

Journal of Inequalities in Pure and Applied Mathematics

INEQUALITIES RELATED TO THE CHEBYCHEV FUNCTIONAL INVOLVING INTEGRALS OVER DIFFERENT INTERVALS

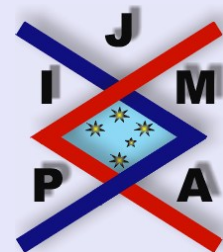
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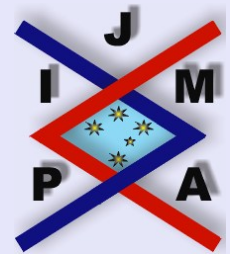
A generalised Chebychev functional involving integral means of functions over different intervals is investigated. Bounds are obtained for which the functions are assumed to be of Hölder type. A weighted generalised Chebychev functional is also introduced and bounds are obtained in terms of weighted Grüss, Chebychev and Lupaş inequalities.

2000 Mathematics Subject Classification: 26D15, 26D20, 26D99.

Key words: Grüss, Chebychev and Lupaş inequalities, Hölder.

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1. Introduction

For two measurable functions $f, g : [a, b] \rightarrow \mathbb{R}$, define the functional, which is known in the literature as Chebychev's functional

$$(1.1) \quad T(f, g; a, b) := \frac{1}{b-a} \int_a^b f(x) g(x) dx - \frac{1}{(b-a)^2} \int_a^b f(x) dx \cdot \int_a^b g(x) dx,$$

provided that the involved integrals exist.

The following inequality is well known as the Grüss inequality [9]

$$(1.2) \quad |T(f, g; a, b)| \leq \frac{1}{4} (M - m) (N - n),$$

provided that $m \leq f \leq M$ and $n \leq g \leq N$ a.e. on $[a, b]$, where m, M, n, N are real numbers. The constant $\frac{1}{4}$ in (1.2) is the best possible.

Another inequality of this type is due to Chebychev (see for example [1, p. 207]). Namely, if f, g are absolutely continuous on $[a, b]$ and $f', g' \in L_\infty[a, b]$ and $\|f'\|_\infty := \text{ess sup}_{t \in [a, b]} |f'(t)|$, then

$$(1.3) \quad |T(f, g; a, b)| \leq \frac{1}{12} \|f'\|_\infty \|g'\|_\infty (b - a)^2$$

and the constant $\frac{1}{12}$ is the best possible.

Finally, let us recall a result by Lupaş (see for example [1, p. 210]), which states that:

$$(1.4) \quad |T(f, g; a, b)| \leq \frac{1}{\pi^2} \|f'\|_2 \|g'\|_2 (b - a),$$



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provided f, g are absolutely continuous and $f', g' \in L_2[a, b]$. The constant $\frac{1}{\pi^2}$ is the best possible here.

For other Grüss type inequalities, see the books [1] and [2], and the papers [3]-[10], where further references are given.

Recently, Cerone and Dragomir [11] have pointed out generalizations of the above results for integrals defined on two different intervals $[a, b]$ and $[c, d]$.

Define the functional (generalised Chebychev functional)

$$(1.5) \quad T(f, g; a, b, c, d) := M(fg; a, b) + M(fg; c, d) \\ - M(f; a, b)M(g; c, d) - M(f; c, d)M(g; a, b),$$

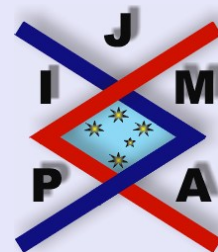
where the integral mean is defined by

$$(1.6) \quad M(f; a, b) := \frac{1}{b-a} \int_a^b f(x) dx.$$

Cerone and Dragomir [11] proved the following result.

