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NONOSCILLATORY SOLUTIONS OF SOME DIFFERENTIAL SYSTEMS

Abstract. For a system of differential inequalities we present exact conditions for non-existence of global solutions of constant sign in subcritical and critical cases. In the supercritical case we indicate one sufficient condition for non-existence of global solutions of constant sign which is, possibly, also exact.

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Introduction

The problem on asymptotic behavior of solutions of second order nonlinear nonautonomous ordinary differential equations attracted attention of a great number of mathematicians at the beginning of the twentieth century in connection with astrophysical investigations of R. Emden in which there appeared the equation of the type

$$u'' \pm t^{\sigma} u^{n} = 0.$$

The detailed qualitative investigation of this equation, called subsequently the Emden-Fowler equation, for different values of parameters σ and n was carried out by R. Fowler.

The interest in the study of asymptotics of solutions of nonlinear second order equations has considerably increased after the appearance of the well-known R. Bellman's monograph [5] in which the author stated all basic results concerning the Emden-Fowler equation.

The qualitative investigation of the Emden-Fowler type equation

$$u'' + a(t)|u|^{n} \operatorname{sign} u = 0, \tag{0.1}$$

where $n \in (0, +\infty)$ and the function $a : [0, +\infty) \to \mathbb{R}$ is summable on each finite segment, was started by F.V. Atkinson [2]. He proved that if $a(t) \geq 0$ and $n > 1$, then the condition

$$\int\limits_{0}^{+\infty} t a(t) dt = +\infty$$

is necessary and sufficient for all proper solutions of the equation (0.1) to be oscillatory.

Š. Belohorec [6] proved that if $a(t) \geq 0$, $0 < n < 1$, then for all proper solutions of the equation (0.1) to be oscillatory, it is necessary and sufficient that

$$\int\limits_{0}^{+\infty} t^{n} a(t) dt = +\infty.$$

The oscillation problem of solutions of the equation (0.1) in case $a(t)$ is a function with alternating signs, has been studied by I.T. Kiguradze [25].

The efficient methods for investigating the asymptotic behavior of proper and singular solutions of the equation (0.1) have been proposed by M.M. Aripov [1], L.A. Beklemisheva [4], Š. Belohorec [6,7], T.A. Chanturia [27,30], V.M. Evtukhov [20], A.G. Katranov [24], I.T. Kiguradze [25–28], L.B. Klebanov [29], A.V. Kostin [32] and others.

The equations of the type

$$u'' + f(t, u) = 0.$$

are also studied in detail. In particular, there were obtained: sufficient conditions for the existence of proper solutions; sufficient conditions for the boundedness and stability in one or another sense; necessary and sufficient conditions for all proper solutions to be oscillatory; sufficient conditions for the existence of at least one oscillatory solution; sufficient conditions for all proper solutions to be nonoscillatory; conditions for the solvability of various boundary value problems.

Similar problems for higher order nonlinear differential equations and systems of nonlinear nonautonomous differential equations have been studied by M. Bartušhek [3], T.A. Chanturia [27,30], Z. Došla [8,9], O. Došly [10–14], A. Elbert [13, 17–19], J. Jaroš [22], I.T. Kiguradze [26–28], T. Kusano [19, 22, 33], A.G. Lomtatidze [14, 23, 34, 35], J.V. Manojlovič [36], J. D. Mirzov [37], I. Nečas [39], B. Půža [41–43], V.A. Rabtsevich [44], B. L. Shekhter [28], Ch.A. Skhalyakho [45, 46] etc.

In the last years a considerable progress has been made by many mathematicians in investigation of problems connected with the existence (non-existence) of solutions of constant signs of nonlinear differential equations and systems of differential equations. Among them we can mention the works of M. Cecchi [8, 9], P. Drabek [15, 16], Yu.V. Egorov [21], V.A. Galaktionov [21], V.A. Kondrat'ev [21], R.G. Koplatadze [30, 31], R. Manasevich [16], M. Marini [8, 9], E. Mitidieri [38] and S.I. Pokhozhaev [38, 40].

Our work is devoted to systems of the type

$$u_2' \operatorname{sign} u_1 \le -a_2(t)|u_1|^{\lambda_2} \le 0 \le a_1(t)|u_2|^{\lambda_1} \le u_1' \operatorname{sign} u_2, \tag{0.2}$$

$$-a_2(t)|u_1|^{\lambda_2} \le u_2' \operatorname{sign} u_1 \le 0 \le u_1' \operatorname{sign} u_2 \le a_1(t)|u_2|^{\lambda_1}, \tag{0.3}$$

where $a_i : (0, +\infty) \to [0, +\infty)$ $(i = 1, 2)$ are the functions which are summable on every finite segment from $(0, +\infty)$, $\lambda_i > 0$ $(i = 1, 2)$.

Below we will indicate exact conditions guaranteeing the non-existence of global solutions of constant signs of systems of the type (0.2), i.e. the solutions such that $u_1(t) \cdot u_2(t) \ne 0$ for $0 < t < +\infty$. The exactness of the conditions is understood in a sense that their violation leads to the existence of global solutions of constant sign of the system (0.3).

For the nonlinear system of the type

$$u_i' = (-1)^{i-1} a_i(t) |u_{3-i}|^{\lambda_i} \operatorname{sign} u_{3-i},$$

where $a_i(t) \ge 0$ $(i = 1, 2)$, $\lambda_1 \cdot \lambda_2 = 1$, we present an original characteristic of the principal solution and new criteria for the non-existence of conjugate points.

The basic method of our investigation is the method of a priori estimates which is widely used by I.T. Kiguradze and his numerous followers.

The results obtained in the paper can be applied to the qualitative theory of ordinary differential equations, to the theory of boundary value problems, in investigating the behavior of solutions of partial differential equations with p-Laplacian, and so on.

It should be noted that the statements given in the present work can, with natural changes, be paraphrased for any interval of the type (a, b), where $-\infty \le a < b \le +\infty$.

