

# Journal of Inequalities in Pure and Applied Mathematics

ON AN OPEN PROBLEM OF BAI-NI GUO AND FENG QI

ŽIVORAD TOMOVSKI AND KOSTADIN TRENČEVSKI

Institute of Mathematics,  
St. Cyril and Methodius University,  
P. O. Box 162, 1000 Skopje,  
MACEDONIA

*E-Mail:* [tomovski@iunona.pmf.ukim.edu.mk](mailto:tomovski@iunona.pmf.ukim.edu.mk)

*E-Mail:* [kostatre@iunona.pmf.ukim.edu.mk](mailto:kostatre@iunona.pmf.ukim.edu.mk)

©2000 Victoria University  
ISSN (electronic): 1443-5756  
101-02



---

volume 4, issue 2, article 29,  
2003.

*Received 17 September, 2002;  
accepted 28 March, 2003.*

*Communicated by: F. Qi*

---

[Abstract](#)

[Contents](#)



[Home Page](#)

[Go Back](#)

[Close](#)

[Quit](#)

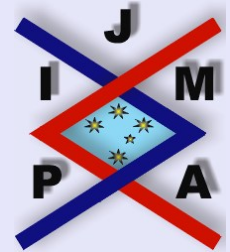
## Abstract

In this paper, an open problem posed respectively by B.-N. Guo and F. Qi in [4, 6, 7] is partially solved: an integral expression and a new double inequality of the generalized Mathieu's series  $\sum_{n=1}^{\infty} \frac{2n}{(n^2+a^2)^{p+1}}$  are established by using some properties of gamma function and Fourier transform inequalities, where  $a > 0, p \in \mathbb{N}$ .

*2000 Mathematics Subject Classification:* Primary 26D15, 33E20; Secondary 40A30  
*Key words:* Integral expression, Inequality, Mathieu's series, Gamma function, Fourier transform inequality.

## Contents

1	Introduction .....	3
2	The Integral Expressions .....	6
3	The Inequality .....	13



---

On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

---

Title Page

Contents



Go Back

Close

Quit

Page 2 of 16

# 1. Introduction

It is well-known that the following

$$(1.1) \quad S(a, 1) \triangleq \sum_{n=1}^{\infty} \frac{2n}{(n^2 + a^2)^2}, \quad a > 0$$

is called the Mathieu's series. The integral expression of Mathieu's series (1.1) was given in [3] as follows

$$(1.2) \quad S(a, 1) = \frac{1}{a} \int_0^{\infty} \frac{x \sin ax}{e^x - 1} dx.$$

The Mathieu' series (1.1) and related inequalities have been studied by many mathematicians for more than a century and there has been a vast amount of literature. Please refer to [4, 6, 7] and the references therein.

The following Fourier transform inequalities can be found in [2, pp. 89–90]: If  $f \in L([0, \infty))$  with  $\lim_{t \rightarrow \infty} f(t) = 0$ , then

$$(1.3) \quad \sum_{k=1}^{\infty} (-1)^k f(k\pi) < \int_0^{\infty} f(t) \cos t dt < \sum_{k=0}^{\infty} (-1)^k f(k\pi),$$

$$(1.4) \quad \sum_{k=0}^{\infty} (-1)^k f\left(\left(k + \frac{1}{2}\right)\pi\right) < \int_0^{\infty} f(t) \sin t dt < f(0) + \sum_{k=0}^{\infty} (-1)^k f\left(\left(k + \frac{1}{2}\right)\pi\right).$$



---

On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

---

Title Page

Contents



Go Back

Close

Quit

Page 3 of 16

By using the integral expression (1.2) and Fourier transform inequality (1.4), Bai-Ni Guo established in [4] the following inequalities for Mathieu's series (1.1).

