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ON MATRIX SOLVABLE CALOGERO MODELS OF B_2 TYPE

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Abstract. We use a method developed earlier to construct new matrix solvable models. We apply this method to the model of the B_2 type and obtain new solvable model of the system with matrix potential.

1. Introduction

Let us consider an eigenvalue problem

$$\mathbf{H}\psi = E\psi$$

where $\mathbf{H}$ is the differential operator of two variables

$$\mathbf{H} = \partial_{11} + \partial_{22} - \mathbf{U}(x_1, x_2) \tag{1}$$

and $\mathbf{U}(x_1, x_2)$ is, say, two by two matrix (we denote $\partial_k \equiv \dfrac{\partial}{\partial x_k}$). If we transform the operator (1) by using similarity transformation $\widehat{\mathbf{H}} = \mathbf{G}^{-1}\mathbf{H}\mathbf{G}$ and transformation of variables $y_1 = y_1(x_1, x_2)$, $y_2 = y_2(x_1, x_2)$ to the form

$$\widehat{\mathbf{H}} = g^{rs}(y)\partial_{rs} + 2\mathbf{b}^r(y)\partial_r + \mathbf{V}(y)$$

(here we of course differentiate with respect to y), for which we know infinite flag of finite dimensional invariant subspaces, it is possible to find the spectrum of $\widehat{\mathbf{H}}$ (and hence $\mathbf{H}$) by diagonalizing $\widehat{\mathbf{H}}$ on these subspaces using standard algebraic methods.

The basic idea used in this paper is to reverse such a process, i.e., to start from the operator $\widehat{\mathbf{H}}$ for which we know invariant subspaces and try to reconstruct the operator $\mathbf{H}$ so that the matrix potential $\mathbf{U}(x_1, x_2)$ is symmetric. There exist some necessary conditions on g^{rs}, $\mathbf{b}^r$ and $\mathbf{V}$, which ensure the existence of the operator $\mathbf{H}$. The detailed description of these conditions can be found in [1].