Theorem 1.1. *Let $f, g : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be measurable on I and the intervals $[a, b], [c, d] \subset I$. Assume that the integrals involved in (2.12) exist. Then we have the inequality*

$$(1.7) \quad |T(f, g; a, b, c, d)| \\ \leq [T(f; a, b) + T(f; c, d) + (M(f; a, b) - M(f; c, d))^2]^{\frac{1}{2}} \\ \times [T(g; a, b) + T(g; c, d) + (M(g; a, b) - M(g; c, d))^2]^{\frac{1}{2}},$$



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where

$$(1.8) \quad T(f; a, b) := \frac{1}{b-a} \int_a^b f^2(x) dx - \left(\frac{1}{b-a} \int_a^b f(x) dx \right)^2$$

and the integrals involved in the right membership of (2.3) exist.

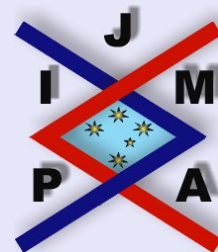
They used a generalisation of the classical identity due to Korkine namely,

$$(1.9) \quad T(f, g; a, b, c, d) \\ = \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d (f(x) - f(y))(g(x) - g(y)) dy dx$$

and the fact that

$$(1.10) \quad T(f, f; a, b, c, d) \\ = T(f; a, b) + T(f; c, d) + (M(f; a, b) - M(f; c, d))^2.$$

In the current article, bounds are obtained for the generalised Chebychev functional (1.5) assuming that f and g are of Hölder type. The special case for which f and g are Lipschitzian is also investigated. A weighted generalised Chebychev functional is treated in Section 3 involving weighted means of functions over different intervals. Grüss, Chebychev and Lupaş results are utilised to obtain bounds for such a functional.



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2. The Results for Functions of Hölder Type

The following lemma will prove to be useful in the subsequent work.

Lemma 2.1. Let $a, b, c, d \in \mathbb{R}$ with $a < b$ and $c < d$. Define

$$(2.1) \quad C_{\theta}(a, b, c, d) := \int_a^b \int_c^d |x - y|^{\theta} dy dx, \quad \theta \geq 0,$$

then

$$(2.2) \quad (\theta + 1)(\theta + 2)C_{\theta}(a, b, c, d) \\ = |b - c|^{\theta+2} - |b - d|^{\theta+2} + |d - a|^{\theta+2} - |c - a|^{\theta+2}.$$

Proof. Let $[\]$ denote the order in which a, b, c, d appear on the real number line. There are six possibilities to consider since we are given that $a < b$ and $c < d$.

Firstly, consider the situation $c = a$ and $d = b$. Then

$$(2.3) \quad D_{\theta}(a, b) = C_{\theta}(a, b, a, b) \\ = \int_a^b \int_a^b |x - y|^{\theta} dy dx, \quad \theta \geq 0 \\ = \int_a^b \left[\int_a^x (x - y)^{\theta} dy + \int_x^b (y - x)^{\theta} dy \right] dx \\ = \frac{1}{\theta + 1} \int_a^b \left[(x - a)^{\theta+1} + (b - x)^{\theta+1} \right] dx$$

and so

$$(2.4) \quad (\theta + 1)(\theta + 2)D_{\theta}(a, b) = 2(b - a)^{\theta+2}.$$



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Now, taking the six possibilities in turn, we have:

(i) For the ordering $[c, d, a, b]$, $y < x$ giving for $C_\theta(a, b, c, d)$

$$\begin{aligned}
 (2.5) \quad I_\theta(a, b, c, d) &:= \int_a^b \int_c^d (x - y)^\theta dy dx \\
 &= \int_a^b \left[\int_c^x (x - y)^\theta dy + \int_x^d (y - x)^\theta dy \right] dx \\
 &= \frac{1}{\theta + 1} \int_a^b \left[(x - c)^{\theta+1} - (x - d)^{\theta+1} \right] dx
 \end{aligned}$$

and so

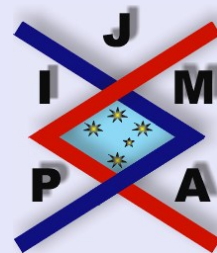
$$\begin{aligned}
 (2.6) \quad (\theta + 1)(\theta + 2) I_\theta(a, b, c, d) \\
 &= (b - c)^{\theta+2} - (a - c)^{\theta+2} + (a - d)^{\theta+2} - (b - d)^{\theta+2} \\
 &= (\theta + 1)(\theta + 2) C_\theta(a, b, c, d), \quad [c, d, a, b].
 \end{aligned}$$