**Theorem A ([4]).** *for  $a > 0$ , then*

$$(1.5) \quad \frac{\pi}{a^3} \sum_{k=0}^{\infty} \frac{(-1)^k \left(k + \frac{1}{2}\right)}{\exp \left[\left(k + \frac{1}{2}\right) \frac{\pi}{a}\right] - 1} < S(a, 1) < \frac{1}{a^2} \left(1 + \frac{\pi}{a} \sum_{k=0}^{\infty} \frac{(-1)^k \left(k + \frac{1}{2}\right)}{\exp \left[\left(k + \frac{1}{2}\right) \frac{\pi}{a}\right] - 1}\right).$$

At the end of the short note [4], B.-N. Guo proposed an open problem: *Let*

$$(1.6) \quad S(a, p) = \sum_{n=1}^{\infty} \frac{2n}{(n^2 + a^2)^{p+1}},$$

*where  $p > 0$  and  $a > 0$ . Can one establish an integral expression of  $S(a, p)$ ?*

Soon after, Feng Qi further proposed in [6, 7] a similar open problem: *Let*

$$(1.7) \quad S(r, t, \alpha) = \sum_{n=1}^{\infty} \frac{2n^{\alpha/2}}{(n^{\alpha} + r^2)^{t+1}}$$

*for  $t > 0$ ,  $r > 0$  and  $\alpha > 0$ . Can one obtain an integral expression of  $S(r, t, \alpha)$ ? Give some sharp inequalities for the series  $S(r, t, \alpha)$ .*

In this paper, using the well-known formula

$$(1.8) \quad \frac{1}{t^{a+1}} = \frac{1}{\Gamma(a+1)} \int_0^{\infty} x^a e^{-xt} dx,$$



On an Open Problem of Bai-Ni Guo and Feng Qi

Živorad Tomovski and Kostadin Trenčevski

Title Page

Contents



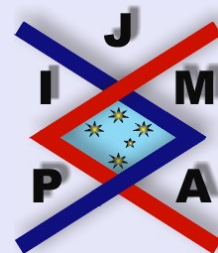
Go Back

Close

Quit

Page 4 of 16

which can be deduced from the definition of a gamma function, and Fourier transform inequalities (1.3) and (1.4), we will establish an integral expression and a new double inequality of the generalized Mathieu's series (1.6) for  $p \in \mathbb{N}$ , the set of all positive integers. Our results partially solve the open problems by B.-N. Guo and F. Qi in [4] and [6, 7] mentioned above.



---

On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

---

Title Page

Contents



Go Back

Close

Quit

Page 5 of 16

## 2. The Integral Expressions

One of our main results is to establish an integral expression of  $S(a, p)$  for  $a > 0$  and  $p \in \mathbb{N}$ , which can be stated as the following.

**Theorem 2.1.** *Let  $a > 0$  and  $p \in \mathbb{N}$ . Then we have*

$$\begin{aligned}
 (2.1) \quad S(a, p) &= \sum_{n=1}^{\infty} \frac{2n}{(n^2 + a^2)^{p+1}} \\
 &= \frac{2}{(2a)^{2p}} \int_0^{\infty} \frac{t^p \cos\left(\frac{p\pi}{2} - at\right)}{e^t - 1} dt \\
 &\quad - 2 \sum_{k=2}^p \frac{(k-1)(2a)^{k-2p-1}}{k!(p-k+1)} \binom{-(p+1)}{p-k} \\
 &\quad \times \int_0^{\infty} \frac{t^k \cos\left[\frac{\pi}{2}(2p-k+1) - at\right]}{e^t - 1} dt.
 \end{aligned}$$

*Proof.* Let  $a_n = \frac{2n}{(n^2+a^2)^{p+1}}$ , where  $a > 0$  and  $p \in \mathbb{N}$ . Then

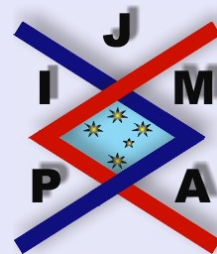
$$a_n = \frac{n + ai + n - ai}{(n + ai)^{p+1}(n - ai)^{p+1}} = b_n + c_n,$$

where

$$b_n = \frac{1}{(n + ai)^p(n - ai)^{p+1}}, \quad c_n = \frac{1}{(n + ai)^{p+1}(n - ai)^p}.$$

By putting  $n + ai = x$ , we obtain

$$b_n = \frac{1}{x^p(x - 2ai)^{p+1}} = \sum_{k=1}^p \frac{A_k}{x^k} + \sum_{k=1}^{p+1} \frac{B_k}{(x - 2ai)^k},$$



On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

Page 6 of 16

where  $A_k$  and  $B_k$  are constants.

