journal of inequalities in pure and applied mathematics
http://jipam.vu.edu.au
issn: 1443-5756
Volume 8 (2007), Issue 4, Article 121, 2 pp.

CORRIGENDUM OF THE PAPER ENTITLED: NOTE ON AN OPEN PROBLEM

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Received 22 July, 2007; accepted 04 December, 2007
Communicated by P.S. Bullen

ABSTRACT. This paper is a corrigendum on a paper published in an earlier volume of JIPAM, ŚNote on an open problemŠ, published in JIPAM, Vol. 8, No. 2. (2007), Article 58. http://jipam.vu.edu.au/article.php?sid=871.

Key words and phrases: Corrigendum.

2000 *Mathematics Subject Classification.* 26D15.

The conditions

$$\int_x^b f(t)dt \le \int_x^b (t-a)dt \quad \left(\text{resp. } \int_x^b f(t)dt \ge \int_x^b (t-a)dt\right), \qquad \forall x \in [a,b],$$

given in Lemma 1.1, Theorem 2.1 and Theorem 2.3 [1] are not sufficient to prove the following results

$$f(x) \le x - a \quad (\text{resp. } f(x) \ge x - a)\,.$$

And clearly, the mistake appears in line 5 of the proof of Lemma 1.1 [1].

It is easy to give counter examples for the above lemma. If we choose, $f(x) = \frac{1}{3}$, $a = 0$ and $b = 1$, then the first part of the assumptions of Lemma 1.1 gives $x \le \frac{1}{3}$ and also, since $f(x) \le x - a$, we have $\frac{1}{3} \le x$. This is a contradiction.

In fact, the following conditions $f'(x) \ge 1$ (resp. $f'(x) \ge 1$), $\forall\, x \in (a,b)$ should be added in the first (resp. second) part of the assumptions of Lemma 1.1, Theorem 2.1 and Theorem 2.3. Therefore, Lemma 1.1 becomes:

Lemma 1. *Let $f(x)$ be a nonnegative function, continuous on $[a,b]$ and differentiable on (a,b). If $\int_x^b f(t)dt \le \int_x^b (t-a)dt,\ \forall x \in [a,b]$, and $f'(x) \ge 1,\ \forall x \in (a,b)$, then,*

$$f(x) \le x - a. \tag{1}$$

I would like to express deep gratitude to Tibor Pogany and Quôc Anh Ngô for their comments and the successful completion of this note.
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If $\int_x^b f(t)dt \geq \int_x^b (t-a)dt,\ \forall x \in [a,b]$, *and* $f'(x) \leq 1,\ \forall x \in (a,b)$, *then*

$$f(x) \geq x - a. \tag{2}$$

Proof. In order to prove (1), set

$$G(x) = \left(\int_x^b [f(t) - (t-a)]dt \right) (x - a - f(x)), \qquad \forall x \in [a,b],$$

we have

$$G'(x) = (x - a - f(x))^2 + \left(\int_x^b [f(t) - (t-a)]dt \right) (1 - f'(x)).$$

If $\int_x^b f(t)dt \leq \int_x^b (t-a)dt$, and $f'(x) \geq 1$, then $G'(x) \geq 0$, $G(x)$ increases and $G(x) \leq 0$, since $G(b) = 0$, that is, $x - a - f(x) \geq 0$, so that $f(x) \leq x - a$.
Similarly, if $\int_x^b f(t)dt \geq \int_x^b (t-a)dt$ and $f'(x) \leq 1$, we obtain $G'(x) \geq 0$, and $G(x) \leq 0$, that is, $f(x) \geq x - a$. □

Remark 1. In the second part of Example 2.1, the function $f(t) = t - \frac{\pi}{2} + \cos t$ should be replaced by $f(t) = t - \frac{\pi}{2} - \sin t$.

REFERENCES

[1] L. BOUGOFFA, Note on an open problem, *J. Inequal. Pure & Appl. Math.*, **8**(2) (2007), Art. 58. [ONLINE http://jipam.vu.edu.au/article.php?sid=871].