1. Systems of Inequalities in the Subcritical Case

In this section, using the results of [37], we find exact conditions ensuring the non-existence of global solutions of constant signs of systems of the type (0.2) with $\lambda_1 \cdot \lambda_2 < 1$, and then we apply the obtained conditions to some partial differential equations.

Theorem 1.1. *Let $\lambda_1 \cdot \lambda_2 < 1$ and for some $t_0 \in [0, +\infty)$*

$$\begin{gathered} \int_0^{t_0} a_1(t)dt < +\infty, \quad \int_0^{t_0} a_2(t)dt = +\infty, \\ \int_{t_0}^{+\infty} a_1(t)dt = +\infty, \quad \int_{t_0}^{+\infty} a_2(t)dt < +\infty. \end{gathered} \tag{1.1}$$

Then for the system (0.2) to have no global solution with the property $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$, it is sufficient that the equality

$$\int_0^{t_0} a_1(t)\bigg(\int_t^{t_0} a_2(\tau)d\tau\bigg)^{\lambda_1} dt + \int_{t_0}^{+\infty} a_2(t)\bigg(\int_{t_0}^{t} a_1(\tau)d\tau\bigg)^{\lambda_2} dt = +\infty \tag{1.2}$$

be fulfilled.

Proof. Assume the contrary, i.e., suppose that the first summand in (1.2) is equal to $+\infty$ and, nevertheless, $u_1(t) \cdot u_2(t) > 0$ for $t \in (0, +\infty)$ for some solution $u_1(t)$, $u_2(t)$ of the system (0.2). Then for $0 < t < +\infty$,

$$|u_1|' \ge a_1(t)|u_2|^{\lambda_1}, \quad |u_2|' \le -a_2(t)|u_1|^{\lambda_2}. \tag{1.3}$$

Hence $|u_2(t)| \ge |u_1(t)|^{\lambda_2} \int_t^{t_0} a_2(\tau)d\tau$ for $0 < t \le t_0$. Therefore

$$|u_1(t)|^{-\lambda_1\lambda_2}|u_1(t)|' \ge a_1(t)\bigg(\int_t^{t_0} a_2(\tau)d\tau\bigg)^{\lambda_1} \quad \text{for} \quad 0 < t \le t_0.$$

Integrating the last inequality from t to t_0 and passing to the limit as $t \to 0+$, we come to the contradiction.

Suppose now that the second summand in (1.2) is equal to $+\infty$ and, nevertheless, $u_1(t) \cdot u_2(t) > 0$ for $t \in (0, +\infty)$ for some solution $u_1(t)$, $u_2(t)$ of the system (0.2). Then (1.3) holds. Consequently, $|u_1(t)| \ge |u_2(t)|^{\lambda_1} \int_{t_0}^t a_1(\tau)d\tau$ for $t \ge t_0$. Therefore $|u_2(t)|^{-\lambda_1 \cdot \lambda_2}|u_2(t)|' \le -a_2(t)\big(\int_{t_0}^t a_1(\tau)d\tau\big)^{\lambda_2}$ for $t \ge t_0$. Integrating the last inequality from t_0 to t and passing to the limit as $t \to +\infty$, we arrive at the contradiction. □

Theorem 1.2. *Let $\lambda_1 \cdot \lambda_2 < 1$ and* (1.1) *be satisfied. Then for the system* (0.3) *to have no global solution with the property $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$, it is necessary that the equality* (1.2) *be fulfilled.*

Proof. Suppose that (1.2) is not fulfilled. Let us show that the system (0.3) has a solution $u_1(t)$, $u_2(t)$ defined on $(0, +\infty)$ with the property $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$.

Consider the solution $u_1(t)$, $u_2(t)$ of the system (0.3) defined by the initial conditions $u_1(t_0) = u_{10}$, $u_2(t_0) = u_{20}$, $u_{10} \cdot u_{20} > 0$, where the numbers u_{10} and u_{20} will be chosen later on. In some neighbourhood of t_0 the inequalities

$$\begin{gathered} u_1(t) \cdot u_2(t) > 0, \\ -a_2(t)|u_1(t)|^{\lambda_2} \leq |u_2(t)|' \leq 0 \leq |u_1(t)|' \leq a_1(t)|u_2(t)|^{\lambda_1} \end{gathered} \tag{1.4}$$

hold. Consequently, for $t \geq t_0$ in the neighbourhood of t_0 we have

$$|u_{20}| - \int_{t_0}^{t} a_2(\tau) \bigg| |u_{10}| + \int_{t_0}^{\tau} a_1(s)|u_2(s)|^{\lambda_1} ds \bigg|^{\lambda_2} d\tau \leq |u_2(t)|.$$

Since $(x+y)^\alpha \leq 2^\alpha(x^\alpha + y^\alpha)$ for any $x > 0$, $y > 0$, $\alpha \geq 0$, the last inequality yields

$$|u_{20}| - 2^{\lambda_2}|u_{10}|^{\lambda_2} \int_{t_0}^{t} a_2(\tau) d\tau -$$

$$-2^{\lambda_2}|u_{20}|^{\lambda_1 \cdot \lambda_2} \int_{t_0}^{t} a_2(\tau) \bigg(\int_{t_0}^{\tau} a_1(s) ds \bigg)^{\lambda_2} d\tau \leq |u_2(t)|.$$

Thus we see that $|u_2(t)| > 0$ for all $t \geq t_0$ if

$$|u_{20}| - 2^{\lambda_2}|u_{10}|^{\lambda_2} \int_{t_0}^{+\infty} a_2(t) dt -$$

$$-2^{\lambda_2}|u_{20}|^{\lambda_1 \cdot \lambda_2} \int_{t_0}^{+\infty} a_2(t) \bigg(\int_{t_0}^{t} a_1(\tau) d\tau \bigg)^{\lambda_2} dt > 0. \tag{1.5}$$