(ii) For the ordering $[c, a, d, b]$ we have

$$\begin{aligned}
 C_\theta(a, b, c, d) \\
 &= \int_a^b \int_c^a (x - y)^\theta dy dx + \int_a^d \int_a^d |x - y|^\theta dy dx + \int_b^d \int_a^d (x - y)^\theta dy dx \\
 &= I_\theta(a, b, c, a) + D_\theta(a, d) + I_\theta(d, b, a, d),
 \end{aligned}$$

where we have used (2.3) and (2.5). Further, utilising (2.4) and (2.6) gives

$$\begin{aligned}
 (2.7) \quad (\theta + 1)(\theta + 2) C_\theta(a, b, c, d) \\
 &= (b - c)^{\theta+2} - (b - d)^{\theta+2} + (d - a)^{\theta+2} - (a - c)^{\theta+2}, \quad [c, a, d, b].
 \end{aligned}$$



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(iii) For the ordering $[a, c, d, b]$

$$\begin{aligned} C_\theta(a, b, c, d) &= \int_a^c \int_c^d (y-x)^\theta dydx + \int_c^d \int_c^d |y-x|^\theta dydx + \int_d^b \int_c^d (x-y)^\theta dydx \\ &= I_\theta(c, d, a, c) + D_\theta(c, d) + I_\theta(d, b, c, d), \end{aligned}$$

giving, on using (2.4) and (2.6)

$$(2.8) \quad (\theta + 1)(\theta + 2) C_\theta(a, b, c, d) = (b-c)^{\theta+2} - (b-d)^{\theta+2} + (d-a)^{\theta+2} - (c-a)^{\theta+2}, \quad [a, c, d, b].$$

(iv) For the ordering $[a, c, b, d]$

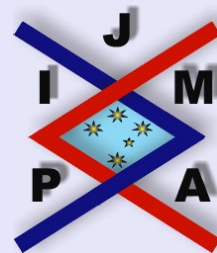
$$\begin{aligned} C_\theta(a, b, c, d) &= \int_a^c \int_c^d (y-x)^\theta dydx + \int_c^b \int_c^b |x-y|^\theta dydx + \int_c^b \int_b^d (y-x)^\theta dydx \\ &= I_\theta(c, d, a, c) + D_\theta(c, b) + I_\theta(b, d, c, b), \end{aligned}$$

giving, from (2.4) and (2.6)

$$(2.9) \quad (\theta + 1)(\theta + 2) C_\theta(a, b, c, d) = (b-c)^{\theta+2} - (d-b)^{\theta+2} + (d-a)^{\theta+2} - (c-a)^{\theta+2}, \quad [a, c, b, d].$$

(v) For the ordering $[a, b, c, d]$

$$(\theta + 1)(\theta + 2) C_\theta(a, b, c, d) = \theta(\theta + 1) I_\theta(c, d, a, b)$$



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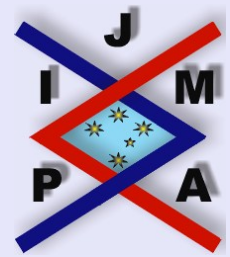


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and so from (2.6)

$$(2.10) \quad (\theta + 1)(\theta + 2)C_{\theta}(a, b, c, d) \\ = (d - a)^{\theta+2} - (c - a)^{\theta+2} + (c - b)^{\theta+2} - (d - b)^{\theta+2}, \quad [a, b, c, d].$$

(vi) For the ordering $[c, a, d, b]$, interchanging a and c and b and d in case (iii) gives

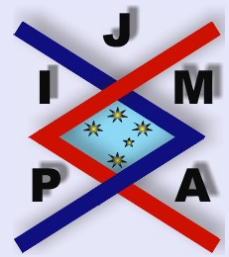
$$(2.11) \quad (\theta + 1)(\theta + 2)C_{\theta}(a, b, c, d) \\ = (d - a)^{\theta+2} - (d - b)^{\theta+2} + (b - c)^{\theta+2} - (a - c)^{\theta+2}, \quad [c, a, b, d].$$