Applying the binomial expansion, we get

$$\begin{aligned} (x - 2ai)^{-(p+1)} &= (-2ai)^{-(p+1)} \left(1 - \frac{x}{2ai}\right)^{-(p+1)} \\ &= (-2ai)^{-(p+1)} \sum_{k=0}^{\infty} \binom{-(p+1)}{k} \left(-\frac{x}{2ai}\right)^k \\ &= (-2ai)^{-(p+1)} \sum_{k=0}^{\infty} \frac{1}{(-2ai)^k} \binom{-(p+1)}{k} x^k \end{aligned}$$

for  $|x| < 2a$ , i.e.

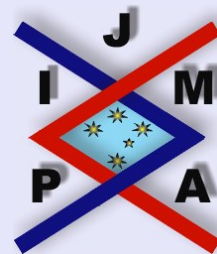
$$\begin{aligned} b_n &\sim (-2ai)^{-(p+1)} \sum_{k=0}^{p-1} \frac{1}{(-2ai)^k} \binom{-(p+1)}{k} x^{k-p} \\ &= (-2ai)^{-(p+1)} \sum_{k=1}^p \frac{1}{(-2ai)^{p-k}} \binom{-(p+1)}{p-k} \frac{1}{x^k}. \end{aligned}$$

Hence,

$$A_k = (-2ai)^{-2p+k-1} \binom{-(p+1)}{p-k}, \quad k = 1, 2, \dots, p.$$

Further, by putting  $n - ai = y$  in  $b_n$ , we obtain

$$\begin{aligned} b_n &= \frac{1}{y^{p+1}(y + 2ai)^p} \\ &\sim \frac{(2ai)^{-p}}{y^{p+1}} \sum_{k=0}^p \binom{-p}{k} \left(\frac{y}{2ai}\right)^k \end{aligned}$$



On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

Page 7 of 16

$$= (2ai)^{-p} \sum_{k=1}^{p+1} \binom{-p}{p-k+1} \frac{1}{(2ai)^{p-k+1}} \frac{1}{y^k}.$$

Hence,

$$B_k = (2ai)^{-2p+k-1} \binom{-p}{p-k+1}, \quad k = 1, 2, \dots, p+1.$$

Analogously,

$$c_n = \frac{1}{x^{p+1}(x-2ai)^p} = \sum_{k=1}^{p+1} \frac{C_k}{x^k} + \sum_{k=1}^p \frac{D_k}{(x-2ai)^k}.$$

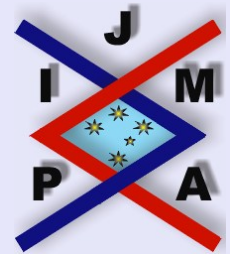
Applying the same technique, for coefficients  $C_k, D_k$  we obtain

$$C_k = (-2ai)^{-2p+k-1} \binom{-p}{p-k+1}, \quad k = 1, 2, \dots, p+1,$$

$$D_k = (2ai)^{-2p+k-1} \binom{-(p+1)}{p-k}, \quad k = 1, 2, \dots, p.$$

Thus

$$\begin{aligned} a_n &= \frac{(2ai)^{-p}}{(n-ai)^{p+1}} + \frac{(-2ai)^{-p}}{(n+ai)^{p+1}} \\ &+ \sum_{k=1}^p (2ai)^{-2p+k-1} \left[ \binom{-p}{p-k+1} + \binom{-(p+1)}{p-k} \right] \frac{1}{(n-ai)^k} \\ &+ \sum_{k=1}^p (-2ai)^{-2p+k-1} \left[ \binom{-p}{p-k+1} + \binom{-(p+1)}{p-k} \right] \frac{1}{(n+ai)^k} \end{aligned}$$



