

Journal of Inequalities in Pure and Applied Mathematics

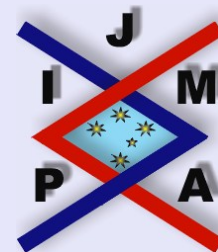
FEKETE-SZEGÖ FUNCTIONAL FOR SOME SUBCLASS OF NON-BAZILEVIĆ FUNCTIONS

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©2000 Victoria University
ISSN (electronic): 1443-5756
108-06



volume 7, issue 3, article 117,
2006.

*Received 18 November, 2005;
accepted 24 March, 2006.*

Communicated by: A. Lupaş

Abstract

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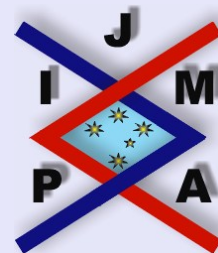
In this present investigation, the authors obtain a sharp Fekete-Szegő's inequality for certain normalized analytic functions $f(z)$ defined on the open unit disk for which $(1 + \beta) \left(\frac{z}{f(z)}\right)^\alpha - \beta f'(z) \left(\frac{z}{f(z)}\right)^{1+\alpha}$, $(\beta \in \mathbb{C}, 0 < \alpha < 1)$ lies in a region starlike with respect to 1 and is symmetric with respect to the real axis. Also, certain applications of our results for a class of functions defined by convolution are given. As a special case of this result, Fekete-Szegő's inequality for a class of functions defined through fractional derivatives is also obtained.

2000 Mathematics Subject Classification: Primary 30C45.

Key words: Analytic functions, Starlike functions, Subordination, Coefficient problem, Fekete-Szegő inequality.

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1. Introduction

Let \mathcal{A} denote the class of all *analytic* functions $f(z)$ of the form

$$(1.1) \quad f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \Delta := \{z \in \mathbb{C} / |z| < 1\})$$

and \mathcal{S} be the subclass of \mathcal{A} consisting of univalent functions. Let $\phi(z)$ be an analytic function with positive real part on Δ with $\phi(0) = 1$, $\phi'(0) > 0$ which maps the unit disk Δ onto a region starlike with respect to 1 and is symmetric with respect to the real axis. Let $S^*(\phi)$ be the class of functions $f \in \mathcal{S}$ for which

$$\frac{zf'(z)}{f(z)} \prec \phi(z), \quad (z \in \Delta)$$

and $C(\phi)$ be the class of functions $f \in \mathcal{S}$ for which

$$1 + \frac{zf''(z)}{f'(z)} \prec \phi(z), \quad (z \in \Delta),$$

where \prec denotes the subordination between analytic functions. These classes were introduced and studied by Ma and Minda [3]. They have obtained the Fekete-Szegő inequality for the functions in the class $C(\phi)$. Since $f \in C(\phi)$ if and only if $zf' \in S^*(\phi)$, we get the Fekete-Szegő inequality for functions in the class $S^*(\phi)$. Recently, Shanmugam and Sivasubramanian [9] obtained Fekete-Szegő inequalities for the class of functions $f \in \mathcal{A}$ such that

$$\frac{zf'(z) + \alpha z^2 f''(z)}{(1 - \alpha)f(z) + \alpha z f'(z)} \prec \phi(z) \quad (0 \leq \alpha < 1).$$



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Also, Ravichandran et al. [7] obtained the Fekete-Szegő inequality for the class of Bazilevič functions. For a brief history of the Fekete-Szegő problem for the class of starlike, convex and close-to-convex functions, see the recent paper by Srivastava *et al.* [11]. Obradovic [4] introduced a class of functions $f \in \mathcal{A}$, such that, for $0 < \alpha < 1$,

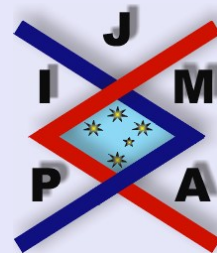
$$\Re \left\{ f'(z) \left(\frac{z}{f(z)} \right)^\alpha \right\} > 0, \quad z \in \Delta.$$

