1,\tag{8}$$

where $\gamma_{1ik}, \gamma_{2ik} \in \mathbb{R}_+$ $(i,k = 1,\dots,n)$, $\frac{1}{\mu} + \frac{2}{\nu} = 1$, $\gamma \in \mathbb{R}_+$, *and the* $2n \times 2n$*-matrix* $\mathcal{H}_0 = (\mathcal{H}_{0jm})_{j,m=1}^2$ *is defined by*

$$\mathcal{H}_{011} = \left((b-a)^{\frac{1}{\nu}} \gamma_{1ik} + \left[\frac{2}{\pi}\,(b-a) \right]^{\frac{1}{\nu}} \|p_{ik}\|_{L^\mu} \right)_{i,k=1}^n,$$

$$\mathcal{H}_{012} = \left((b-a)^{\frac{1}{\nu}} \gamma_{2ik} + \left[(b-a) \sum_{l=1}^{m_0} \alpha_{li} \right]^{\frac{1}{\nu}} \left(\sum_{l=1}^{m_0} |p_{ik}(\tau_l)|^\mu \right)^{\frac{1}{\mu}} \right)_{i,k=1}^n,$$

$$\mathcal{H}_{021} = \left(\left(\sum_{l=1}^{m_0} \alpha_{li} \right)^{\frac{1}{\nu}} \gamma_{1ik} + \left[(b-a) \sum_{l=1}^{m_0} \alpha_{li} \right]^{\frac{1}{\nu}} \|p_{ik}\|_{L^\mu} \right)_{i,k=1}^n,$$

$$\mathcal{H}_{022} = \left(\left(\sum_{l=1}^{m_0} \alpha_{li} \right)^{\frac{1}{\nu}} \gamma_{2ik} + \left(\frac{1}{4}\,\mu_i \mu_k \sin^{-2} \frac{\pi}{4n_k+2} \right)^{\frac{1}{\nu}} \left(\sum_{l=1}^{m_0} |p_{ik}(\tau_l)|^\mu \right)^{\frac{1}{\mu}} \right)_{i,k=1}^n;$$

here $\mu_i = \max\{\alpha_{li} : l = 1,\dots,m_0\}$, *and* $n_k = n_{\alpha_k 2}$ *is the quantity of nonzero numbers from the sequence* $\alpha_{1k},\dots,\alpha_{m_0 k}$. *Then the problem* (1), (2); (3) *is solvable.*

Remark 1. The $2n \times 2n$-matrix $\mathcal{H}$ appearing in Corollary 1 can be replaced by the $n \times n$-matrix

$$\Bigg(\max \Bigg\{ \left[(b-a)^{\frac{1}{\nu}} + \left(\sum_{l=1}^{m_0} \alpha_{li} \right)^{\frac{1}{\nu}} \right] \gamma_{1ik} +$$
$$+ \left(\left[\frac{2}{\pi}\,(b-a) \right]^{\frac{2}{\nu}} + \left[(b-a) \sum_{l=1}^{m_0} \alpha_{li} \right]^{\frac{1}{\nu}} \right) \|p_{ik}\|_{L^\mu},\ \left[(b-a)^{\frac{1}{\nu}} + \left(\sum_{l=1}^{m_0} \alpha_{li} \right)^{\frac{1}{\nu}} \right] \gamma_{2ik} +$$
$$+ \left(\left[(b-a) \sum_{l=1}^{m_0} \alpha_{li} \right]^{\frac{1}{\nu}} + \left(\frac{1}{4}\,\mu_i \mu_k \sin^{-2} \frac{\pi}{4n_k+2} \right)^{\frac{1}{\nu}} \right) \left(\sum_{l=1}^{m_0} |p_{lk}(\tau_l)|^\mu \right)^{\frac{1}{\mu}} \Bigg\} \Bigg)_{i,k=1}^n.$$