On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

Page 8 of 16



$$\begin{aligned}
&= \frac{(2ai)^{-p}}{p!} \int_0^\infty t^p e^{-(n-ai)t} dt + \frac{(-2ai)^{-p}}{p!} \int_0^\infty t^p e^{-(n+ai)t} dt \\
&+ \sum_{k=1}^p (2ai)^{-2p+k-1} \left[ \binom{-p}{p-k+1} + \binom{-(p+1)}{p-k} \right] \frac{1}{k!} \int_0^\infty t^k e^{-(n-ia)t} dt \\
&+ \sum_{k=1}^p (-2ai)^{-2p+k-1} \left[ \binom{-p}{p-k+1} + \binom{-(p+1)}{p-k} \right] \frac{1}{k!} \int_0^\infty t^k e^{-(n+ia)t} dt.
\end{aligned}$$

Since

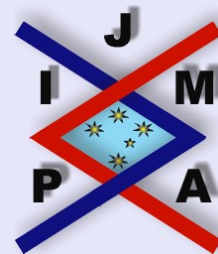
$$\sum_{n=1}^{\infty} e^{-nt} = \frac{1}{e^t - 1}$$

and

$$\binom{-p}{p-k+1} + \binom{-(p+1)}{p-k} = \binom{-(p+1)}{p-k} \frac{1-k}{p-k+1},$$

we obtain

$$\begin{aligned}
\sum_{n=1}^{\infty} a_n &= \frac{(2ai)^{-p}}{p!} \int_0^\infty \frac{t^p}{e^t - 1} e^{iat} dt + \frac{(-2ai)^{-p}}{p!} \int_0^\infty \frac{t^p}{e^t - 1} e^{-iat} dt \\
&+ \sum_{k=1}^p (2ai)^{-2p+k-1} \binom{-(p+1)}{p-k} \frac{1-k}{k!(p-k+1)} \int_0^\infty \frac{t^k}{e^t - 1} e^{iat} dt \\
&+ \sum_{k=1}^p (-2ai)^{-2p+k-1} \binom{-(p+1)}{p-k} \frac{1-k}{k!(p-k+1)} \int_0^\infty \frac{t^k}{e^t - 1} e^{-iat} dt.
\end{aligned}$$



On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

Page 9 of 16

Let  $z = (2ai)^{-p}e^{iat}$  and  $u = (2ai)^{-2p+k-1}e^{iat}$ . Then

$$\begin{aligned} z + \bar{z} &= \frac{2}{(2a)^p} \operatorname{Re} \left[ \left( \cos \frac{p\pi}{2} - i \sin \frac{p\pi}{2} \right) (\cos at + i \sin at) \right] \\ &= \frac{2}{(2a)^p} \cos \left( \frac{p\pi}{2} - at \right) \end{aligned}$$

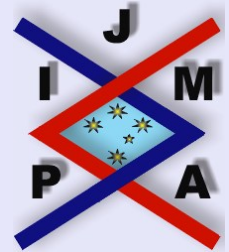
and

$$\begin{aligned} u + \bar{u} &= \frac{2 \operatorname{Re} \left\{ \left[ \cos \frac{(2p-k+1)\pi}{2} - i \sin \frac{(2p-k+1)\pi}{2} \right] (\cos at + i \sin at) \right\}}{(2a)^{2p+1-k}} \\ &= \frac{2}{(2a)^{2p-k+1}} \cos \left[ \frac{(2p-k+1)\pi}{2} - at \right]. \end{aligned}$$