He called this class of function as "Non-Bazilevič" type. Tuneski and Darus [14] obtained the Fekete-Szegő inequality for the non-Bazilevič class of functions. Using this non-Bazilevič class, Wang et al.[15] studied many subordination results for the class $N(\alpha, \beta, A, B)$ defined as

$$N(\alpha, \beta, A, B) := \left\{ f \in \mathcal{A} : (1 + \beta) \left(\frac{z}{f(z)} \right)^\alpha - \beta f'(z) \left(\frac{z}{f(z)} \right)^{1+\alpha} \prec \frac{1 + Az}{1 + Bz} \right\},$$

where $\beta \in \mathbb{C}$, $-1 \leq B \leq 1$, $A \neq B$, $0 < \alpha < 1$.

In the present paper, we obtain the Fekete-Szegő inequality for functions in a more general class $N_{\alpha, \beta}(\phi)$ of functions which we define below. Also we give applications of our results to certain functions defined through convolution (or Hadamard product) and in particular we consider a class $N_{\alpha, \beta}^\lambda(\phi)$ of functions defined by fractional derivatives. The aim of this paper is to give a generalization the Fekete-Szegő inequalities for some subclass of Non-Bazilevič functions



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Definition 1.1. Let $\phi(z)$ be an univalent starlike function with respect to 1 which maps the unit disk Δ onto a region in the right half plane which is symmetric with respect to the real axis, $\phi(0) = 1$ and $\phi'(0) > 0$. A function $f \in \mathcal{A}$ is in the class $N_{\alpha,\beta}(\phi)$ if

$$(1 + \beta) \left(\frac{z}{f(z)} \right)^\alpha - \beta f'(z) \left(\frac{z}{f(z)} \right)^{1+\alpha} \prec \phi(z), \quad (\beta \in \mathbb{C}, 0 < \alpha < 1).$$

For fixed $g \in \mathcal{A}$, we define the class $N_{\alpha,\beta}^g(\phi)$ to be the class of functions $f \in \mathcal{A}$ for which $(f * g) \in N_{\alpha,\beta}(\phi)$.

Remark 1. $N_{\alpha,-1} \left(\frac{1+z}{1-z} \right)$ is the class of Non-Bazilevič functions introduced by Obradovic [4].

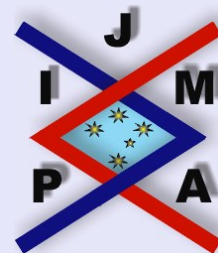
Remark 2. $N_{\alpha,-1} \left(\frac{1+(1-2\gamma)z}{1-z} \right)$, $0 \leq \gamma < 1$ is the class of Non-Bazilevič functions of order γ introduced and studied by Tuneski and Darus [14].

Remark 3. We call $N_{\alpha,\beta} \left\{ 1 + \frac{2}{\pi^2} \left(\log \frac{1+\sqrt{z}}{1-\sqrt{z}} \right)^2 \right\}$ the class of "Non-Bazilevič parabolic starlike functions".

To prove our main result, we need the following:

Lemma 1.1 ([3]). If $p_1(z) = 1 + c_1z + c_2z^2 + \dots$ is an analytic function with a positive real part in Δ , then

$$|c_2 - vc_1^2| \leq \begin{cases} -4v + 2 & \text{if } v \leq 0, \\ 2 & \text{if } 0 \leq v \leq 1, \\ 4v - 2 & \text{if } v \geq 1. \end{cases}$$



