Mem. Differential Equations Math. Phys. 42 (2007), 145–152

Short Communications

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ON SOLVABILITY AND WELL-POSEDNESS OF INITIAL–BOUNDARY VALUE PROBLEMS FOR HIGHER ORDER NONLINEAR HYPERBOLIC EQUATIONS

Abstract. The sufficient conditions for unique local solvability, global solvability and of well-posedness of initial-boundary value problems for higher order nonlinear hyperbolic equations are studied.

[illegible]

2000 Mathematics Subject Classification: 35L35, 35B10.
Key words and phrases: Initial–boundary, well–posed, higher order, nonlinear hyperbolic equation.

Let $b > 0$, I be a compact interval containing zero, $\Omega = I \times [0, b]$, m and n be natural numbers and $f : \Omega \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^{m\times n} \to \mathbb{R}$ be a continuous function. In the rectangle Ω consider the nonlinear hyperbolic equation

$$u^{(m,n)} = f\big(x, y, u^{(m,0)}, \dots, u^{(m,n-1)}, u^{(0,n)}, \dots, u^{(m-1,n)}, \mathcal{D}^{m-1,n-1}[u]\big) \quad (1)$$

with the initial–boundary conditions

$$\begin{aligned} &u^{(j,0)}(0, y) = \varphi_j(y) \qquad (j = 0, \dots, m-1),\\ &h_k(u^{(m,0)}(x, \cdot))(x) = \psi_k(x) \qquad (k = 1, \dots, n). \end{aligned} \quad (2)$$

Here for any j and k

$$u^{(j,k)}(x, y) = \frac{\partial^{j+k} u(x, y)}{\partial x^j \partial y^k}, \quad \mathcal{D}^{m-1,n-1}[u](x, y) = \Big(u^{(j-1,k-1)}(x, y)\Big)_{1,1}^{m,n},$$

$\varphi_j \in C^n([0, b])$, $\psi_k \in C(I)$ and $h_k : C^{n-1}([0, b]) \to C(I)$ is a linear bounded operator.

Reported on the Tbilisi Seminar on Qualitative Theory of Differential Equations on July 16, 2007.

The linear case of problem (1),(2), i.e., the linear hyperbolic equation

$$u^{(m,n)} = \sum_{k=0}^{n-1} p_{mk}(x,y)u^{(m,k)} + \sum_{j=0}^{m-1}\sum_{k=0}^{n} p_{jk}(x,y)u^{(j,k)} + q(x,y) \qquad (3)$$

with conditions (2) is studied in [3] and [4] . In [3] necessary and sufficient conditions of well–posedness and so–called μ–well–posedness of problem (3),(2) are established. In [4] a complete description of problem (3),(2) in the ill–posed case is given.

For the history of the matter see [2–5] and the references quoted therein.

The general initial–boundary value problem (1),(2) has been little investigated. Namely this problem is investigated in the present paper.

Throughout the paper we will use the following notations.

$\mathbb{R}$ is the set of real numbers; $\mathbb{R}^{m\times n}$ is the space of real $m \times n$ matrices

$$Z = (z_{ij})_{1,1}^{m,n} = \begin{pmatrix} z_{11} & \dots & z_{1n} \\ \cdot & \cdots & \cdot \\ z_{m1} & \dots & z_{mn} \end{pmatrix}$$

with the norm $\|Z\| = \sum_{i=1}^{m}\sum_{j=1}^{n}|z_{ij}|$.

$C(I)$ and $C(\Omega)$, respectively, are the Banach spaces of continuous functions $z : I \to \mathbb{R}$ and $u : \Omega \to \mathbb{R}$, with the norms

$$\|z\|_{C(I)} = \max\{|z(x)| : x \in I\}, \quad \|u\|_{C(\Omega)} = \max\{|u(x,y)| : (x,y) \in \Omega\}.$$

$C(I;\mathbb{R}^{m\times n})$ is the Banach space of continuous matrix functions $Z : I \to \mathbb{R}^{m\times n}$ with the norm $\|z\|_{C(I;\mathbb{R}^{m\times n})} = \max\{\|Z(x)\| : x \in I\}$.