Finally, we get

$$\begin{aligned} S(a, p) &= \sum_{n=1}^{\infty} a_n \\ &= \frac{2(2a)^{-p}}{p!} \int_0^{\infty} \frac{t^p}{e^t - 1} \cos \left( \frac{p\pi}{2} - at \right) dt \\ &\quad + \sum_{k=1}^p \frac{2}{(2a)^{2p-k+1}} \binom{-(p+1)}{p-k} \frac{1-k}{k!(p-k+1)} \\ &\quad \times \int_0^{\infty} \frac{t^k}{e^t - 1} \cos \left[ (2p-k+1) \frac{\pi}{2} - at \right] dt. \end{aligned}$$

The proof is complete. □



On an Open Problem of Bai-Ni Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

Page 10 of 16

**Remark 2.1.** Using the well-known formula for the polygamma function (see [1])

$$\psi^{(n)}(z) = (-1)^{n+1} \int_0^\infty \frac{t^n e^{-zt}}{1 - e^{-t}} dt \quad (n = 1, 2, 3, \dots, \operatorname{Re} z > 0),$$

where  $\psi(z) = \frac{d \ln \Gamma(z)}{dz}$ , we obtain

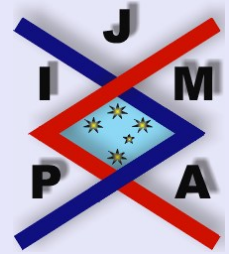
$$\begin{aligned} & \int_0^\infty \frac{t^p}{e^t - 1} \cos\left(\frac{p\pi}{2} - at\right) dt \\ &= \frac{e^{i\frac{p\pi}{2}}}{2} \int_0^\infty \frac{t^p e^{-t(1+ia)}}{1 - e^{-t}} dt + \frac{e^{-i\frac{p\pi}{2}}}{2} \int_0^\infty \frac{t^p e^{-(1-ia)t}}{1 - e^{-t}} dt \\ &= \frac{e^{i\frac{p\pi}{2}}}{2} \psi^{(p)}(1 + ia) + \frac{e^{-i\frac{p\pi}{2}}}{2} \psi^{(p)}(1 - ia) \\ &= \operatorname{Re}[e^{ip\pi/2} \psi^{(p)}(1 + ia)]. \end{aligned}$$

Analogously,

$$\int_0^\infty \frac{t^p}{e^t - 1} \cos\left[(2p - k + 1)\frac{\pi}{2} - at\right] dt = \operatorname{Re} \left[ e^{i(2p-k+1)\pi/2} \psi^{(p)}(1 + ia) \right].$$

So for  $S(a, p)$  we have the following expression

$$\begin{aligned} (2.2) \quad S(a, p) &= \frac{2}{p!(2a)^p} \operatorname{Re} \left[ e^{ip\pi/2} \psi^{(p)}(1 + ia) \right] \\ &+ \sum_{k=1}^p \frac{2(1-k)}{(2a)^{2p-k+1} k! (p-k+1)} \binom{-(p+1)}{p-k} \\ &\quad \times \operatorname{Re} \left[ e^{i(2p-k+1)\pi/2} \psi^{(p)}(1 + ia) \right]. \end{aligned}$$



On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

Page 11 of 16

**Remark 2.2.** If  $p > 0$ ,  $p \in \mathbb{R}$ , then we have

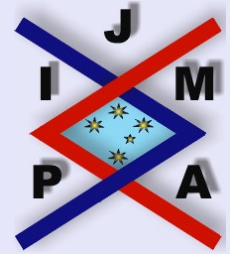
$$\frac{2n}{(n^2 + a^2)^{p+1}} = \frac{2}{\Gamma(p+1)} \int_0^\infty t^p n e^{-(n^2+a^2)t} dt.$$

Using the Cauchy integration test, we obtain that  $\sum_{n=1}^\infty n e^{-n^2 t}$  is convergent for all  $t > 0$ , i.e.  $f(t) = \sum_{n=1}^\infty n e^{-n^2 t}$ . Thus

$$\begin{aligned} (2.3) \quad S(a, p) &= \frac{2}{\Gamma(p+1)} \int_0^\infty t^p e^{-a^2 t} \left( \sum_{n=1}^\infty n e^{-n^2 t} \right) dt \\ &= \frac{2}{\Gamma(p+1)} \int_0^\infty t^p e^{-a^2 t} f(t) dt. \end{aligned}$$