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When $v < 0$ or $v > 1$, the equality holds if and only if $p_1(z)$ is $(1+z)/(1-z)$ or one of its rotations. If $0 < v < 1$, then the equality holds if and only if $p_1(z)$ is $(1+z^2)/(1-z^2)$ or one of its rotations. If $v = 0$, the equality holds if and only if

$$p_1(z) = \left(\frac{1}{2} + \frac{1}{2}\lambda\right) \frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{1}{2}\lambda\right) \frac{1-z}{1+z} \quad (0 \leq \lambda \leq 1)$$

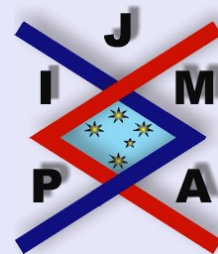
or one of its rotations. If $v = 1$, the equality holds if and only if p_1 is the reciprocal of one of the functions such that equality holds in the case of $v = 0$.

Also the above upper bound is sharp, and it can be improved as follows when $0 < v < 1$:

$$|c_2 - vc_1^2| + v|c_1|^2 \leq 2 \quad (0 < v \leq 1/2)$$

and

$$|c_2 - vc_1^2| + (1-v)|c_1|^2 \leq 2 \quad (1/2 < v \leq 1).$$



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2. Fekete-Szegő Problem

Our main result is the following:

Theorem 2.1. Let $\phi(z) = 1 + B_1z + B_2z^2 + B_3z^3 + \dots$. If f given by (1.1) belongs to $N_{\alpha,\beta}(\phi)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} -\frac{B_2}{(\alpha+2\beta)} - \frac{\mu B_1^2}{2(\alpha+\beta)^2} + \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2 & \text{if } \mu \leq \sigma_1; \\ -\frac{B_1}{(\alpha+2\beta)} & \text{if } \sigma_1 \leq \mu \leq \sigma_2; \\ \frac{B_2}{(\alpha+2\beta)} + \frac{\mu B_1^2}{2(\alpha+\beta)^2} - \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2 & \text{if } \mu \geq \sigma_2, \end{cases}$$

where,

$$\sigma_1 := \frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 - B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2},$$

$$\sigma_2 := \frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 + B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}.$$

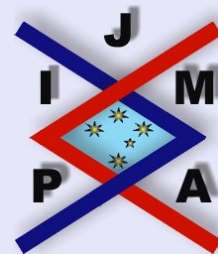
The result is sharp.

Proof. For $f \in N_{\alpha,\beta}(\phi)$, let

$$(2.1) \quad p(z) := (1+\beta) \left(\frac{z}{f(z)} \right)^\alpha - \beta f'(z) \left(\frac{z}{f(z)} \right)^{1+\alpha} = 1 + b_1z + b_2z^2 + \dots$$

From (2.1), we obtain

$$-(\alpha + \beta)a_2 = b_1$$



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$$(2\beta + \alpha) \left(\frac{\alpha + 1}{2} a_2^2 - a_3 \right) = b_2.$$

Since $\phi(z)$ is univalent and $p \prec \phi$, the function

$$p_1(z) = \frac{1 + \phi^{-1}(p(z))}{1 - \phi^{-1}(p(z))} = 1 + c_1 z + c_2 z^2 + \dots$$

is analytic and has a positive real part in Δ . Also we have

$$(2.2) \quad p(z) = \phi \left(\frac{p_1(z) - 1}{p_1(z) + 1} \right)$$

and from this equation (2.2), we obtain

$$b_1 = \frac{1}{2} B_1 c_1$$

and

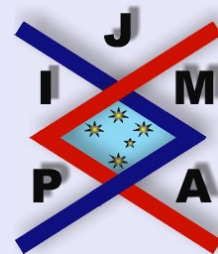
$$b_2 = \frac{1}{2} B_1 \left(c_2 - \frac{1}{2} c_1^2 \right) + \frac{1}{4} B_2 c_1^2.$$

Therefore we have

$$a_3 - \mu a_2^2 = -\frac{B_1}{2(2\beta + \alpha)} \{c_2 - \nu c_1^2\}$$

where

$$\nu := \frac{1}{2} \left[1 - \frac{B_2}{B_1} + \frac{(2\beta + \alpha)(\alpha + 1 - 2\mu)}{2(\beta + \alpha)^2} B_1 \right].$$