Combining (2.6) – (2.11) produces the results (2.1) – (2.2) and the lemma is proved. \square

Remark 2.1. It may be noticed from (2.1) – (2.2) that (2.4) is recaptured of $c = a$ and $d = b$. Further, the answer appears in terms of differences between a limit of one integral and the other integral. The differences between a top and bottom limit is associated with a positive sign while the difference between the two bottom limits or the two top limits is associated with a negative sign. The order of the differences depends on the order of the limits on the real number line and is taken in such a way that the difference is positive.

Theorem 2.2. Let $f, g : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be measurable on I and the intervals $[a, b], [c, d] \subset I$. Further, suppose that f and g are of Hölder type so that for $x \in [a, b], y \in [c, d]$

$$(2.12) \quad |f(x) - f(y)| \leq H_1 |x - y|^r \quad \text{and} \quad |g(x) - g(y)| \leq H_2 |x - y|^s,$$



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where $H_1, H_2 > 0$ and $r, s \in (0, 1]$ are fixed. The following inequality then holds on the assumption that the integrals involved exist. Namely,

$$(2.13) \quad (\theta + 1)(\theta + 2) |T(f, g; a, b, c, d)| \\ \leq \frac{H_1 H_2}{(b-a)(d-c)} \left[|b-c|^{\theta+2} - |b-d|^{\theta+2} + |d-a|^{\theta+2} - |c-a|^{\theta+2} \right],$$

where $\theta = r + s$ and $T(f, g; a, b, c, d)$ is as defined by (1.5) and (1.6).

Proof. From the Hölder assumption (2.12), we have

$$|(f(x) - f(y))(g(x) - g(y))| \leq H_1 H_2 |x - y|^{r+s}$$

for all $x \in [a, b], y \in [c, d]$.

Hence,

$$\left| \int_a^b \int_c^d (f(x) - f(y))(g(x) - g(y)) dy dx \right| \\ \leq \int_a^b \int_c^d |(f(x) - f(y))(g(x) - g(y))| dy dx \\ \leq H_1 H_2 \int_a^b \int_c^d |x - y|^{r+s} dy dx = H_1 H_2 C_{r+s}(a, b, c, d),$$

where $C_\theta(a, b, c, d)$ is as given by (2.2).

Now, from identity (1.9) and the above inequality readily produces (2.13) and the theorem is thus proved. \square

Corollary 2.3. Let $f, g : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be measurable on I and the intervals $[a, b], [c, d] \subset I$. Further, suppose that f and g are Lipschitzian mappings such that for $x \in [a, b]$ and $y \in [c, d]$

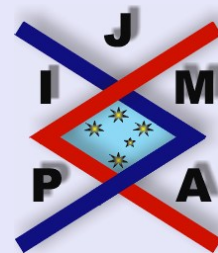
$$|f(x) - f(y)| \leq L_1|x - y| \quad \text{and} \quad |g(x) - g(y)| \leq L_2|x - y|,$$

where $L_1, L_2 > 1$. Assuming that the integrals involved exist, then the following inequality holds. That is,

$$\begin{aligned} & |T(f, g; a, b, c, d)| \\ & \leq \frac{L_1 L_2}{12(b-a)(d-c)} [(b-c)^4 - (c-a)^4 + (d-a)^4 - (b-d)^4]. \end{aligned}$$

Proof. Taking $r = s = 1$ in Theorem 2.2 and $L_1 = H_1, L_2 = H_2$, then from (2.13) we obtain the above inequality. \square

Remark 2.2. The situation in which f is of Hölder type and g is Lipschitzian may be handled by taking $s = 1$ and $H_2 = L_2$. Further, taking $d = b$ and $c = a$ recaptures the results of Dragomir [7].