**Remark 2.3.** In addition we set an open problem for summing up the functional series

$$\sum_{n=1}^\infty n e^{-n^2 t} \text{ for all } t > 0.$$




---

On an Open Problem of Bai-Ni Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

---

Title Page

Contents



Go Back

Close

Quit

Page 12 of 16

### 3. The Inequality

Another one of our main results is to obtain a double inequality of  $S(a, p)$  for  $a > 0$  and  $p \in \mathbb{N}$  by using Fourier transform inequalities (1.3) and (1.4).

**Theorem 3.1.** For  $a > 0$  and  $p \in \mathbb{N}$ , we have

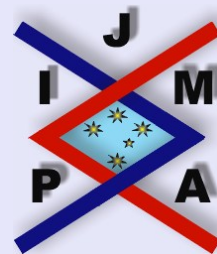
$$(3.1) \quad |S(a, p)| \leq \frac{2}{a^{p+1}(2a)^{pp}!} \left[ \sum_{k=0}^{\infty} \frac{(-1)^k (k\pi)^p}{\exp \frac{k\pi}{a} - 1} + \sum_{k=0}^{\infty} (-1)^k \frac{((k + \frac{1}{2})\pi)^p}{\exp((k + \frac{1}{2})\frac{\pi}{a}) - 1} \right] + \sum_{k=1}^p \frac{2(k-1)(2a)^{-2p+k-1}}{k!(p-k+1)a^{k+1}} \left| \binom{-(p+1)}{p-k} \right| \times \left[ \sum_{j=0}^{\infty} \frac{(-1)^j (j\pi)^k}{\exp \frac{j\pi}{a} - 1} + \sum_{j=0}^{\infty} \frac{(-1)^j [(j + \frac{1}{2})\pi]^k}{\exp [(j + \frac{1}{2})\frac{\pi}{a}] - 1} \right].$$

*Proof.* For all  $k = 1, 2, \dots, p$ , let

$$I(a, k) = \int_0^{\infty} \frac{t^k \cos at}{e^t - 1} dt \quad \text{and} \quad J(a, k) = \int_0^{\infty} \frac{t^k \sin at}{e^t - 1} dt.$$

Then

$$S(a, p) = \frac{2}{(2a)^{pp}!} \left[ I(a, p) \cos \frac{p\pi}{2} + J(a, p) \sin \frac{p\pi}{2} \right] + \sum_{k=1}^p \frac{2(1-k)(2a)^{k-2p-1}}{k!(p-k+1)} \binom{-(p+1)}{p-k}$$



On an Open Problem of Bai-Ni Guo and Feng Qi

Živorad Tomovski and Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

Page 13 of 16

$$\times \left[ I(a, k) \cos \frac{(2p - k + 1)\pi}{2} + J(a, k) \sin \frac{(2p - k + 1)\pi}{2} \right].$$

Since

$$I(a, k) = \frac{1}{a^{k+1}} \int_0^\infty \frac{t^k \cos t}{e^{t/a} - 1} dt,$$

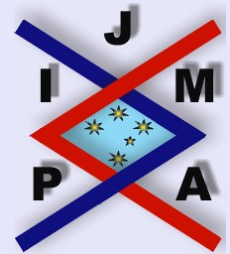
$$J(a, k) = \frac{1}{a^{k+1}} \int_0^\infty \frac{t^k \sin t}{e^{t/a} - 1} dt$$

for fixed  $a > 0$  and  $k = 1, 2, \dots, p$ , and

$$f_k \in L([0, \infty)), \quad \lim_{t \rightarrow \infty} f_k(t) = \lim_{t \rightarrow \infty} \frac{t^k}{e^{t/a} - 1} = 0, \quad \lim_{t \rightarrow 0} f_k(t) = 0,$$