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Our result now follows by an application of Lemma 1.1. To show that the bounds are sharp, we define the functions $K_{\alpha,\beta}^{\phi_n}$ ($n = 2, 3, \dots$) by

$$(1 + \beta) \left(\frac{z}{K_{\alpha,\beta}^{\phi_n}(z)} \right)^\alpha - \beta \left(K_{\alpha,\beta}^{\phi_n} \right)'(z) \left(\frac{z}{K_{\alpha,\beta}^{\phi_n}(z)} \right)^{1+\alpha} = \phi(z^{n-1}),$$

$$K_{\alpha,\beta}^{\phi_n}(0) = 0 = [K_{\alpha,\beta}^{\phi_n}]'(0) - 1$$

and the function $F_{\alpha,\beta}^\lambda$ and $G_{\alpha,\beta}^\lambda$ ($0 < \alpha < 1$) by

$$(1 + \beta) \left(\frac{z}{F_{\alpha,\beta}^\lambda(z)} \right)^\alpha - \beta [F_{\alpha,\beta}^\lambda]'(z) \left(\frac{z}{F_{\alpha,\beta}^\lambda(z)} \right)^{1+\alpha} = \phi(z^{n-1}),$$

$$[F_{\alpha,\beta}^\lambda](0) = 0 = [F_{\alpha,\beta}^\lambda]'(0) - 1$$

and

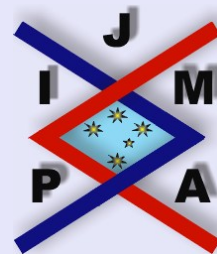
$$(1 + \beta) \left(\frac{z}{G_{\alpha,\beta}^\lambda} \right)^\alpha - \beta [G_{\alpha,\beta}^\lambda]'(z) \left(\frac{z}{G_{\alpha,\beta}^\lambda(z)} \right)^{1+\alpha} = \phi(z^{n-1}),$$

$$[G_{\alpha,\beta}^\lambda](0) = 0 = [G_{\alpha,\beta}^\lambda]'(0) - 1.$$

Clearly, the functions $K_{\alpha,\beta}^{\phi_n}$, $[F_{\alpha,\beta}^\lambda]$ and $[G_{\alpha,\beta}^\lambda] \in N_{\alpha,\beta}(\phi)$

Also we write $K_{\alpha,\beta}^\phi := K_{\alpha,\beta}^{\phi_2}$.

If $\mu < \sigma_1$ or $\mu > \sigma_2$, then the equality holds if and only if f is $K_{\alpha,\beta}^\phi$ or one of its rotations. When $\sigma_1 < \mu < \sigma_2$, the equality holds if and only if f is $K_{\alpha,\beta}^{\phi_3}$ or one of its rotations. If $\mu = \sigma_1$ then the equality holds if and only if f is $F_{\alpha,\beta}^\lambda$



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or one of its rotations. If $\mu = \sigma_2$ then the equality holds if and only if f is $G_{\alpha,\beta}^\lambda$ or one of its rotations. \square

Corollary 2.2. Let $\phi(z) = 1 + \frac{2}{\pi^2} \left(\log \frac{1+\sqrt{z}}{1-\sqrt{z}} \right)^2$. If f given by (1.1) belongs to $N_{\alpha,\beta}(\phi)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} -\frac{8}{3\pi^2(\alpha+2\beta)} - \frac{8\mu}{\pi^4(\alpha+\beta)^2} + \frac{(1+\alpha)}{(\alpha+\beta)^2} \frac{8}{\pi^4} & \text{if } \mu \leq \sigma_1 \\ -\frac{4}{\pi^2(\alpha+2\beta)} & \text{if } \sigma_1 \leq \mu \leq \sigma_2 \\ \frac{8}{3\pi^2(\alpha+2\beta)} + \frac{8\mu}{\pi^4(\alpha+\beta)^2} - \frac{(1+\alpha)}{(\alpha+\beta)^2} \frac{8}{\pi^4} & \text{if } \mu \geq \sigma_2 \end{cases}$$