For $t \leq t_0$, in the neighbourhood of t_0 according to (1.4) we have

$$|u_{10}| \leq |u_1(t)| + \int_{t}^{t_0} a_1(\tau) \Big| |u_{20}| + \int_{\tau}^{t_0} a_2(s)|u_1(s)|^{\lambda_2} ds \Big|^{\lambda_1} d\tau \leq |u_1(t)| +$$

$$+2^{\lambda_1}|u_{20}|^{\lambda_1} \int_{t}^{t_0} a_1(\tau) d\tau + 2^{\lambda_1}|u_{10}|^{\lambda_1 \cdot \lambda_2} \int_{t}^{t_0} a_1(\tau) \bigg(\int_{\tau}^{t_0} a_2(s) ds \bigg)^{\lambda_1} d\tau.$$

Hence, if

$$|u_{10}| - 2^{\lambda_1}|u_{20}|^{\lambda_1}\int\limits_0^{t_0} a_1(t)dt -$$

$$-2^{\lambda_1}|u_{10}|^{\lambda_1\cdot\lambda_2}\int\limits_0^{t_0} a_1(t)\bigg(\int\limits_t^{t_0} a_2(s)ds\bigg)^{\lambda_1} dt > 0, \tag{1.6}$$

then $|u_1(t)| > 0$ for all $t \in (0, t_0]$.

To complete the proof of the theorem we have to show that the system of inequalities (1.5), (1.6) has at least one solution $|u_{10}|$, $|u_{20}|$. Suppose $|u_{20}| = |u_{10}|^\gamma$, where $\lambda_2 < \gamma < \frac{1}{\lambda_1}$. It is obvious that for sufficiently large $|u_{10}|$ the inequalities (1.5), (1.6) are fulfilled. Therefore for any solution of the system (0.3) defined by the above-mentioned initial values we have $u_1(t)\cdot u_2(t) > 0$ for $0 < t < +\infty$. □

From Theorems 1.1 and 1.2 we immediately arrive at

Theorem 1.3. *Let $\lambda_1\cdot\lambda_2 < 1$ and* (1.1) *be satisfied. Then for the system*

$$\begin{aligned} a_1(t)|u_2|^{\lambda_1} &\le u_1' \operatorname{sign} u_2 \le Ma_1(t)|u_2|^{\lambda_1},\\ -Ma_2(t)|u_1|^{\lambda_2} &\le u_2' \operatorname{sign} u_1 \le -a_2(t)|u_1|^{\lambda_2}, \end{aligned} \tag{1.7}$$

where $M \ge 1$, to have no global solution with the property $u_1(t)\cdot u_2(t) > 0$ for $0 < t < +\infty$, it is necessary and sufficient that the condition (1.2) *be fulfilled.*

Theorem 1.4. *Let $\lambda_1\cdot\lambda_2 < 1$ and for some $t_0 \in (0, +\infty)$,*

$$\begin{aligned} &\int\limits_0^{t_0} a_1(t)dt = +\infty, \quad \int\limits_0^{t_0} a_2(t)dt < +\infty,\\ &\int\limits_{t_0}^{+\infty} a_1(t)dt < +\infty, \quad \int\limits_{t_0}^{+\infty} a_2(t)dt = +\infty. \end{aligned} \tag{1.8}$$

Then for the system (0.2) *to have no global solution with the property $u_1(t)\cdot u_2(t) < 0$ for $0 < t < +\infty$, it is sufficient that the equality*

$$\int\limits_0^{t_0} a_2(t)\bigg(\int\limits_t^{t_0} a_1(\tau)d\tau\bigg)^{\lambda_2} dt + \int\limits_{t_0}^{+\infty} a_1(t)\bigg(\int\limits_{t_0}^{t} a_2(\tau)d\tau\bigg)^{\lambda_1} dt = +\infty \tag{1.9}$$

be fulfilled.

Proof. Assume $a_i = b_{3-i}$, $\lambda_i = \mu_{3-i}$, $u_i = (-1)^{i-1}v_{3-i}$ $(i = 1, 2)$ and make use of Theorem 1.1. □

Theorem 1.5. *Let $\lambda_1 \cdot \lambda_2 < 1$ and (1.8) be satisfied. Then for the system (0.3) to have no global solution with the property $u_1(t) \cdot u_2(t) < 0$ for $0 < t < +\infty$, it is necessary that the condition (1.9) hold.*

The proof is similar to that of the previous theorem and follows directly from Theorem 1.2.

From Theorems 1.4 and 1.5 follows

Theorem 1.6. *Let $\lambda_1 \cdot \lambda_2 < 1$ and (1.8) be satisfied. Then for the system (1.7) to have no solution with the property $u_1(t) \cdot u_2(t) < 0$ for $0 < t < +\infty$, it is necessary and sufficient that the condition (1.9) hold.*

At the end of this section we give some applications of the obtained results to the partial differential equations of the type

$$\operatorname{div}(|\bigtriangledown u|^{m-2} \bigtriangledown u) + f(|x|)|u|^{n-2}u = 0, \tag{1.10}$$

where the function $f : (0, +\infty) \to [0, +\infty)$ is summable on every finite segment of the interval $(0, +\infty)$, $m > 1$, $n > 1$, $x = (x_1, x_2, \dots, x_N) \in \mathbb{R}^N$, $N \geq 2$, $\bigtriangledown u = \left(\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \dots, \frac{\partial u}{\partial x_N}\right)$.

