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CHARGE OF D-BRANES ON SINGULAR VARIETIES

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Abstract. Considering the D-branes on a variety Z as the objects of the derived category $D^b(Z)$, we propose a definition for the charge of D-branes on not necessarily smooth varieties. We define the charge $Q(\mathcal{G})$ of $\mathcal{G} \in D^b(Z)$ as an element of the homology of Z, so that the mapping Q is compatible with the pushforward by proper maps between varieties.

Given a generic anticanonical hypersurface Y of a toric variety X defined by a reflexive polytope, we express the charge of a line bundle on Y defined by a divisor D of X in terms of intersections of D with cycles determined by the polytope faces.

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1. Introduction

The D-branes of type B on a Calabi-Yau variety Z are the objects of $D^b(Z)$, the bounded derived category of algebraic coherent sheaves on Z (see monograph [2]). Given a brane $\mathcal{G}$ on Z, when Z is a *smooth* variety, the charge of $\mathcal{G}$ is the element of the cohomology $H^*(Z)$ defined by the cup product of the Chern character of $\mathcal{G}$ and the root square of the Todd class of Z [1]

$$\mathrm{ch}(\mathcal{G})\sqrt{\mathrm{td}(Z)}. \tag{1}$$

For some branes $\mathcal{G}$, the integration of this cohomology class gives the index of a differential Dirac operator.

Obviously, when the variety Z is singular its Todd class is not defined. On the other hand, in the singular case there are not locally free resolutions of the coherent sheaves, so the Chern character for vector bundles has not a direct extension to coherent sheaves.

In Section 2 of this paper, we propose a definition for the charge of a brane on a projective variety Z. This charge will be an element of H_*Z, the rational homology of Z. When Z is not necessarily smooth, it is not possible to demand the

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