$C^k(I)$ is the Banach space of k–times continuously differentiable functions $z : I \to \mathbb{R}$, with the norm

$$\|z\|_{C^k(I)} = \sum_{i=0}^{k} \|z^{(i)}\|_{C(I)}.$$

$C^{m,n}(\Omega)$ is the Banach space of functions $u : \Omega \to \mathbb{R}$, having continuous partial derivatives $u^{(j,k)}$ $(j = 0,\dots,m;\ k = 0,\dots,n)$, with the norm

$$\|u\|_{C^{m,n}(\Omega)} = \sum_{j=0}^{m}\sum_{k=0}^{n} \|u^{(j,k)}\|_{C(\Omega)}.$$

$\widetilde{C}^{m,n}(\Omega)$ is the Banach space of functions $u : \Omega \to \mathbb{R}$, having continuous partial derivatives $u^{(j,k)}$ $(j = 0,\dots,m;\ k = 0,\dots,n;\ j+k < m+n)$, with the norm

$$\|u\|_{\widetilde{C}^{m,n}(\Omega)} = \sum_{k=0}^{n-1} \|u^{(m,k)}\|_{C(\Omega)} + \sum_{j=0}^{m-1}\sum_{k=0}^{n} \|u^{(j,k)}\|_{C(\Omega)}.$$

If $u \in \widetilde{C}^{m,n}(\Omega)$ and $r_0 > 0$, then $\widetilde{\mathcal{B}}^{m,n}(z;\Omega,r_0) = \{\zeta \in \widetilde{C}^{m,n}(\Omega) : \|\zeta - z\|_{\widetilde{C}^{m,n}} \le r_0\}$.

It will be assumed that $(x, y, z_1, \dots, z_{n+m}, Z) \to f(x, y, z_1, \dots, z_{n+m}, Z)$ is continuous in $\Omega \times \mathbb{R}^{n+m} \times \mathbb{R}^{m\times n}$ and *continuously differentiable* with respect to $z_1, \dots, z_{n+m}$.

Let $I_0 \subset I$ be an arbitrary (not necessarily compact) set containing zero. By a *solution of problem* (1),(2) *in the rectangle* $\Omega_0 = I_0 \times [0, b]$ we understand a classical solution, i.e., a function $u : \Omega_0 \to \mathbb{R}$ having the continuous partial derivatives $u^{(i,k)}$ $(i = 0, \dots, m;\ k = 0, \dots, n)$ and satisfying (1) and (2) at every point of Ω_0.

Definition 1. A solution u of problem (1),(2) defined on $\Omega_0 = I_0 \times [0, b]$ is called *continuable to the right* (*to the left*), if there exists an interval $I_1 \supset I_0$ and a solution u_1 of this problem in $\Omega_1 = I_1 \times [0, b]$ such that $\sup I_1 > \sup I_0$ $(\inf I_1 < \inf I_0)$ and

$$u_1(x, y) = u(x, y) \quad \text{for} \quad (x, y) \in \Omega_0.$$

u is called *non–continuable* if it is non-continuable both to the right and to the left.

Definition 2. A solution u of problem (1),(2) defined on $I_0 \times [0, b]$ is a called *global solution* (*local solution*) if $I_0 = I$ ($I_0 \neq I$ is a compact interval such that $[-\varepsilon, \varepsilon] \cap I \subset I_0$ for any sufficiently small $\varepsilon > 0$). Problem (1),(2) is called globally solvable (locally solvable), if it has a global (local) solution.

Along with (1),(2) consider the perturbed problem

$$v^{(m,n)} = f\big(x, y, v^{(m,0)}, \dots, v^{(m,n-1)}, v^{(0,n)}, \dots, v^{(m-1,n)}, \mathcal{D}^{m-1,n-1}[v]\big) + \\ + q(x, y), \tag{4}$$

$$\begin{aligned} v^{(j,0)}(0, y) &= \varphi_j(y) + \widetilde{\varphi}_j(y) \quad (j = 0, \dots, m-1), \\ h_k(v^{(m,0)}(x, \cdot))(x) &= \psi_k(x) + \widetilde{\psi}_k(x) \quad (k = 1, \dots, n). \end{aligned} \tag{5}$$