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3. A Weighted Generalised Chebychev Functional

Define the weighted generalised Chebychev Functional by

$$(3.1) \quad \mathfrak{I}(f, g; a, b, c, d) = \mathfrak{M}(fg; a, b) + \mathfrak{M}(fg; c, d) \\ - \mathfrak{M}(f; a, b)\mathfrak{M}(g; c, d) - \mathfrak{M}(f; c, d)\mathfrak{M}(g; a, b),$$

where the w -weighted integral mean is given by

$$(3.2) \quad \mathfrak{M}(f; a, b) = \frac{1}{\int_a^b w(x) dx} \int_a^b w(x) f(x) dx$$

with $w : [a, b] \rightarrow [0, \infty)$ is integrable and $0 < \int_a^b w(x) dx < \infty$.

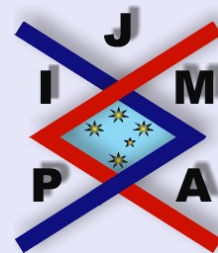
Theorem 3.1. *Let $f, g, w : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be measurable on I and the intervals $[a, b], [c, d] \subset I$. Assuming that the integrals involved in (3.1) exist and $\int_I w(x) dx > 0$, then we have*

$$(3.3) \quad |\mathfrak{I}(f, g; a, b, c, d)| \\ \leq [\mathfrak{I}(f; a, b) + \mathfrak{I}(f; c, d) + (\mathfrak{M}(f; a, b) - \mathfrak{M}(f; c, d))^2]^{\frac{1}{2}} \\ \times [\mathfrak{I}(g; a, b) + \mathfrak{I}(g; c, d) + (\mathfrak{M}(g; a, b) - \mathfrak{M}(g; c, d))^2]^{\frac{1}{2}},$$

where

$$(3.4) \quad \mathfrak{I}(f; a, b) := \mathfrak{M}(f^2; a, b) - \mathfrak{M}^2(f; a, b)$$

and the integrals involved in the right hand side of (3.1) exist.



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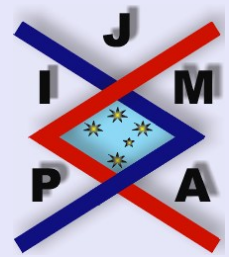


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Proof. It is easily demonstrated that the identity

$$(3.5) \quad \mathfrak{F}(f, g; a, b, c, d) = \frac{1}{\int_a^b w(x) dx \int_c^d w(y) dy} \int_a^b \int_c^d w(x) w(y) \\ \times (f(x) - f(y))(g(x) - g(y)) dx dy$$

holds, which is a weighted generalised Korkine type identity involving integrals over different intervals.

Using the Cauchy-Buniakowski-Schwartz inequality for double integrals gives

$$(3.6) \quad |\mathfrak{F}(f, g; a, b, c, d)|^2 \leq \mathfrak{F}(f, f; a, b, c, d) \mathfrak{F}(g, g; a, b, c, d),$$

where from (3.1)

$$\mathfrak{F}(f, f; a, b, c, d) = \mathfrak{M}(f^2; a, b) + \mathfrak{M}(f^2; c, d) - 2\mathfrak{M}(f; a, b) \mathfrak{M}(f; c, d)$$

and using (3.4) gives

$$(3.7) \quad \mathfrak{F}(f, f; a, b, c, d) = \mathfrak{F}(f; a, b) + \mathfrak{F}(f; c, d) + (\mathfrak{M}(f; a, b) - \mathfrak{M}(f; c, d))^2.$$

A similar identity to (3.7) holds for g and thus from (3.6) and (3.2), the result (3.3) is obtained and the theorem is thus proved. \square

Corollary 3.2. *Let the conditions of Theorem 3.1 hold. Moreover, assume that*

$$m_1 \leq f \leq M_1, \text{ a.e. on } [a, b], \quad m_2 \leq f \leq M_2, \text{ a.e. on } [c, d]$$

and

$$n_1 \leq g \leq N_1, \text{ a.e. on } [a, b], \quad n_2 \leq g \leq N_2, \text{ a.e. on } [c, d].$$

The inequality

$$\begin{aligned} & |\mathfrak{F}(f, g; a, b, c, d)| \\ & \leq \frac{1}{4} [(M_1 - m_1)^2 + (M_2 - m_2)^2 + 4(\mathfrak{M}(f; a, b) - \mathfrak{M}(f; c, d))^2]^{\frac{1}{2}} \\ & \quad \times [(N_1 - n_1)^2 + (N_2 - n_2)^2 + 4(\mathfrak{M}(g; a, b) - \mathfrak{M}(g; c, d))^2]^{\frac{1}{2}} \end{aligned}$$

holds.