Remark 2. In the Corollary 1, as a matrix-function $C = (c_{il})_{i,l=1}^n$ we take

$$c_{il}(t) \equiv p_{il} t + \alpha_{il}(t) \ \ (i,l = 1,\dots,n).$$

Corollary 2. *Let the conditions* (5) *and* (6) *be fulfilled on* $\mathbb{R}^n$, *where* $p_{ii} \in L^\mu([a,b],\mathbb{R})$, $p_{il} \in L^\mu([a,b],\mathbb{R}_+)$ $(i \neq l;\ i,l = 1,\dots,n)$, $q_i \in L([a,b],\mathbb{R}_+)$ $(i = 1,\dots,n)$, $h_{kil} = \alpha_{kl} p_{il}(\tau_k)$ $(k = 1,\dots,m_0;\ i,l = 1,\dots,n)$, $1 \le \mu \le +\infty$, $\alpha_{ik} \in \mathbb{R}_+$ $(i = 1,\dots,n;\ k = 1,\dots,m_0)$. *Let, moreover, the inequality* (8) *hold, where the* $2n \times 2n$*-matrix* $\mathcal{H}_0 = (\mathcal{H}_{0jm})_{j,m=1}^2$ *is defined by*

$$\mathcal{H}_{011} = \left(\left[\frac{2}{\pi}\,(b-a) \right]^{\frac{1}{\nu}} \|h_{ik}\|_{L^\mu} \right)_{i,k=1}^n,$$

$$\mathcal{H}_{012} = \left(\left[(b-a) \sum_{l=1}^{m_0} \alpha_{li} \right]^{\frac{1}{\nu}} \left(\sum_{l=1}^{m_0} |h_{ik}(\tau_l)|^\mu \right)^{\frac{1}{\mu}} \right)_{i,k=1}^n,$$

$$\mathcal{H}_{021} = \left(\left[(b-a) \sum_{l=1}^{m_0} \alpha_{li} \right]^{\frac{1}{\nu}} \|h_{ik}\|_{L^\mu} \right)_{i,k=1}^n,$$

$$\mathcal{H}_{022}=\bigg(\Big(\frac{1}{4}\,\mu_i\mu_k\sin^{-2}\frac{\pi}{4n_k+2}\Big)^{\frac{1}{\nu}}\Big(\sum_{l=1}^{m_0}|h_{ik}(\tau_l)|^{\mu}\Big)^{\frac{1}{\mu}}\bigg)_{i,k=1}^{n};$$

here $\frac{1}{\mu}+\frac{2}{\nu}=1$, $\mu_i=\max\{\alpha_{li}: l=1,\dots,m_0\}$, *and* $n_k=n_{\alpha_k 2}$ *is the quantity of nonzero numbers from the sequence* $\alpha_{1k},\dots,\alpha_{m_0k}$. *Then the problem* $(1),(2);(4)$ *is solvable.*

Remark 3. The $2n\times 2n$-matrix $\mathcal{H}_0$ appearing in Corollary 2 can be replaced by the $n\times n$-matrix

$$\Bigg(\max\bigg\{\Big(\Big[\frac{2}{\pi}\,(b-a)\Big]^{\frac{2}{\nu}}+\Big[(b-a)\sum_{l=1}^{m_0}\alpha_{li}\Big]^{\frac{1}{\nu}}\Big)\|p_{ik}\|_{L^{\mu}},$$

$$\Big(\Big[(b-a)\sum_{l=1}^{m_0}\alpha_{li}\Big]^{\frac{1}{\nu}}+\Big(\frac{1}{4}\,\mu_i\mu_k\sin^{-2}\frac{\pi}{4n_k+2}\Big)^{\frac{1}{\nu}}\Big)\Big(\sum_{l=1}^{m_0}|p_{lk}(\tau_l)|^{\mu}\Big)^{\frac{1}{\mu}}\bigg\}\Bigg)_{i,k=1}^{n}.$$

By Remark 3, the Corollary 2 has the following form for $p_{il}(t)\equiv p_{il}=const$ $(i,l=1,\dots,n)$ and $\mu=+\infty$.

Corollary 3. *Let the conditions* (6) *and*

$$f_i(t,x_1,\dots,x_n)\,\mathrm{sgn}[(t-t_i)x_i]\le\sum_{l=1}^{n}p_{il}|x_l|+q_i(t)$$

$$\text{for almost every } t\in[a,b]\setminus\{\tau_k\}_{k=1}^{m_0}\ \ (i=1,\dots,n)\qquad(9)$$