Let $I_0 \subset I$ be a compact interval containing zero, u be a solution of problem (1),(2) in $\Omega_0 = I_0 \times [0, b]$, and let r_0 be a positive constant. Introduce the following

Definition 3. Problem (1),(2) is called $(u; r_0)$ *well–posed* if there exist positive constants δ and r such that for any $\widetilde{\varphi}_j \in C^n([0, b])$ $(j = 0, \dots, m-1)$, $\widetilde{\psi}_k \in C(I)$ $(k = 1, \dots, n)$, and $q \in C(\Omega_0)$ satisfying the inequality

$$\sum_{j=0}^{m-1} \|\widetilde{\varphi}_j\|_{C^n([0,b])} + \sum_{k=1}^{n} \|\widetilde{\psi}_k\|_{C(I_0)} + \|q\|_{C(\Omega_0)} \leq \delta, \tag{6}$$

problem (4),(5) in the ball $\widetilde{\mathcal{B}}^{m,n}(u; \Omega_0, r_0)$ has a unique solution v and the inequality

$$\|u - v\|_{\widetilde{C}^{m,n}(J\times[0,b])} \leq r\Big(\sum_{j=0}^{m-1} \|\widetilde{\varphi}_j\|_{C^n([0,b])} + \sum_{k=1}^{n} \|\widetilde{\psi}_k\|_{C(J)} + \|q\|_{C(J\times[0,b)}\Big) \tag{7}$$

holds for every compact subinterval $J \subset I_0$ containing zero.

Definition 4. Problem (1),(2) is called *well–posed* if there exist positive constants δ and r such that for any $\widetilde{\varphi}_j \in C^n([0,b])$ $(j = 0,\dots,m-1)$, $\widetilde{\psi}_k \in C(I_0)$ $(k = 1,\dots,n)$, and $q \in C(\Omega_0)$ satisfying (6) problem (4),(5) has a unique solution v in Ω and estimate (7) is valid for every compact subset $J \subset I$ containing zero.

The proposed method of investigation of problem (1),(2) is based on the theory of boundary value problems for ordinary differential equations (see, e.g. [1]). For the boundary value problem

$$z^{(n)} = p(y, z, \dots, z^{(n-1)}); \quad l_k(z) = c_k \ \ (k = 1,\dots,n), \tag{8}$$

where $l_k : C^{n-1}([0,b]) \to \mathbb{R}$ $(k = 1,\dots,n)$ are linear bounded functionals and $p : [0,b] \times \mathbb{R}^n \to \mathbb{R}$ is a continuous function having continuous partial derivatives

$$p_k(y, z_1, \dots, z_n) = \frac{\partial p(y, z_1, \dots, z_n)}{\partial z_k} \quad (k = 1,\dots,n),$$

we introduce a definition of a strongly isolated solution, which is a modification of the definition from [1].

Definition 5. A solution z of problem (8) is called *strongly isolated* if the problem

$$\zeta^{(n)} = \sum_{j=1}^{n} p_j\big(y, z(y), \dots, z^{(n-1)}(y)\big)\zeta^{(j-1)}; \quad l_k(\zeta) = 0 \ \ (k = 1,\dots,n)$$

has only a trivial solution.

Set

$$\Phi(y) = \big(\varphi_{j-1}^{(k-1)}(y)\big)_{1,1}^{m,n},$$

$$p_0(y, z_1, \dots, z_n) = f\big(0, y, z_1, \dots, z_n, \varphi_0^{(n)}(y), \dots, \varphi_{m-1}^{(n)}(y), \Phi(y)\big),$$

$$p[u](x, y, z_1, \dots, z_n)$$
$$= f\Big(x, y, z_1, \dots, z_n, u^{(0,n)}(x,y), \dots, u^{(m-1,n)}(x,y), \mathcal{D}^{m-1,n-1}[u](x,y)\Big).$$

Theorem 1. *Let z_0 be a strongly isolated solution of the problem*

$$z^{(n)} = p_0(y, z, \dots, z^{(n-1)}), \quad h_k(z)(0) = \psi_k(0) \ \ (k = 1,\dots,n). \tag{9}$$