where,

$$\sigma_1 := \frac{(1+\alpha)(2\beta+\alpha)\frac{16}{\pi^4} - 2\left(\frac{8}{3\pi^2} - \frac{4}{\pi^2}\right)(\beta+\alpha)^2}{2(2\beta+\alpha)\frac{16}{\pi^4}}$$

$$\sigma_2 := \frac{(1+\alpha)(2\beta+\alpha)\frac{16}{\pi^4} - 2\left(\frac{8}{3\pi^2} + \frac{4}{\pi^2}\right)(\beta+\alpha)^2}{2(2\beta+\alpha)\frac{16}{\pi^4}}.$$

The result is sharp.

Corollary 2.3. For $\beta = -1$, $\phi(z) = \frac{1+(1-2\gamma)z}{1-z}$, $0 \leq \gamma < 1$ in Theorem 2.1, we get the results obtained by Tuneski and Darus [14].

Remark 4. If $\sigma_1 \leq \mu \leq \sigma_2$, then, in view of Lemma 1.1, Theorem 2.1 can be improved. Let σ_3 be given by

$$\sigma_3 := \frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2B_2(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}.$$



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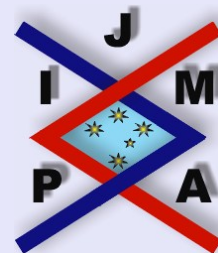
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If $\sigma_1 \leq \mu \leq \sigma_3$, then

$$|a_3 - \mu a_2^2| - \frac{(\beta + \alpha)^2}{(2\beta + \alpha)B_1^2} \left[B_1 - B_2 + B_1^2 \frac{(\alpha + 1 - 2\mu)(2\beta + \alpha)}{2(\beta + \alpha)^2} \right] |a_2|^2 \leq -\frac{B_1}{(2\beta + \alpha)}.$$

If $\sigma_3 \leq \mu \leq \sigma_2$, then

$$|a_3 - \mu a_2^2| - \frac{(\beta + \alpha)^2}{(2\beta + \alpha)B_1^2} \left[B_1 + B_2 - B_1^2 \frac{(\alpha + 1 - 2\mu)(2\beta + \alpha)}{2(\beta + \alpha)^2} \right] |a_2|^2 \leq -\frac{B_1}{(2\beta + \alpha)}.$$



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3. Applications to Functions Defined by Fractional Derivatives

In order to introduce the class $N_{\alpha,\beta}^\lambda(\phi)$, we need the following:

Definition 3.1 (see [5, 6]; see also [12, 13]). *Let f be analytic in a simply connected region of the z -plane containing the origin. The fractional derivative of f of order λ is defined by*

$$D_z^\lambda f(z) := \frac{1}{\Gamma(1-\lambda)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\lambda} d\zeta \quad (0 \leq \lambda < 1)$$

where the multiplicity of $(z-\zeta)^\lambda$ is removed by requiring that $\log(z-\zeta)$ is real for $z-\zeta > 0$.