It is known that the function $u(x) = y(|x|) = y(t)$ is a solution of (1.10) if and only if $y(t)$ satisfies the ordinary differential equation [33]

$$(t^{N-1}|y'|^{m-2}y')' + t^{N-1}f(t)|y|^{n-2}y = 0. \tag{1.11}$$

Theorem 1.7. *Let $n < m$, $N < m$ and for some $t_0 \in (0, +\infty)$,*

$$\int_0^{t_0} t^{N-1}f(t)dt = +\infty, \quad \int_{t_0}^{+\infty} t^{N-1}f(t)dt < +\infty.$$

Then for the equation (1.10) to have no global radial positive (negative), increasing (decreasing) with respect to the radial variable solution, it is necessary and sufficient that

$$\int_0^{t_0} t^{-\frac{N-1}{m-1}} \left(\int_t^{t_0} \tau^{N-1} f(\tau)d\tau \right)^{\frac{1}{m-1}} dt + \int_{t_0}^{+\infty} t^{N-1+\frac{(m-N)(n-1)}{m-1}} f(t)dt = +\infty.$$

Proof. We rewrite the equation (1.11) in the form of the system

$$u_1' = t^{-\frac{N-1}{m-1}}|u_2|^{\frac{1}{m-1}} \operatorname{sign} u_2, \quad u_2' = -t^{N-1}f(t)|u_1|^{n-1} \operatorname{sign} u_1$$

and make use of Theorem 1.3. □

Theorem 1.8. *Let $n < m$, $N > m$ and for some $t_0 \in (0, +\infty)$,*

$$\int_0^{t_0} t^{N-1}f(t)dt < +\infty, \quad \int_{t_0}^{+\infty} t^{N-1}f(t)dt = +\infty.$$

Then for the equation (1.10) *to have no global radial positive* (*negative*), *decreasing* (*increasing*) *with respect to the radial variable solution, it is necessary and sufficient that*

$$\int_0^{t_0} t^{N-1+\frac{(m-N)(n-1)}{m-1}} f(t)dt + \int_{t_0}^{+\infty} t^{-\frac{N-1}{m-1}} \bigg(\int_{t_0}^{t} \tau^{N-1} f(\tau)d\tau \bigg)^{\frac{1}{m-1}} dt = +\infty.$$

The proof follows directly from Theorem 1.6.

2. Systems of Inequalities in the Critical Case

In this section we obtain exact conditions for the non-existence of global solution of constant sign of the system (0.2) for $\lambda_1 \cdot \lambda_2 = 1$.

First we give some auxiliary statements.

Lemma 2.1. *Let* $\lambda_1 \cdot \lambda_2 = 1$, $\sigma \in \{-1, 1\}$ *the system* (0.2) *have a solution defined on* $[t_0, +\infty)$ $((0, t_0])$ *and possessing the property* $\sigma u_1(t) \cdot u_2(t) > 0$ *for* $t \geq t_0$ $(t \leq t_0)$. *Then the system*

$$v_1' = a_1(t)|v_2|^{\lambda_1} \operatorname{sign} v_2, \quad v_2' = -a_2(t)|v_1|^{\lambda_2} \operatorname{sign} v_1 \tag{2.1}$$

has also a solution defined on $[t_0, +\infty]$ $((0, t_0])$ *and possessing the property* $\sigma v_1(t) \cdot v_2(t) > 0$ *for* $t \geq t_0$ $(t \leq t_0)$.

Proof. For the sake of definiteness, we assume that $\sigma = 1$. Suppose that the system (0.2) has the solution $u_1(t)$, $u_2(t)$, defined on $[t_0, +\infty)$ and possessing the property $u_1(t) \cdot u_2(t) > 0$ for $t \geq t_0$. Consider a solution $v_1(t)$, $v_2(t)$ of the system (2.1) whose initial values satisfy the inequality

$$\frac{v_2(t_0) \operatorname{sign} v_1(t_0)}{|v_1(t_0)|^{\lambda_1}} \geq \frac{u_2(t_0) \operatorname{sign} u_1(t_0)}{|u_1(t_0)|^{\lambda_2}}.$$

From (0.2) it follows that

$$\Big(\frac{u_2 \operatorname{sign} u_1}{|u_1|^{\lambda_2}} \Big)' \leq -\lambda_2 a_1(t) \Big(\frac{|u_2|}{|u_1|^{\lambda_2}} \Big)^{1+\lambda_1} - a_2(t) \quad \text{for} \quad t \geq t_0.$$

Therefore, by virtue of the lemma on differential inequalities (see, e.g., [26], p. 42), we obtain

$$\frac{v_2(t) \operatorname{sign} v_1(t)}{|v_1(t)|^{\lambda_2}} \geq \frac{u_2(t) \operatorname{sign} u_1(t)}{|u_1(t)|^{\lambda_2}} \quad \text{for} \quad t \geq t_0,$$

where it is clear that the function appearing in the left-hand side of the inequality is non-increasing on $[t_0, +\infty)$.

Let us prove the second part of the statement of the lemma. Suppose that the system (0.2) has a solution $u_1(t)$, $u_2(t)$ defined on $(0, t_0]$ and possessing the property $u_1(t) \cdot u_2(t) > 0$ for $t \leq t_0$. Consider the solution $v_1(t)$, $v_2(t)$ of the system (2.1) whose initial values satisfy the inequality

$$\frac{v_1(t_0) \operatorname{sign} v_2(t_0)}{|v_2(t_0)|^{\lambda_1}} \geq \frac{u_1(t_0) \operatorname{sign} u_2(t_0)}{|u_2(t_0)|^{\lambda_1}}.$$

From (0.2) it follows that

$$\Big(\frac{u_1 \operatorname{sign} u_2}{|u_2|^{\lambda_1}}\Big)' \geq \lambda_1 a_2(t)\Big(\frac{|u_1|}{|u_2|^{\lambda_1}}\Big)^{1+\lambda_2} + a_1(t) \quad \text{for} \quad 0 < t \leq t_0.$$

Therefore by virtue of the above-mentioned lemma on differential inequalities, we get

$$\frac{v_1(t) \operatorname{sign} v_2(t)}{|v_2(t)|^{\lambda_1}} \geq \frac{u_1(t) \operatorname{sign} u_2(t)}{|u_2(t)|^{\lambda_1}} \quad \text{for} \quad 0 < t \leq t_0,$$

where the function appearing in the left-hand side of the inequality does not decrease on $(0, t_0]$. The case where $\sigma = -1$ is reduced by means of the substitution $u_i = (-1)^{i-1} w_{3-i}$, $v_i = (-1)^{i-1} z_{3-i}$, $a_i = b_{3-i}$, $\lambda_i = \mu_{3-i}$ $(i = 1, 2)$ (since the systems (0.2), (0.3) and (2.1) are invariant with respect to the above-mentioned substitution) to the previous one. □

Lemma 2.2. *Let* $\lambda_1 \cdot \lambda_2 = 1$, $\sigma \in \{-1, 1\}$, *the system* (2.1) *have a solution defined on* $[t_0, +\infty)$ $((0, t_0])$ *and possessing the property* $\sigma v_1(t) \cdot v_2(t) > 0$ *for* $t \geq t_0$ $(t \leq t_0)$. *Then the system* (0.3) *has also a solution defined on* $[t_0, +\infty)$ $((0, t_0])$ *and possessing the property* $\sigma u_1(t) \cdot u_2(t) > 0$ *for* $t \geq t_0$ $(t \leq t_0)$.