Then problem (1), (2) *has a local solution u satisfying the condition*

$$u^{(m,0)}(0,y) = z_0(y) \quad \textit{for} \quad y \in [0,b].$$

Furthermore, if $f(x, y, z_1, \dots, z_{n+m}, Z)$ is locally Lipschitz continuous with respect to Z, then problem (1), (2) *is $(u; r_0)$–well–posed for some sufficiently small $r_0 > 0$.*

Remark 1. In Theorem 1 the requirement of strong isolation of a solution z to problem (9) is essential and it cannot be replaced by the requirement of uniqueness of a solution. Indeed, consider the problem

$$u^{(1,1)} = (u^{(1,0)})^2 + x^2; \quad u(0,y) = 0, \quad u^{(1,0)}(x,0) = u^{(1,0)}(x,b), \qquad (10)$$

for which problem (9) has the form

$$z' = z^2; \quad z(0) = z(b). \qquad (11)$$

It is clear that problem (10) has no solution. On the other hand problem (11) has only a trivial solution which is not strongly isolated.

Remark 2. Under the conditions of Theorem 1 problem (1),(2) may have an infinite set of solutions even for smooth f. Indeed, consider the problem

$$\begin{gathered} u^{(1,1)} = \sin(u^{(1,0)}) + x\, f_0(x,y,u^{(1,0)},u^{(0,1)},u), \\ u(0,y) = 0, \ \ u^{(1,0)}(x,0) = u^{(1,0)}(x,b), \end{gathered} \qquad (12)$$

where $f_0 : \Omega \times \mathbb{R}^2 \to \mathbb{R}$ is a continuously differentiable function. For (12) problem (9) has the form

$$z' = \sin z; \quad z(0) = z(b).$$

The latter problem has a countable set of strongly isolated solutions $z_k = k\pi$ $(k = 0, \pm 1, \dots)$. By Theorem 1, for every integer k there exists positive ε_k such that in $\Omega_k = I_k \times [0,b]$, where $I_k = [-\varepsilon_k, \varepsilon_k] \cap I$, problem (12) has a unique solution u_k satisfying the condition

$$u_k^{(1,0)}(0,y) = k\pi \quad \text{for} \quad y \in [0,b].$$

Theorem 2. *Let u be a a non-continuable solution of problem* (1), (2) *defined in $\Omega_0 = I_0 \times [0,b]$. Furthermore, let for any $x_0 \in I_0$ the function $z(y) = u^{(m,0)}(x_0,y)$ be a strongly isolated solution of the problem*

$$\begin{gathered} z^{(n)} = p[u](x_0,y,z,z',\dots,z^{(n-1)}), \\ h_k(z)(x_0) = \psi_k(x_0) \quad (k = 1,\dots,n). \end{gathered} \qquad (13)$$

Then I_0 is an open set in I. Moreover, if $a^* = \sup I_0 \notin I_0$, then

$$\limsup_{x \to a^*} \Big(\|u^{(m,0)}(x,\cdot)\|_{C^{n-1}([0,b])} + \sum_{j=0}^{m-1} \|u^{(j,0)}(x,\cdot)\|_{C^n([0,b])} \Big) = +\infty, \qquad (14)$$

and if $a_* = \inf I_0 \notin I_0$, then

$$\liminf_{x \to a_*} \Big(\|u^{(m,0)}(x,\cdot)\|_{C^{n-1}([0,b])} + \sum_{j=0}^{m-1} \|u^{(j,0)}(x,\cdot)\|_{C^n([0,b])} \Big) = +\infty. \qquad (15)$$

Remark 3. In Theorem 2 the requirement of strong isolation of the solution $z(y) = u^{(m,0)}(x_0,y)$ of problem (13) for every $x_0 \in I_0$ is essential and it cannot be weakened. As an example in the rectangle $[-2,2] \times [0,b]$ consider the problem

$$u^{(1,1)} = |u| u^{(1,0)} + u, \quad u(0,y) = 1, \quad u^{(1,0)}(x,0) = u^{(1,0)}(x,b).$$