be fulfilled on $\mathbb{R}^n$, *where* $p_{ii}\in\mathbb{R}$, $p_{il}\ge 0$ $(i\ne l;\ i,l=1,\dots,n)$, $h_{kil}=\alpha_k p_{il}$ $(k=1,\dots,m_0;\ i,l=1,\dots,n)$, $\alpha_k\ge 0$ $(i=1,\dots,n;\ k=1,\dots,m_0)$, $q_i\in L([a,b],\mathbb{R}_+)$ $(i=1,\dots,n)$. *Let, moreover, the inequality* (8) *hold, where* $H_0=\rho_0 H$, $H=(h_il)_{i,l=1}^{n}$, *and*

$$\rho_0=\Big((b-a)\sum_{l=1}^{m_0}\alpha_l\Big)^{\frac{1}{2}}+\max\bigg\{\frac{2}{\pi}\,(b-a),\frac{1}{2}\,\mu_\alpha\sin^{-1}\frac{\pi}{4n_\alpha+2}\bigg\},$$

$\mu_\alpha=\max\{\alpha_l: l=1,\dots,m_0\}$, n_α *is the quantity of nonzero numbers from the sequence* $\alpha_1,\dots,\alpha_{m_0}$. *Then the problem* $(1),(2);(4)$ *is solvable.*

Corollary 4. *Let the conditions* (5) *and* (6) *be fulfilled on* $\mathbb{R}^n$, *the inequalities*

$$\big|\varphi_i(x_1,\dots,x_n)\big|\le\mu_i|x_i(s_i)|+\gamma\ \ (i=1,\dots,n)\qquad(10)$$

be fulfilled on the set $\widetilde{C}_{loc}([a,b]\setminus\{\tau_k\}_{k=1}^{m_0},\mathbb{R}^n)\cap \mathrm{BV}_s([a,b],\mathbb{R}^n)$ *and let*

$$\mu_i\gamma_i(s_i,t_i)<1\ \ (i=1,\dots,n),\qquad(11)$$

where $p_{il}(t)\equiv h_{il}\beta_i(t)+\beta_{il}(t)$, $q_i\in L([a,b],\mathbb{R}_+)$ $(i=1,\dots,n)$, $h_{kil}=h_{il}\beta_{ki}+\beta_{kil}$ $(i,l=1,\dots,n;\ k=1,\dots,m_0)$, $h_{ii}<0$, $h_{il}\ge 0$ $(i\ne l;\ i,l=1,\dots,n)$; $\mu_i\in\mathbb{R}_+$, $s_i\in[a,b]$, $s_i\ne t_i$ $(i=1,\dots,n)$; $\gamma\in\mathbb{R}_+$, $\beta_{ii}\in L([a,b],\mathbb{R}_+)$ $(i=1,\dots,n)$; β_{il}, $\beta_i\in L([a,b];\mathbb{R})$ $(i\ne l;\ i,l=1,\dots,n)$ *are such that* $\beta_{il}(t)\ge 0$ $(i\ne l)$ *and* $\beta_i(t)\ge 0$ *for a.e.* $t\in[a,t_i[\cup]t_i,b]$;

$\beta_{kii} \in \mathbb{R}_+$ $(i = 1, \dots, n;\ k = 1, \dots, m_0)$; β_{kil}, $\beta_{ki} \in \mathbb{R}$ $(i \neq l;\ i, l = 1, \dots, n;$ $k = 1, \dots, m_0)$ *are such that* $\beta_{kil} \geq 0$ *and* $\beta_{ki} \geq 0$ *if* $\tau_k \neq t_i$, $\beta_{ki} \leq 0$ *if* $\tau_k = t_i$, *and* $0 \leq \beta_{ki} < |\eta_i|^{-1}$ *if* $\tau_k > t_i$, $\eta_i < 0$; $\gamma_i(t,t) = 1$, $\gamma_i(t,s) = \gamma_i^{-1}(s,t)$ *for* $t < s$ *and*

$$\gamma_i(t,s) = \exp\left(\eta_i \int_s^t \beta_i(\tau)\, d\tau\right) \prod_{s \leq \tau_k < t} (1 + \eta_i \beta_{ki}) \ \text{ for } \ t > s \quad (i = 1, \dots, n).$$