where  $f_k(t) = \frac{t^k}{e^{t/a} - 1}$ , then, using inequalities (1.3) and (1.4), we have

$$\begin{aligned} |S(a, p)| &\leq \frac{2}{p!(2a)^p} [I(a, p) + J(a, p)] \\ &+ \sum_{k=1}^p \frac{2(k-1)}{k!(p-k+1)(2a)^{2p-k+1}} \left| \binom{-(p+1)}{p-k} \right| [I(a, k) + J(a, k)] \\ &\leq \frac{2(2a)^{-p}}{p!a^{p+1}} \left[ \sum_{k=0}^\infty \frac{(-1)^k (k\pi)^p}{\exp \frac{k\pi}{a} - 1} + \sum_{k=0}^\infty \frac{(-1)^k \left[ (k + \frac{1}{2}) \pi \right]^p}{\exp \left[ (k + \frac{1}{2}) \frac{\pi}{a} \right] - 1} \right] \\ &+ \sum_{k=1}^p \frac{2(k-1)(2a)^{-2p+k-1}}{k!a^{k+1}(p-k+1)} \left| \binom{-(p+1)}{p-k} \right| \end{aligned}$$



On an Open Problem of Bai-Ni  
Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

Title Page

Contents



Go Back

Close

Quit

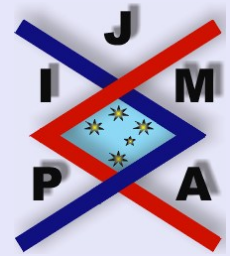
Page 14 of 16

$$\times \left[ \sum_{j=0}^{\infty} \frac{(-1)^j (j\pi)^k}{\exp \frac{j\pi}{a} - 1} + \sum_{j=0}^{\infty} \frac{(-1)^j \left[ \left( j + \frac{1}{2} \right) \pi \right]^k}{\exp \left[ \left( j + \frac{1}{2} \right) \frac{\pi}{a} \right] - 1} \right].$$

The proof is complete. □

## Acknowledgements

The authors would like to thank Professor Feng Qi and the anonymous referee for some valuable suggestions which have improved the final version of this paper.




---

On an Open Problem of Bai-Ni Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

---

Title Page

Contents



Go Back

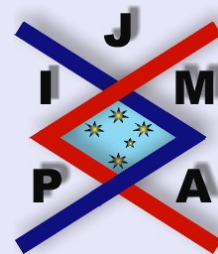
Close

Quit

Page 15 of 16

## References

- [1] M. ABRAMOWITZ AND I.A. STEGUN (Eds), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series **55**, 4th printing, Washington, 1965.
- [2] P.S. BULLEN, *A Dictionary of Inequalities*, Pitman Monographs and Surveys in Pure and Applied Mathematics **97**, Addison Wesley Longman Limited, 1998.
- [3] O.E. EMERSLEBEN, Über die reihe  $\sum_{k=1}^{\infty} k(k^2 + c^2)^{-2}$ , *Math. Ann.*, **125** (1952), 165–171.
- [4] B.-N. GUO, Note on Mathieu's inequality, *RGMA Res. Rep. Coll.*, **3**(3) (2000), Art. 5, 389–392. Available online at <http://rgmia.vu.edu.au/v3n3.html>.
- [5] E. MATHIEU, *Traité de physique mathématique, VI–VII: Théorie de l'élasticité des corps solides*, Gauthier-Villars, Paris, 1890.
- [6] F. QI, *Inequalities for Mathieu's series*, *RGMA Res. Rep. Coll.*, **4**(2) (2001), Art. 3, 187–193. Available online at <http://rgmia.vu.edu.au/v4n2.html>.
- [7] F. QI AND Ch.-P. CHEN, Notes on double inequalities of Mathieu's series, *Internat. J. Pure Appl. Math.*, (2003), in press.



---

On an Open Problem of Bai-Ni Guo and Feng Qi

Živorad Tomovski and  
Kostadin Trenčevski

---

Title Page

Contents



Go Back

Close

Quit

Page 16 of 16