Proof. From (3.3) and using the fact that for the Grüss inequality involving weighted means (see for example, Dragomir [7]), then

$$\mathfrak{F}(f; a, b) \leq \left(\frac{M_1 - m_1}{2}\right)^2, \quad \mathfrak{F}(f; c, d) \leq \left(\frac{N_1 - n_1}{2}\right)^2$$

and similar results for the mapping g readily produces the results as stated. \square

Corollary 3.3. Let f and g be absolutely continuous on \dot{I} . In addition, assume that $f', g' \in L_\infty(\dot{I})$ and $[a, b], [c, d] \subseteq \dot{I}$ (\dot{I} is the closure of I). Then we have the inequality

$$\begin{aligned} & |\mathfrak{F}(f, g; a, b, c, d)| \\ & \leq \left[S(a, b) \|f'\|_{\infty, [a, b]} + S(c, d) \|f'\|_{\infty, [c, d]} + (\mathfrak{M}(f; a, b) - \mathfrak{M}(f; c, d))^2 \right]^{\frac{1}{2}} \\ & \times \left[S(a, b) \|g'\|_{\infty, [a, b]} + S(c, d) \|g'\|_{\infty, [c, d]} + 12(\mathfrak{M}(g; a, b) - \mathfrak{M}(g; c, d))^2 \right]^{\frac{1}{2}}, \end{aligned}$$



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where $\|f'\|_{\infty,[a,b]} := \text{ess sup}_{x \in [a,b]} |f'(x)|$,

$$S(a, b) = \frac{W_2(a, b)}{W_0(a, b)} - \left(\frac{W_1(a, b)}{W_0(a, b)} \right)^2 \quad \text{and} \quad W_n(a, b) = \int_a^b x^n w(x) dx.$$

Proof. Using (3.3) and the fact that the weighted Chebychev inequality (see [7] for example) is such that

$$\mathfrak{T}(f; a, b) \leq S(a, b) \|f'\|_{\infty,[a,b]}$$

then, the stated result is readily produced. \square

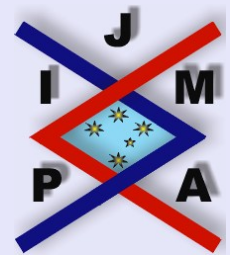
Finally, using a weighted generalisation of the Lupaş inequality of G.V and I.Z. Milovanić [12], namely, for $w^{-\frac{1}{2}}f' \in L_2[a, b]$

$$\mathfrak{T}(f; a, b) \leq \frac{W_0(a, b)}{\pi^2} \left\| w^{-\frac{1}{2}}f' \right\|_2^2$$

produces the following corollary.

Corollary 3.4. *Let f and g be absolutely continuous on \hat{I} , $f', g' \in L_2(\hat{I})$ and $[a, b], [c, d] \subset \hat{I}$. The following inequality then holds*

$$|\mathfrak{T}(f, g; a, b, c, d)| \leq \frac{1}{\pi} \left[W_0^2(a, b) \left\| w^{-\frac{1}{2}}f' \right\|_{2,[a,b]}^2 + W_0^2(c, d) \left\| w^{-\frac{1}{2}}f' \right\|_{2,[c,d]}^2 + \pi^2 (\mathfrak{M}(f; a, b) - \mathfrak{M}(f; c, d))^2 \right]^{\frac{1}{2}}$$