Proof. For the sake of definiteness, we assume $\sigma = 1$. Suppose that the system (2.1) has a solution $v_1(t)$, $v_2(t)$ defined on $[t_0, +\infty)$ and possessing the property $v_1(t) \cdot v_2(t) > 0$ for $t \geq t_0$. Consider the solution $u_1(t)$, $u_2(t)$ of the system (0.3) whose initial values satisfy the inequality

$$\frac{u_2(t_0) \operatorname{sign} u_1(t_0)}{|u_1(t_0)|^{\lambda_2}} \geq \frac{v_2(t_0) \operatorname{sign} v_1(t_0)}{|v_1(t_0)|^{\lambda_2}}.$$

It follows from (0.3) that

$$\Big(\frac{u_2 \operatorname{sign} u_1}{|u_1|^{\lambda_2}}\Big)' \geq -\lambda_2 a_1(t)\Big(\frac{|u_2|}{|u_1|^{\lambda_2}}\Big)^{1+\lambda_1} - a_2(t)$$

in some right half-neighbourhood of t_0, and the function appearing under the sign of the derivative does not increase in the above-mentioned half-neighbourhood. According to the lemma on differential inequalities,

$$\frac{u_2(t) \operatorname{sign} u_1(t)}{|u_1(t)|^{\lambda_2}} \geq \frac{v_2(t) \operatorname{sign} v_1(t)}{|v_1(t)|^{\lambda_2}}$$

for all $t \geq t_0$.

Let us prove the second part of the lemma. Let the system (2.1) have a solution $v_1(t)$, $v_2(t)$ defined on $(0, t_0]$ and possessing the property $v_1(t) \cdot v_2(t) > 0$ for $t \leq t_0$. Consider the solution $u_1(t)$, $u_2(t)$ of the system (0.3) whose initial values satisfy the inequality

$$\frac{u_1(t_0) \operatorname{sign} u_2(t_0)}{|u_2(t_0)|^{\lambda_1}} \geq \frac{v_1(t_0) \operatorname{sign} v_2(t_0)}{|v_2(t_0)|^{\lambda_1}}.$$

It follows from (0.3) that

$$\Big(\frac{u_1 \operatorname{sign} u_2}{|u_2|^{\lambda_1}}\Big)' \le \lambda_1 a_2(t)\Big(\frac{|u_1|}{|u_2|^{\lambda_1}}\Big)^{1+\lambda_2} + a_1(t)$$

in some left half-neighbourhood of t_0, and the function appearing under the sign of the derivative does not decrease in the above-mentioned half-neighbourhood. According to the lemma on differential inequalities,

$$\frac{u_1(t)\operatorname{sign} u_2(t)}{|u_2(t)|^{\lambda_1}} \ge \frac{v_1(t)\operatorname{sign} v_2(t)}{|v_2(t)|^{\lambda_1}}$$

for all $t \in (0, t_0]$. For $\sigma = -1$, the statement of the lemma remains valid. □

Theorem 2.1. *Suppose that* $\lambda_1 \cdot \lambda_2 = 1$, (1.1) *holds and the condition*

$$\lim_{n\to+\infty}\Big(\int_0^{t_0} a_2(t)\rho_n^{1+\lambda_2}(t)dt + \int_{t_0}^{+\infty} a_1(t) r_n^{1+\lambda_1}(t)dt\Big) < +\infty \tag{2.2}$$

is satisfied, where

$$r_0(t) = \int_t^{+\infty} a_2(\tau)d\tau > 0, \quad r_n(t) =$$

$$= \lambda_2 \int_t^{+\infty} a_1(\tau) r_{n-1}^{1+\lambda_1}(\tau)d\tau + \int_t^{+\infty} a_2(\tau)d\tau \ \text{ for } \ t_0 \le t < +\infty, \tag{2.3}$$

$$\rho_0(t) = \int_0^t a_1(\tau)d\tau > 0, \ \rho_n(t) =$$

$$= \lambda_1 \int_0^t a_2(\tau)\rho_{n-1}^{1+\lambda_2}(\tau)d\tau + \int_0^t a_1(\tau)d\tau \ \text{ for } \ 0 < t \le t_0. \tag{2.4}$$

Then for the system (2.1) *to have no global solution with the property* $v_1(t) \cdot v_2(t) > 0$ *for* $0 < t < +\infty$*, it is necessary and sufficient that*

$$r(t_0) \cdot \rho^{\lambda_2}(t_0) > 1, \tag{2.5}$$

where $r(t) = \lim_{n\to+\infty} r_n(t)$ *and* $\rho(t) = \lim_{n\to+\infty} \rho_n(t)$.