Let, moreover, the condition

$$g_{ii} < 1 \ \ (i = 1, \dots, n) \tag{12}$$

hold and the real part of every characteristic value of the matrix $(\xi_{il})_{i,l=1}^n$ *be negative, where*

$$\xi_{il} = h_{il}\big(\delta_{il} + (1 - \delta_{il}) h_i\big) - h_{ii} g_{il} \ \ (i, l = 1, \dots, n),$$
$$g_{il} = \mu_i \big(1 - \mu_i \gamma_i(s_i, t_i)\big)^{-1} \gamma_{il}(\tau_i) + \max\big\{\gamma_{il}(a), \gamma_{il}(b)\big\} \ \ (i, l = 1, \dots, n),$$
$$\gamma_{il}(t) = \big|\alpha_{il}(t) - \alpha_{il}(t_i)\big| \ \text{ for } \ t \leq t_i,$$
$$\gamma_{il}(t) = \big|\alpha_{il}(t) - \alpha_{il}(t_i)\big| \ \text{ for } \ t > t_i \ \text{ if } \ t_i \notin \{\tau_1, \dots, \tau_{m_0}\},$$
$$\gamma_{il}(t) = \big|\alpha_{il}(t) - \alpha_{il}(t_i)\big| - (1 - \delta_{il}) \beta_{ki}$$
$$\text{for } \ t > t_i \ \text{ if } \ t_i \in \{\tau_1, \dots, \tau_{m_0}\} \ \ (i, l = 1, \dots, n),$$
$$h_i = 1 \ \text{ if } \ 0 \leq \mu_i \leq 1,$$
$$h_i = 1 + (\mu_i - 1)\big(1 - \mu_i \gamma_i(s_i, t_i)\big)^{-1} \ \text{ if } \ \mu_i > 1 \ \ (i = 1, \dots, n),$$
$$\alpha_{il}(t) \equiv \int_a^t \beta_{il}(\tau)\, d\tau + \sum_{a \leq \tau_k < t} \beta_{kil} \ \ (i, l = 1, \dots, n).$$

Then the problem (1), (2); (4) *is solvable.*

Corollary 4 has the following form if we assume therein that $\beta_i(1 \equiv 1$, $\beta_{il}(t) \equiv 0$, $\beta_{ki} = 0$, $h_{kil} = \beta_{kil}$ $(i, l = 1, \dots, n;\ k = 1, \dots, m_0$, and $p_{il}(t) \equiv p_{il} = const$, where $p_{il} = h_{il}$ $(i, l = 1, \dots, n)$.

Corollary 5. *Let the conditions* (6) *and* (9) *be fulfilled on* $\mathbb{R}^n$, *the inequalities* (10) *be fulfilled on the set* $\widetilde{C}_{loc}([a,b] \setminus \{\tau_k\}_{k=1}^{m_0}, \mathbb{R}^n) \cap \mathrm{BV}_s([a,b], \mathbb{R}^n)$ *and let the condition* (11) *hold, where* $p_{ii} < 0$, $p_{il} \geq 0$ $(i \neq l;\ i, l = 1, \dots, n)$; $h_{kii} \in \mathbb{R}_+$, $h_{kil} \in \mathbb{R}$ $(k = 1, \dots, m_0;\ i \neq l;\ i, l = 1, \dots, n)$, $q_i \in L([a,b], \mathbb{R}_+)$ $(i = 1, \dots, n)$, μ_i *and* $\gamma \in \mathbb{R}_+$, $s_i \in [a,b]$ *and* $s_i \neq t_i$ $(i = 1, \dots, n)$, $\gamma_i(s_i, t_i) = \exp(\eta_i |s_i - t_i|)$ $(i = 1, \dots, n)$, $\eta_i < 0$ $(i = 1, \dots, n)$, *and* $q_i \in L([a,b], \mathbb{R}_+)$ $(i = 1, \dots, n)$. *Let, moreover, the condition* (12) *hold and the real part of every characteristic value of the matrix* $(\xi_{il})_{i,l=1}^n$ *be negative, where*

$$\xi_{il} = p_{il}\big(\delta_{il} + (1 - \delta_{il}) h_i\big) - p_{ii} g_{il} \quad (i, l = 1, \dots, n),$$
$$g_{il} = \mu_i (1 - \mu_i \gamma_i(s_i, t_i))^{-1} \gamma_{il}(s_i) + \max\{\gamma_{il}(a), \gamma_{il}(b)\} \quad (i, l = 1, \dots, n),$$

$$\gamma_{il}(t) \equiv |\alpha_{il}(t) - \alpha_{il}(t_i)| \ \ (i,l = 1,\dots,n),$$
$$h_i = 1 \ \ for \ \ 0 \le \mu_i \le 1,$$
$$h_i = 1 + (\mu_i - 1)\big(1 - \mu_i \gamma_i(s_i,t_i)\big)^{-1} \ \ for \ \ \mu_i > 1 \ \ (i = 1,\dots,n),$$
$$\alpha_{il}(a) = 0, \ \ \alpha_{il}(t) = \sum_{a<\tau_k<t} h_{kil} \ \ for \ \ t \in]a,b] \ \ (i,l = 1,\dots,n).$$