This problem has a non-continuable solution $u(x,y) = 1-x$ defined on the set $[-2,1]\times[0,b]$. Indeed, supposing that u can be continued to the right, by continuity of u and $u^{(1,0)}$ we will have

$$u^{(1,0)}(x,y) < 0, \;\; u(x,y) < 0 \qquad \text{for} \quad (x,y) \in (1, 1+\delta] \times [0,b]$$

for some sufficiently small $\delta > 0$. Consequently

$$u^{(1,1)}(x,y) = |u(x,y)|u^{(1,0)}(x,y) + u(x,y) < 0 \qquad \text{for} \quad (x,y) \in (1, 1+\delta]\times[0,b].$$

But the latter inequality contradicts to the periodicity of $u^{(1,0)}$ with respect to the second argument. Consequently (14) does not hold for u. The reason for this is that problem (13) has the form

$$z' = |1-x_0|\, z + 1 - x_0, \qquad v(0) = v(b),$$

and $z(y) = -1$ is a strongly isolated solution of this problem for every $x_0 < 1$, but not for $x_0 = 1$.

Definition 6. We say that the function f belongs to the set $S_{h_1,\ldots,h_n}$ if there exist functions $p_{ik} \in C(\Omega)$ $(i=1,2;\; k=1,\ldots,n)$ such that:
(i)

$$p_{1i}(x,y) \le f_{z_i}(x,y,z_1,\ldots,z_{n+m},Z) \le p_{2i}(x,y)$$
$$\text{for} \quad (x,y)\in\Omega \;\; (i=1,\ldots,n);$$

(ii) for any $x \in I$ and measurable functions $p_i : [0,b] \to \mathbb{R}$ $(i=1,\ldots,n)$ satisfying inequalities $p_{1i}(x,y) \le p_i(y) \le p_{2i}(x,y)$ for $(x,y)\in\Omega$ $(i = 1,\ldots,n)$ the problem

$$\zeta^{(n)} = \sum_{j=1}^{n} f_j(y)\zeta^{(j-1)}; \qquad h_k(\zeta)(x) = 0 \;\; (k=1,\ldots,n)$$

has only a trivial solution.

Theorem 3. *Let there exist a positive constant l_0 such that*

$$f \in S_{h_1,\ldots,h_n}, \tag{16}$$

$$|f(x,y,z_1,\ldots,z_{n+m},Z)| \le l_0\Big(1 + \sum_{k=1}^{n+m}|z_k| + \|Z\|\Big). \tag{17}$$

Then problem (1), (2) *is globally solvable. Furthermore, if $f(x,y,z_1,\ldots,z_{n+m},Z)$ is locally Lipschitz continuous with respect to Z, then problem* (1), (2) *is well–posed.*

Remark 4. In Theorem 3 condition (16) is optimal and it cannot be weakened. Indeed, in the rectangle $[-\pi,\pi]\times[0,b]$ consider the problem

$$\begin{aligned} &u^{(1,1)} = \arctan(u^{(1,0)}) - \arctan(1+u^2);\\ &u(0,y) = 0, \quad u^{(1,0)}(x,0) = u^{(1,0)}(x,b), \end{aligned} \tag{18}$$

for which condition (17) holds but condition (16) is violated. As a result problem (18) has a unique solution $u(x, y) \equiv \tan(x)$, which cannot be continued outside the rectangle $(-\frac{\pi}{2}, \frac{\pi}{2}) \times [0, b]$.

Below separately consider the case, where the righthand side of equation (1.1) does not contain the derivatives $u^{(m,k)}$ $(k = 1, \dots, n-1)$, i.e., where equation (1.1) has the form

$$u^{(m,n)} = g(x, y, u^{(m,0)}, u^{(0,n)}, \dots, u^{(m-1,n)}, \mathcal{D}^{m-1,n-1}[u]), \tag{19}$$

where $(x, y, z_1, \dots, z_{m+1}, Z) \to g(x, y, z_1, \dots, z_{m+1}, Z)$ is continuous in $\Omega \times \mathbb{R}^{m+1} \times \mathbb{R}^{m \times n}$ and *continuously differentiable* with respect to $z_1, \dots, z_{m+1}$. We also assume that the function g is *sublinear*, i.e., for some constant $l_0 > 0$ g satisfies the inequality

$$|g(x, y, z_1, \dots, z_{m+1}, Z)| \le l_0 \Big(1 + \sum_{k=1}^{m+1} |z_k| + \|Z\|\Big)$$

in $\Omega \times \mathbb{R}^{m+1} \times \mathbb{R}^{m \times n}$.