Proof. The necessity. Let the system (2.1) have no solution with the property $v_1(t) \cdot v_2(t) > 0$ for $0 < t < +\infty$. It follows from (2.2) that there exist the limits

$$r(t) = \lim_{n\to+\infty} r_n(t) \ \text{ for } \ t_0 \le t < +\infty, \quad \rho(t) = \lim_{n\to+\infty} \rho_n(t) \ \text{ for } \ 0 < t \le t_0.$$

Clearly, $r(t)$ is a minimal solution of the equation

$$r' = -\lambda_2 a_1(t)|r|^{1+\lambda_1} - a_2(t) \tag{2.6}$$

defined on $[t_0, +\infty)$ (see [37], p. 65), and $\rho(t)$ is a minimal solution of the equation

$$\rho' = \lambda_1 a_2(t)|\rho|^{1+\lambda_2} + a_1(t) \tag{2.7}$$

defined on $(0, t_0]$; note that $r(t) > 0$ for $t_0 \le t < +\infty$ and $\rho(t) > 0$ for $0 < t \le t_0$. Consequently, $\rho^{-\lambda_2}(t)$ is a maximal solution of the equation (2.6) defined on $(0, t_0]$. As far as we have assumed that the system (2.1) has no solution defined on $(0, +\infty)$ and possessing the property $v_1(t) \cdot v_2(t) > 0$ for $0 < t < +\infty$, the equation (2.6) has no solution defined on $(0, +\infty)$. Therefore $\rho^{-\lambda_2}(t_0) < r(t_0)$, i.e. (2.5) holds.

The sufficiency. Let (2.5) hold. Let us show that the system (2.1) has no global solution possessing the property $v_1(t) \cdot v_2(t) > 0$ for $0 < t < +\infty$. Suppose that this is not the case, i.e., we suppose that the system (2.1) has a solution with the above-mentioned property. Then the function $\frac{v_2(t) \operatorname{sign} v_1(t)}{|v_1(t)|^{\lambda_2}}$ is a solution of the equation (2.6), defined on $(0, +\infty)$. As far as $r(t)$ is a minimal solution of the equation (2.6) on $[t_0, +\infty)$, we get

$$\frac{v_2(t_0) \operatorname{sign} v_1(t_0)}{|v_1(t_0)|^{\lambda_2}} \ge r(t_0),$$

and also, taking into account the fact that $\rho^{-\lambda_2}(t)$ is a maximal solution of the equation (2.6), defined on $(0, t_0]$, we have

$$\rho^{-\lambda_2}(t_0) \ge \frac{v_2(t_0) \operatorname{sign} v_1(t_0)}{|v_1(t_0)|^{\lambda_2}}.$$

Consequently, $r(t_0) \cdot \rho^{\lambda_2}(t_0) \le 1$, which contradicts (2.5). □

Theorem 2.2. *Suppose that* $\lambda_1 \cdot \lambda_2 = 1$, (1.8) *holds and the condition*

$$\lim_{n\to+\infty} \left(\int_0^{t_0} a_1(t)\rho_n^{1+\lambda_1}(t)dt + \int_{t_0}^{+\infty} a_2(t) r_n^{1+\lambda_2}(t)dt \right) < +\infty, \tag{2.8}$$

is satisfied, where

$$r_0(t) = \int_t^{+\infty} a_1(\tau)d\tau > 0, \;\; r_n(t) = \lambda_1 \int_t^{+\infty} a_2(\tau) r_{n-1}^{1+\lambda_2}(\tau)d\tau +$$
$$+ \int_t^{+\infty} a_1(\tau)d\tau \quad for \quad t_0 \le t < +\infty \tag{2.9}$$

and

$$\rho_0(t) = \int_0^{t} a_2(\tau)d\tau > 0, \;\; \rho_n(t) = \lambda_2 \int_0^{t} a_1(\tau) \rho_{n-1}^{1+\lambda_1}(\tau)d\tau +$$
$$+ \int_0^{t} a_2(\tau)d\tau \quad for \quad 0 < t \le t_0. \tag{2.10}$$

Then for the system (2.1) *to have no global solution with the property* $v_1(t) \cdot v_2(t) < 0$ *for* $0 < t < +\infty$, *it is necessary and sufficient that*

$$r(t_0) \cdot \rho^{\lambda_1}(t_0) > 1, \tag{2.11}$$

where $r(t) = \lim_{n\to+\infty} r_n(t)$ *and* $\rho(t) = \lim_{n\to+\infty} \rho_n(t)$.

Proof. We put $v_i = (-1)^{i-1} w_{3-i}$, $a_i = b_{3-i}$, $\lambda_i = \mu_{3-i}$ $(i = 1, 2)$ and make use of Theorem 2.1. □

Theorem 2.3. *Let* $\lambda_1 \cdot \lambda_2 = 1$ *and the conditions* (1.1), (2.2)–(2.4) *be satisfied. If the inequality* (2.5) *is fulfilled, then the system* (0.2) *has no global solution with the property* $u_1(t) \cdot u_2(t) > 0$ *for* $0 < t < +\infty$*; if the system* (0.3) *has no global solution with the property* $u_1(t) \cdot u_2(t) > 0$ *for* $0 < t < +\infty$, *then the inequality* (2.5) *holds.*

Proof. Let the inequality (2.5) hold. Make sure that the system (0.2) has no solution with the property $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$. Indeed, if we assume the contrary that the system (0.2) has a solution with the property $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$, then this by Lemma 2.1 will mean that the system (2.1) has a solution with the property $v_1(t) \cdot v_2(t) > 0$ for $0 < t < +\infty$. But this is impossible because by Theorem 2.1 the condition (2.5) is sufficient for the non-existence of solutions with the above-mentioned property.

Let the system (0.3) have no solution with the property $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$. Let us show that in this case the inequality (2.5) is satisfied. Indeed, if this is not the case, then by Theorem 2.1 system (2.1) has a solution with the property $v_1(t) \cdot v_2(t) > 0$ for $0 < t < +\infty$. But then by Lemma 2.1, the system (0.3) has also a solution with the property $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$, which contradicts our supposition. □

Theorem 2.4. *Let* $\lambda_1 \cdot \lambda_2 = 1$, *and let the conditions* (1.8), (2.8)–(2.10) *be satisfied. If the inequality* (2.11) *is fulfilled, then the system* (0.2) *has no global solution with the property* $u_1(t) \cdot u_2(t) < 0$ *for* $0 < t < +\infty$*; if the system* (0.3) *has no global solution with the property* $u_1(t) \cdot u_2(t) < 0$ *for* $0 < t < +\infty$, *then the inequality* (2.11) *holds.*

Proof. Put $u_i = (-1)^{i-1} v_{3-i}$, $a_i = b_{3-i}$, $\lambda_i = \mu_{3-i}$ $(i = 1, 2)$ and make use of Theorem 2.3. □

In conclusion, we present new criteria for the non-existence of conjugate points and an original characteristic of the principal solution (for the definition see [37], p. 95) of the system (2.1).