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$$\times \frac{1}{\pi} \left[W_0^2(a, b) \left\| w^{-\frac{1}{2}} g' \right\|_{2,[a,b]}^2 + W_0^2(c, d) \left\| w^{-\frac{1}{2}} g' \right\|_{2,[c,d]}^2 + \pi^2 (\mathfrak{M}(g; a, b) - \mathfrak{M}(g; c, d))^2 \right]^{\frac{1}{2}},$$

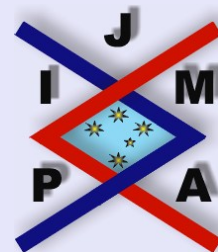
where

$$\left\| w^{-\frac{1}{2}} f' \right\|_{2,[a,b]} := \left(\int_a^b w^{-\frac{1}{2}} |f'(x)|^2 dx \right)^{\frac{1}{2}}$$

and $W_0(a, b)$ is the zeroth moment of $w(\cdot)$ over (a, b) .

Remark 3.1. If $c = a$ and $d = b$ then prior results are recaptured.

Remark 3.2. If f and g are assumed to be of Hölder type, then bounds along similar lines to those obtained in Section 2 could also be obtained for the weighted Chebychev functional utilising identity (3.5). This will however not be pursued further.



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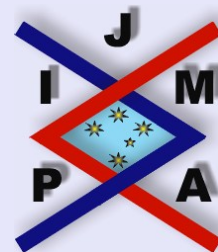
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References

- [1] J. PEČARIĆ, F. PROSCHAN AND Y. TONG, *Convex Functions, Partial Orderings and Statistical Applications*, Academic Press, San Diego, 1992.
- [2] D.S. MITRINOVIĆ, J.E. PEČARIĆ AND A.M. FINK, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, Dordrecht, 1993.
- [3] S.S. DRAGOMIR, Grüss inequality in inner product spaces, *The Australian Math Soc. Gazette*, **26**(2) (1999), 66–70.
- [4] S.S. DRAGOMIR, A generalization of Grüss' inequality in inner product spaces and applications, *J. Math. Anal. Appl.*, **237** (1999), 74–82.
- [5] S.S. DRAGOMIR AND I. FEDOTOV, An inequality of Grüss' type for Riemann-Stieltjes integral and applications for special means, *Tamkang J. of Math.*, **29**(4) (1998), 286–292.
- [6] S.S. DRAGOMIR, A Grüss type integral inequality for mappings of r -Hölder's type and applications for trapezoid formula, *Tamkang J. of Math.*, **31**(1), (2000), 43–47.
- [7] S.S. DRAGOMIR, Some integral inequalities of Grüss type, *Indian J. of Pure and Appl. Math.*, **31**(4), (2000), 397–415.
- [8] S.S. DRAGOMIR AND G.L. BOOTH, On a Grüss-Lupaş type inequality and its applications for the estimation of p -moments of guessing mappings, *Mathematical Communications*, **5** (2000), 117–126.



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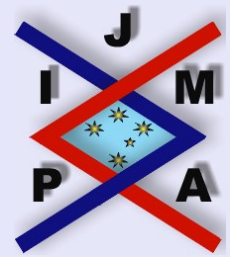


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- [9] G. GRÜSS, Über das Maximum des absoluten Betrages von $\frac{1}{b-a} \int_a^b f(x)g(x)dx - \frac{1}{(b-a)^2} \int_a^b f(x)dx \int_a^b g(x)dx$, *Math. Z.* , **39**(1935), 215–226.
- [10] A.M. FINK, A treatise on Grüss' inequality, *Analytic and Geometric Inequalities and Applications*, Math. Appl., **478** (1999), Kluwer Academic Publishers, Dordrecht, 93–113.
- [11] P. CERONE AND S. DRAGOMIR, Generalisations of the Grüss, Chebyshev and Lupaş inequalities for integrals over different intervals, *Int. J. Appl. Math.* , **6**(2) (2001), 117–128.
- [12] G.V. MILOVANIĆ AND I.Z. MILOVANIĆ, On a generalization of certain results of A. Ostrowski and A. Lupaş, *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz.*, No. 634-677 (1979), 62–69.

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