Corollaries 1—3 concern the case, where (2) is either the initial–Dirichlet

$$\begin{gathered} u^{(j,0)}(0, y) = \varphi_j(y) \quad (j = 0, \dots, m-1), \\ u^{(m,i-1)}(x, y_1(x)) = \psi_{1i}(x) \quad (i = 1, \dots, n^*), \\ u^{(m,k-1)}(x, y_2(x)) = \psi_{2k}(x) \quad (k = 1, \dots, n - n^*), \end{gathered} \tag{20}$$

or the initial–periodic conditions

$$\begin{gathered} u^{(j,0)}(0, y) = \varphi_j(y) \quad (j = 0, \dots, m-1), \\ u^{(m,k-1)}(x, y_1(x)) = u^{(m,k-1)}(x, y_2(x)) + \psi_k(x) \ \ (k = 1, \dots, n), \end{gathered} \tag{21}$$

where n^* is the integer part of $n/2$, $\varphi_j \in C^n([0, b])$, $\psi_k \in C(I)$, ψ_{1k}, $\psi_{2k} \in C(I)$, y_1, $y_2 \in C(I)$, $0 \le y_1(x) < y_2(x) \le b$ for $x \in I$.

Corollary 1. *Let there exist a nonnegative function $p_0 \in C(\Omega)$ and a positive number ε such the condition*

$$\begin{gathered} -p_0(x, y) \le (-1)^{n-n^*} (y - y_1(x))^{n-2n^*} g_{z_1}(x, y, z_1, \dots, z_{m+1}, Z) \le \\ \le \frac{\alpha_n - \varepsilon}{4} \Big(\frac{2\pi}{y_2(x) - y_1(x)} \Big)^{2n^*} \end{gathered}$$

holds in $\Omega \times \mathbb{R}^{m+1} \times \mathbb{R}^{m \times n}$, where $\alpha_n = 1$ for $n = 2n^*$, and $\alpha_n = n/2$ for $n = 2n^* + 1$. Then problem (19), (20) *is globally solvable. Furthermore, if $f(x, y, z_1, \dots, z_{m+1}, Z)$ is locally Lipschitz continuous with respect to Z, then problem* (19), (20) *is well–posed.*

Corollary 2. *Let there exist nonnegative functions $p_i \in C(\Omega)$ $(i = 0, 1)$ such that*

$$\int_{y_1(x)}^{y_2(x)} p_1(x, y)\, dy > 0 \quad \textit{for} \quad x \in I,$$

and the condition

$$-p_0(x,y) \le \sigma\, g_{z_1}(x,y,z_1,\dots,z_{m+1},Z) \le -p_1(x,y),$$

holds in $\Omega \times \mathbb{R}^{m+1} \times \mathbb{R}^{m\times n}$, *where*

$$\sigma = (-1)^{n^*} \quad \textit{for} \quad n = 2n^*, \quad \text{and} \quad \sigma \in \{-1,1\} \quad \textit{for} \quad n = 2n^*+1.$$

Then problem (19), (21) *is globally solvable. Furthermore, if* $g(x,y,z_1,\dots,z_{m+1},Z)$ *is locally Lipschitz continuous with respect to* Z, *then problem* (19), (21) *is well–posed.*

Corollary 3. *Let* $n = 2n^*$, *and let there exist a positive number* ε *and a nonnegative function* $p_1 \in C(\Omega)$ *satisfying inequality* (1.41) *such the condition*

$$p_1(x,y) \le (-1)^{n^*} g_{z_1}(x,y,z_1,\dots,z_{m+1},Z) \le \Big(\frac{2\pi-\varepsilon}{y_2(x)-y_1(x)}\Big)^n,$$

holds in $\Omega \times \mathbb{R}^{m+1} \times \mathbb{R}^{m\times n}$. *Then problem* (19), (21) *is globally solvable. Furthermore, if* $g(x,y,z_1,\dots,z_{m+1},Z)$ *is locally Lipschitz continuous with respect to* Z, *then problem* (19), (21) *is well–posed.*

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(Received 12.06.2007)

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