Using the above Definition 3.1 and its known extensions involving fractional derivatives and fractional integrals, Owa and Srivastava [5] introduced the operator $\Omega^\lambda : \mathcal{A} \rightarrow \mathcal{A}$ defined by

$$(\Omega^\lambda f)(z) = \Gamma(2-\lambda) z^\lambda D_z^\lambda f(z), \quad (\lambda \neq 2, 3, 4, \dots).$$

The class $N_{\alpha,\beta}^\lambda(\phi)$ consists of functions $f \in \mathcal{A}$ for which $\Omega^\lambda f \in N_{\alpha,\beta}(\phi)$. Note that $N_{\alpha,\beta}^\lambda(\phi)$ is the special case of the class $N_{\alpha,\beta}^g(\phi)$ when

$$(3.1) \quad g(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)} z^n.$$

Let $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$ ($g_n > 0$). Since $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in N_{\alpha,\beta}^g(\phi)$ if and only if $(f * g) = z + \sum_{n=2}^{\infty} g_n a_n z^n \in N_{\alpha,\beta}(\phi)$, we obtain the



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coefficient estimate for functions in the class $N_{\alpha,\beta}^g(\phi)$, from the corresponding estimate for functions in the class $N_{\alpha,\beta}(\phi)$. Applying Theorem 2.1 for the function $(f * g)(z) = z + g_2 a_2 z^2 + g_3 a_3 z^3 + \dots$, we get the following Theorem 3.1 after an obvious change of the parameter μ :

Theorem 3.1. *Let the function ϕ be given by $\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \dots$. If f given by (1.1) belongs to $N_{\alpha,\beta}^g(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{1}{g_3} \left\{ -\frac{B_2}{(\alpha+2\beta)} - \frac{\mu g_3 B_1^2}{g_2^2 2(\alpha+\beta)^2} + \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2 \right\} & \text{if } \mu \leq \sigma_1; \\ -\frac{1}{g_3} \frac{B_1}{(\alpha+2\beta)} & \text{if } \sigma_1 \leq \mu \leq \sigma_2; \\ \frac{1}{g_3} \left\{ \frac{B_2}{(\alpha+2\beta)} + \frac{\mu g_3 B_1^2}{2(\alpha+\beta)^2 g_2^2} - \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2 \right\} & \text{if } \mu \geq \sigma_2, \end{cases}$$

where

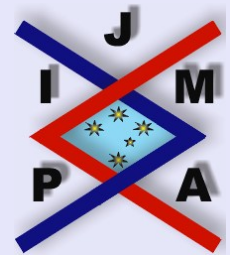
$$\sigma_1 := \frac{g_3 (1 + \alpha)(2\beta + \alpha) B_1^2 - 2(B_2 - B_1)(\beta + \alpha)^2}{g_2^2 2(2\beta + \alpha) B_1^2}$$

$$\sigma_2 := \frac{g_3 (1 + \alpha)(2\beta + \alpha) B_1^2 - 2(B_2 + B_1)(\beta + \alpha)^2}{g_2^2 2(2\beta + \alpha) B_1^2}.$$

The result is sharp.

Since

$$(\Omega^\lambda f)(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)} a_n z^n,$$



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we have

$$(3.2) \quad g_2 := \frac{\Gamma(3)\Gamma(2-\lambda)}{\Gamma(3-\lambda)} = \frac{2}{2-\lambda}$$

and

$$(3.3) \quad g_3 := \frac{\Gamma(4)\Gamma(2-\lambda)}{\Gamma(4-\lambda)} = \frac{6}{(2-\lambda)(3-\lambda)}.$$

For g_2 and g_3 given by (3.2) and (3.3), Theorem 3.1 reduces to the following:

Theorem 3.2. *Let the function ϕ be given by $\phi(z) = 1 + B_1z + B_2z^2 + B_3z^3 + \dots$. If f given by (1.1) belongs to $N_{\alpha,\beta}^g(\phi)$, then*