Theorem 2.5. *Let* $\lambda_1 \cdot \lambda_2 = 1$, *and let* (1.1), (2.2)–(2.4) *hold. Then for every solution* $v_1(t)$, $v_2(t)$ *of the system* (2.1) *the components* $v_i(t)$ $(i = 1, 2)$ *have on* $(0, +\infty)$ *not more than one zero if and only if*

$$r(t_0) \cdot \rho^{\lambda_2}(t_0) \le 1. \tag{2.12}$$

Proof. *The necessity.* Let for every solution $v_1(t)$, $v_2(t)$ of the system (2.1) the components $v_i(t)$ $(i = 1, 2)$ have on $(0, +\infty)$ not more than one zero. Let us show that (2.12) holds. Indeed, if we assume that this is not the case, then (2.5) is fulfilled. Let us take $\alpha \in (\rho^{-\lambda_2}(t_0), r(t_0))$ and consider the solution $x(t)$ of the equation (2.6) which is defined by the initial condition $x(t_0) = \alpha$. Since $r(t)$ is a minimal solution of the equation (2.6) defined on $[t_0, +\infty)$, and $\rho^{-\lambda_2}(t)$ is a maximal solution of the equation (2.6) defined on $(0, t_0]$, there exist numbers $t_1 \in (0, t_0)$ and $t_2 \in (t_0, +\infty)$ such that $\lim_{t\to t_1+} x(t) = +\infty$ and $\lim_{t\to t_2-} x(t) = -\infty$. The latter means that for the solution $v_1(t)$, $v_2(t)$ of the system (2.1) which corresponds to the solution $x(t)$ of the equation (2.6), the component $v_1(t)$ has two zeros t_1 and t_2, which contradicts our assumption.

Analogously, let us take $\beta \in (r^{-\lambda_1}(t_0), \rho(t_0))$ and consider the solution $y(t)$ of the equation (2.7) defined by the initial condition $y(t_0) = \beta$. Since $r^{-\lambda_1}(t)$ is a maximal solution of the equation (2.7) defined on $[t_0, +\infty)$, and $\rho(t)$ is a minimal solution of the same equation defined on $(0, t_0]$, there exist numbers $t_1 \in (0, t_0)$ and $t_2 \in (t_0, +\infty)$ such that $\lim_{t\to t_1+} y(t) = -\infty$ and $\lim_{t\to t_2-} y(t) = +\infty$. But this implies that for the solution $v_1(t)$, $v_2(t)$ of the system (2.1) which corresponds to the solution $y(t)$ of the equation (2.7), the component $v_2(t)$ has two zeros t_1 and t_2, which contradicts our assumption.

The sufficiency. Let (2.12) hold. Then the solution $x(t)$ of the equation (2.6) defined by the initial value $x(t_0) \in [r(t_0), \rho^{-\lambda_1}(t_0)]$ will be given on $(0, +\infty)$. If we assume that for some solution $v_1(t)$, $v_2(t)$ of the system (2.1) the component $v_1(t)$ has two different zeros $t_1 < t_2$ on the interval $(0, +\infty)$, then this will imply that the function $\frac{v_2(t)\operatorname{sign} v_1(t)}{|v_1(t)|^{\lambda_2}}$ is a solution of the equation (2.6) and

$$\lim_{t\to t_1+} \frac{v_2(t)\operatorname{sign} v_1(t)}{|v_1(t)|^{\lambda_2}} = +\infty, \quad \lim_{t\to t_2-} \frac{v_2(t)\operatorname{sign} v_1(t)}{|v_1(t)|^{\lambda_2}} = -\infty.$$

But then there exists a point $\tau \in (t_1, t_2)$ such that

$$x(\tau) = \frac{v_2(\tau)\operatorname{sign} v_1(\tau)}{|v_1(\tau)|^{\lambda_2}}.$$

This contradicts the uniqueness of a solution of the Cauchy problem for the equation (2.6).

Analogously, if we assume that for some solution of the system (2.1) the component $v_2(t)$ has two different zeros $t_1 < t_2$ on the interval $(0, +\infty)$, then the function $\frac{v_1(t)\operatorname{sign} v_2(t)}{|v_2(t)|^{\lambda_1}}$ is a solution of the equation (2.7) and

$$\lim_{t\to t_1+} \frac{v_1(t)\operatorname{sign} v_2(t)}{|v_2(t)|^{\lambda_1}} = -\infty, \quad \lim_{t\to t_2-} \frac{v_1(t)\operatorname{sign} v_2(t)}{|v_2(t)|^{\lambda_1}} = +\infty.$$

Therefore for the solution $y(t)$ of the equation (2.7) defined by the initial value $y(t_0) \in [\rho(t_0), r^{-\lambda_2}(t_0)]$ and given on $(0, +\infty)$ there exists a point

$\tau \in (t_1, t_2)$ such that

$$y(\tau) = \frac{v_1(\tau)\operatorname{sign} v_2(\tau)}{|v_2(\tau)|^{\lambda_1}}.$$

As before, we have a contradiction since the solution of the Cauchy problem for the equation (2.7) is unique. □

Remark. Note that (2.12) is a necessary and sufficient condition for any component of each solution of the system (2.1) to have not more than one zero on $(0, +\infty)$.

Theorem 2.6. *Let $\lambda_1 \cdot \lambda_2 = 1$ and* (1.8), (2.8)–(2.10) *hold. Then for every solution $v_1(t)$, $v_2(t)$ of the system* (2.1) *the components $v_i(t)$ $(i = 1, 2)$ have on $(0, +\infty)$ not more than one zero if and only if*

$$r(t_0) \cdot \rho^{\lambda_1}(t_0) \le 1.$$

Proof follows from Theorem 2.5 if we make the substitution $v_i = (-1)^{i-1} \times w_{3-i}$, $a_i = b_{3-i}$, $\lambda_i = \mu_{3-i}$ $(i = 1, 2)$.