Then the problem (1), (2), (3) *is solvable.*

Corollary 4 has the following form if we assume therein that $\beta_i(1 \equiv 1$, $\beta_{il}(t) \equiv 0$, $\beta_{ki} = 1$, $\beta_{kil} = 0$ $(i,l = 1,\dots,n;\ k = 1,\dots,m_0$, and $p_{il}(t) \equiv p_{il} = const$, where $p_{il} = h_{il}$ $(i,l = 1,\dots,n)$.

Corollary 6. *Let* $\tau_k \ne t_i (i = 1,\dots,n; k = 1,\dots,m_0)$, *the conditions* (9) *and*

$$I_{ki}(x_1,\dots,x_n)\operatorname{sgn}\big[(\tau_k - t_i)x_i\big] \le$$
$$\le \sum_{l=1}^{n} p_{il}|x_l| + q_i(\tau_k) \ \ (k = 1,\dots,m_0; \ \ i = 1,\dots,n)$$

be fulfilled on $\mathbb{R}^n$, *the inequalities* (10) *be fulfilled on the set* $\widetilde{C}_{loc}([a,b] \setminus \{\tau_k\}_{k=1}^{m_0}, \mathbb{R}^n) \cap \mathrm{BV}_s([a,b],\mathbb{R}^n)$ *and let the condition* (11) *hold, where* $p_{ii} < 0$, $p_{il} \ge 0$ $(i \ne l;\ i,l = 1,\dots,n)$; μ_i *and* $\gamma \in \mathbb{R}_+$, $q_i \in L([a,b],\mathbb{R}_+)$ $(i = 1,\dots,n)$, $s_i \in [a,b]$ *and* $s_i \ne t_i$ $(i = 1,\dots,n)$, $q_i \in L([a,b],\mathbb{R}_+)$ $(i = 1,\dots,n)$, $\gamma_i(s_i,t_i) = (1+\eta_i)^{\nu(s_i)-\nu(t_i)} \exp(\eta_i|s_i - t_i|)$ $(i = 1,\dots,n)$, *and* $-1 < \eta_i < 0$ $(i = 1,\dots,n)$. *Let, moreover, the condition* (12) *hold and the real part of every characteristic value of the matrix* $(\xi_{il})_{i,l=1}^n$ *be negative, where*

$$\xi_{il} = p_{il}\big(\delta_{il} + (1-\delta_{il})h_i\big) - p_{ii} g_{il} \quad (i,l = 1,\dots,n),$$
$$g_{il} = \mu_i(1 - \mu_i\gamma_i(s_i,t_i))^{-1}\gamma_{il}(s_i) + \max\{\gamma_{il}(a), \gamma_{il}(b)\} \ \ (i,l = 1,\dots,n),$$
$$\gamma_{il}(t) = 0 \ \ for \ \ t \le t_i, \ \ \gamma_{il}(t) = 0 \ \ for \ \ t > t_i \ \ if \ \ t_i \notin \{\tau_1,\dots,\tau_{m_0}\},$$
$$\gamma_{il}(t) = \delta_{il} - 1 \ \ for \ \ t > t_i \ \ if \ \ t_i \in \{\tau_1,\dots,\tau_{m_0}\} \ \ (i,l = 1,\dots,n),$$
$$h_i = 1 \ \ for \ \ 0 \le \mu_i \le 1,$$
$$h_i = 1 + (\mu_i - 1)\big(1 - \mu_i\gamma_i(s_i,t_i)\big)^{-1} \ \ for \ \ \mu_i > 1 \ \ (i = 1,\dots,n).$$

