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RANDOM SPHERES AND OPERADS

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Abstract. We define a stochastic diffeology over the n-fold loop-space of a n-connected manifold. We perform a stochastic cohomology in the sense of Chen and Souriau and we show that the stochastic cohomology groups are equal to the classical de Rham cohomology groups of the n-fold loop space. We give a stochastic analogue of the classical fact that the homology groups of the n-fold loop space are an algebra for the little cube operad.

1. Introduction

In algebraic topology (see [8], [13], [16], [40]), people consider as models on the n-fold loop space $\Omega^n_x(M)$, the space of maps from $\mathbb{R}^n$ into the compact manifold M which are equal to x outside a cube (which can depends on the map). The n-fold loop space is not the same as the set of applications from the n-dimensional sphere $\mathbb{S}^n$ to M which sends the north pole of the sphere to x. The interest of the n-fold loop space is that there are many different structures. For instance, it carries an action of the little cube operad [40], such that its homology appears as an algebra associated to this operad or in a more sophisticated way, as a n-algebra [13].

The goal of this article is to define a stochastic n-fold loop space and to define a stochastic homology theory which is compatible with the action of the little cube operad.

There is something analogous in one dimension: the Moore loop space. Taking into consideration as a model of the loop space, the based loop space endowed with Brownian bridge measure, a tentative description of the stochastic Moore loop space was produced in [21] which is involved with the long time behavior of the heat kernel. The cohomology of the smooth Moore loop space is an Hopf algebra, but our program in [21] fails in the definition of suitable tensor products of Sobolev forms if we consider Sobolev cohomology of the stochastic Moore loop space.