Theorem 2.7. *Let $\lambda_1 \cdot \lambda_2 = 1$ and the conditions*

$$\int_{t_0}^{+\infty} a_1(t)dt = +\infty, \quad 0 \le \int_{t}^{+\infty} a_2(\tau)d\tau < +\infty,$$

$$r_0(t) = \int_{t}^{+\infty} a_2(\tau)d\tau, \quad r_n(t) = \lambda_2 \int_{t}^{+\infty} a_1(\tau) r_{n-1}^{1+\lambda_1}(\tau)d\tau + \int_{t}^{+\infty} a_2(\tau)d\tau,$$

$$\lim_{n\to+\infty} r_n(t) = r(t) \quad for \quad t \ge t_0$$

be satisfied (note that in this theorem $a_2(t)$ may be a function with alternating signs on $[t_0, +\infty)$). Then the solution $v_1(t)$, $v_2(t)$ of the system (2.1) *is principal if and only if the equality*

$$\frac{v_2(t)\operatorname{sign} v_1(t)}{|v_1(t)|^{\lambda_2}} = r(t) \quad for \quad t \ge t_0 \tag{2.13}$$

is fulfilled or, what comes to the same thing, that

$$v_1(t) = v_1(t_0) \exp \int_{t_0}^{t} a_1(\tau) r^{\lambda_1}(\tau)d\tau \quad for \quad t \ge t_0. \tag{2.14}$$

Proof. The function $r(t)$ is a minimal solution of the equation (2.6) given on $[t_0, +\infty)$, and hence the bounding solution of that equation [37]. Consequently, the solution $v_1(t)$, $v_2(t)$ of the system (2.1) is principal if and only if (2.13) holds. By virtue of (2.1), the equality (2.14) follows from (2.13). □

3. Systems of Inequalities in the Supercritical Case

In this section we present sufficient conditions for the non-existence of global solutions of constant sign of the system (0.2) in the case $\lambda_1 \cdot \lambda_2 > 1$.

Theorem 3.1. *Let $\lambda_1 \cdot \lambda_2 > 1$ and (1.1) be satisfied. Then for the system (0.2) to have no global solution with the property $u_1(t)u_2(t) > 0$ for $0 < t < +\infty$, it is sufficient that the equality*

$$\int_0^{t_0} a_2(t)\bigg(\int_0^t a_1(\tau)d\tau\bigg)^{\lambda_2} dt + \\ + \int_{t_0}^{+\infty} a_1(t)\bigg(\int_t^{+\infty} a_2(\tau)d\tau\bigg)^{\lambda_1} dt = +\infty \tag{3.1}$$

be fulfilled.

Proof. Assume the contrary, i.e., suppose that the first summand in (3.1) is equal to $+\infty$ and, nevertheless, $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$ for some solution $u_1(t)$, $u_2(t)$ of the system (0.2). Then (1.3) holds. Therefore

$$|u_2(t)|^{-\lambda_1\cdot\lambda_2}|u_2(t)|' \le -a_2(t)\bigg(\int_0^t a_1(\tau)d\tau\bigg)^{\lambda_2} \quad \text{for} \quad 0 < t \le t_0.$$

Integrating the last inequality from t to t_0 and passing to the limit as $t \to 0+$, we come to the contradiction.

Assume now that the second summand in (3.1) is equal to $+\infty$ and, nevertheless, $u_1(t) \cdot u_2(t) > 0$ for $0 < t < +\infty$ for some solution of the system (0.2). Then from (1.3) it follows that

$$|u_1(t)|^{-\lambda_1\cdot\lambda_2}|u_1(t)|' \ge a_1(t)\bigg(\int_t^{+\infty} a_2(\tau)d\tau\bigg)^{\lambda_1} \quad \text{for} \quad t \ge t_0.$$

Integrating the last inequality from t_0 to t and passing to limit as $t \to +\infty$, we arrive at the contradiction. □

Theorem 3.2. *Let $\lambda_1 \cdot \lambda_2 > 1$ and (1.8) be satisfied. Then for the system (0.2) to have no global solutions with the property $u_1(t) \cdot u_2(t) < 0$ for $0 < t < +\infty$, it is sufficient that*

$$\int_0^{t_0} a_1(t)\bigg(\int_0^t a_2(\tau)d\tau\bigg)^{\lambda_1} dt + \int_{t_0}^{+\infty} a_2(t)\bigg(\int_t^{+\infty} a_1(\tau)\,d\tau\bigg)^{\lambda_2} dt = +\infty. \tag{3.2}$$

Proof. Put $u_i = (-1)^{i-1}v_{3-i}$, $a_i = b_{3-i}$, $\lambda_i = \mu_{3-i}$ $(i = 1, 2)$ and make use of Theorem 3.1. □

We do not know whether the conditions (3.1) or (3.2) are necessary for the system (0.3) to have no global solution of constant sign on $(0, +\infty)$.

Note that Theorems 1.2 and 1.5 hold true for $\lambda_1 \cdot \lambda_2 > 1$ as well. To see that this is so, we have to repeat the proofs of these theorems by putting $\frac{1}{\lambda_1} < \gamma < \lambda_2$ and choosing $|u_{10}|$ sufficiently small.

It should also be noted that the conditions (1.2) or (1.9) in the case $\lambda_1 \cdot \lambda_2 > 1$ are not sufficient for the system (0.2) to have no global solutions of constant sign on $(0, +\infty)$.

As an example, consider the system

$$u_1' = t^{-2} u_2, \quad u_2' = -3t^2 |u_1|^5 \operatorname{sign} u_1$$

for which the conditions (1.8) and (1.9) are fulfilled. Nevertheless, this system has a global solution of constant sign on $(0, +\infty)$:

$$u_1(t) = (1+t^2)^{-\frac{1}{2}}, \quad u_2(t) = -t^3 (1+t^2)^{-\frac{3}{2}}.$$

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