Then the problem (1), (2), (3) *is solvable.*

Theorem 3. *Let the conditions*

$$\big[f_i(t,x_1,\dots,x_n) - f_i(t,y_1,\dots,y_n)\big]\operatorname{sgn}[(t-t_i)(x_i-y_i)] \le$$
$$\le \sum_{l=1}^{n} p_{il}(t)|x_l - y_l| \ \ for\ almost\ every \ \ t \in [a,b] \setminus \{\tau_k\}_{k=1}^{m_0} \ \ (i = 1,\dots,n) \qquad (13)$$

and

$$\big[I_{ki}(x_1,\dots,x_n)-I_{ki}(y_1,\dots,y_n)]\theta(\tau_k-t_i)\,\mathrm{sgn}(x_i-y_i)]\le$$
$$\le\sum_{l=1}^{n}h_{kil}|x_l-y_l|\ \ (k=1,\dots,m_0;\ \ i=1,\dots,n)\quad(14)$$

be fulfilled on R^n, the inequalities

$$\big|\varphi_i(x_1,\dots,x_n)-\varphi_i(y_1,\dots,y_n)\big|\le$$
$$\le\varphi_{0i}\big(|x_-y_1|,\dots,|x_n-y_n|\big)\ \ (i=1,\dots,n)$$

be fulfilled on the set $\widetilde{C}_{loc}([a,b]\setminus\{\tau_k\}_{k=1}^{m_0},\mathbb{R}^n)\cap \mathrm{BV}_s([a,b],\mathbb{R}^n)$, where p_{il},h_{kil} and φ_{0i} $(i,l=1,\dots,n;\,k=1,\dots;m_0)$ are such that the condition (7) *holds. Then the problem* (1), (2), (3) *has one and only one solution.*

Corollary 7. *Let the conditions* (13) *and* (14) *be fulfilled on R^n, where $p_{ii}\in L^\mu([a,b],\mathbb{R})$, $p_{il}\in L^\mu([a,b],\mathbb{R}_+)$ $(i\neq l;\,i,l=1,\dots,n)$, $q_i\in L([a,b],\mathbb{R}_+)$ $(i=1,\dots,n)$, $h_{kil}=\alpha_{kl}p_{il}(\tau_k)$ $(k=1,\dots,m_0;\,i,l=1,\dots,n)$, $1\le\mu\le+\infty$, $\alpha_{ik}\in\mathbb{R}_+$ $(i=1,\dots,n;\,k=1,\dots,m_0)$. Let, moreover, the conditions* (8) *and*

$$\big|\varphi_i(x_1,\dots,x_n)-\varphi_i(y_1,\dots,y_n)\big|\le$$
$$\le\sum_{k=l}^{n}\left(\gamma_{1il}\|x_l-y_l\|_{L^\nu}+\gamma_{2il}\Big[\sum_{k=1}^{m_0}|x_l(\tau_l-y_l|^\nu\Big]^{\frac{1}{\nu}}\right)$$

be fulfilled on the set $\widetilde{C}_{loc}([a,b]\setminus\{\tau_k\}_{k=1}^{m_0},\mathbb{R}^n)\cap \mathrm{BV}_s([a,b],\mathbb{R}^n)$ $(i=1,\dots,n)$, where $\gamma_{1ik},\gamma_{2ik}\in\mathbb{R}_+$ $(i,k=1,\dots,n)$, $\frac{1}{\mu}+\frac{2}{\nu}=1$, $\gamma\in\mathbb{R}_+$, and $\mathcal{H}_0=(\mathcal{H}_{0jm})_{j,m=1}^2$ is the $2n\times 2n$-matrix defined in Corollary 1. *Then the problem* (1), (2), (3) *has one and only one solution.*

Corollary 8. *Let the conditions* (13) *and* (14) *be fulfilled on $\mathbb{R}^n$, where $p_{ii}\in L^\mu([a,b],\mathbb{R})$, $p_{il}\in L^\mu([a,b],\mathbb{R}_+)$ $(i\neq l;\,i,l=1,\dots,n)$, $h_{kil}=\alpha_{kl}p_{il}(\tau_k)$ $(k=1,\dots,m_0;\,i,l=1,\dots,n)$, $1\le\mu\le+\infty$, $\alpha_{ik}\in\mathbb{R}_+$ $(i=1,\dots,n;\,k=1,\dots,m_0)$. Let, moreover, the inequality* (8) *hold, where $\mathcal{H}_0=(\mathcal{H}_{0jm})_{j,m=1}^2$ is the $2n\times 2n$-matrix is defined in Corollary* 2. *Then the problem* (1), (2), (3) *has one and only one solution.*