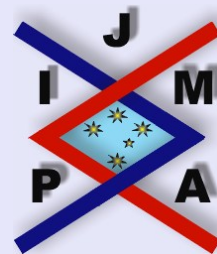
$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(2-\lambda)(3-\lambda)}{6} \left\{ -\frac{B_2}{(\alpha+2\beta)} - \frac{\mu g_3 B_1^2}{g_2^2 2(\alpha+\beta)^2} + \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2 \right\} & \text{if } \mu \leq \sigma_1; \\ -\frac{(2-\lambda)(3-\lambda)}{6} \frac{B_1}{(\alpha+2\beta)} & \text{if } \sigma_1 \leq \mu \leq \sigma_2; \\ \frac{(2-\lambda)(3-\lambda)}{6} \left\{ \frac{B_2}{(\alpha+2\beta)} + \frac{\mu g_3 B_1^2}{2(\alpha+\beta)^2 g_2^2} - \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2 \right\} & \text{if } \mu \geq \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{2(3-\lambda)}{3(2-\lambda)} \frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 - B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2},$$

$$\sigma_2 := \frac{2(3-\lambda)}{3(2-\lambda)} \frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 + B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}.$$

The result is sharp.



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- [1] B.C. CARLSON AND D.B. SHAFFER, Starlike and prestarlike hypergeometric functions, *SIAM J. Math. Anal.*, **15** (1984), 737–745.
- [2] A.W. GOODMAN, Uniformly convex functions, *Ann. Polon. Math.*, **56** (1991), 87–92.
- [3] W. MA AND D. MINDA, A unified treatment of some special classes of univalent functions, in: *Proceedings of the Conference on Complex Analysis*, Z. Li, F. Ren, L. Yang, and S. Zhang (Eds.), Int. Press (1994), 157–169.
- [4] M. OBRADOVIC, A class of univalent functions, *Hokkaido Math. J.*, **27**(2) (1998), 329–335.
- [5] S. OWA AND H.M. SRIVASTAVA, Univalent and starlike generalized hypergeometric functions, *Canad. J. Math.*, **39** (1987), 1057–1077.
- [6] S. OWA, On the distortion theorems I, *Kyungpook Math. J.*, **18** (1978), 53–58.
- [7] V. RAVICHANDRAN, A. GANGADHARAN AND M. DARUS, Fekete-Szegő inequality for certain class of Bazilevic functions, *Far East J. Math. Sci. (FJMS)*, **15**(2) (2004), 171–180.
- [8] F. RØNNING, Uniformly convex functions and a corresponding class of starlike functions, *Proc. Amer. Math. Soc.*, **118** (1993), 189–196.



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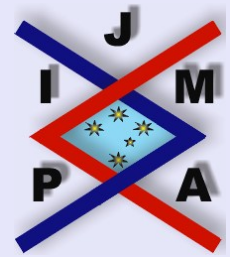
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- [9] T.N. SHANMUGAM AND S. SIVASUBRAMANIAN, On the Fekete-Szegö problem for some subclasses of analytic functions, *J. Inequal. Pure and Appl. Math.*, **6**(3) (2005), Art. 71, 6 pp.
- [10] H.M. SRIVASTAVA AND A.K. MISHRA, Applications of fractional calculus to parabolic starlike and uniformly convex functions, *Computer Math. Appl.*, **39** (2000), 57–69.
- [11] H.M. SRIVASTAVA, A.K. MISHRA AND M.K. DAS, The Fekete-Szegö problem for a subclass of close-to-convex functions, *Complex Variables, Theory Appl.*, **44** (2001), 145–163.
- [12] H.M. SRIVASTAVA AND S. OWA, An application of the fractional derivative, *Math. Japon.*, **29** (1984), 383–389.
- [13] H.M. SRIVASTAVA AND S. OWA, *Univalent functions, Fractional Calculus, and their Applications*, Halsted Press/John Wiley and Sons, Chichester/New York, (1989).
- [14] N. TUNESKI AND M. DARUS, Fekete-Szegö functional for non-Bazilevič functions, *Acta Mathematica Academia Paedagogicae Nyiregyhaziensis*, **18** (2002), 63–65.
- [15] Z. WANG, C. GAO AND M. LIAO, On certain generalized class of non-Bazilevič functions, *Acta Mathematica Academia Paedagogicae Nyiregyhaziensis*, **21** (2005), 147–154.



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