Corollary 9. *Let the conditions* (14) *and*

$$\big[f_i(t,x_1,\dots,x_n)-f_i(t,y_1,\dots,y_n)\big]\,\mathrm{sgn}[(t-t_i)(x_i-y_i)]\le$$
$$\le\sum_{l=1}^{n}p_{il}|x_l-y_l|\ \ \text{for almost every}\ \ t\in[a,b]\setminus\{\tau_k\}_{k=1}^{m_0}\ \ (i=1,\dots,n)\quad(15)$$

be fulfilled on $\mathbb{R}^n$, where $p_{ii}\in\mathbb{R}$, $p_{il}\ge 0$ $(i\neq l;\,i,l=1,\dots,n)$, $h_{kil}=\alpha_k p_{il}$ $(k=1,\dots,m_0;\,i,l=1,\dots,n)$, $\alpha_k\ge 0$ $(i=1,\dots,n;\,k=1,\dots,m_0)$. Let, moreover, the inequality (8) *hold, where the matrix H_0 is defined as in Corollary* 3. *Then the problem* (1), (2), (4) *has one and only one solution.*

Corollary 10. *Let the conditions* (14) *and* (15) *be fulfilled on* $\mathbb{R}^n$ *and let the condition* (11) *hold, where* $p_{ii} < 0$, $p_{il} \ge 0$ $(i \ne l;\ i,l = 1,\dots,n)$; $h_{kii} \in \mathbb{R}_+$, $h_{kil} \in \mathbb{R}$ $(k = 1,\dots,m_0;\ i \ne l;\ i,l = 1,\dots,n)$, $s_i \in [a,b]$ *and* $s_i \ne t_i$ $(i = 1,\dots,n)$, $\gamma_i(s_i,t_i) = \exp(\eta_i|s_i - t_i|)$ $(i = 1,\dots,n)$, $\eta_i < 0$ $(i = 1,\dots,n)$, *and* $\mu_i \in \mathbb{R}_+)$ $(i = 1,\dots,n)$. *Let, moreover, the condition* (12) *hold and the real part of every characteristic value of the matrix* $(\xi_{il})_{i,l=1}^n$ *be negative, where the matrix* $(\xi_{il})_{i,l=1}^n$ *is defined as in Corollary* 5. *Then the system* (1), (2) *has one and only one solution under the condition*

$$x_i(t_i) = \lambda_i x_i(s_i) + \beta_i \ \ (i = 1,\dots,n)$$

for every $\gamma_i \in [-\mu_i, \mu_i]$ *and* $\beta_i \in \mathbb{R}$ $(i = 1,\dots,n)$.

Corollary 11. *Let* $\tau_k \ne t_i$ $(i = 1,\dots,n;\ k = 1,\dots,m_0)$, *the conditions* (15) *and*

$$\big[I_{ki}(x_1,\dots,x_n) - I_{ki}(y_1,\dots,y_n)\big]\operatorname{sgn}[(\tau_k - t_i)(x_i - y_i)] \le \\ \le \sum_{l=1}^n p_{il}|x_l - y_l| \ \ (k = 1,\dots,m_0;\ \ i = 1,\dots,n)$$

be fulfilled on R^n *and let the condition* (11) *hold, where* $p_{ii} < 0$, $p_{il} \ge 0$ $(i \ne l;\ i,l = 1,\dots,n)$; $\mu_i\mathbb{R}_+$ $(i = 1,\dots,n)$, $s_i \in [a,b]$ *and* $s_i \ne t_i$ $(i = 1,\dots,n)$, $\gamma_i(s_i,t_i) = (1+\eta_i)^{\nu(s_i)-\nu(t_i)}\exp(\eta_i|s_i - t_i|)$ $(i = 1,\dots,n)$, *and* $-1 < \eta_i < 0$ $(i = 1,\dots,n)$. *Let, moreover, the condition* (12) *hold and the real part of every characteristic value of the matrix* $(\xi_{il})_{i,l=1}^n$ *be negative, where the matrix* $(\xi_{il})_{i,l=1}^n$ *is defined as in Corollary* 6. *Then the statement of Corollary* 10 *is true.*

Remarks 1–3 given above are true for the uniqueness case, as well.

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